A COMPARISON OF APPROXIMATIONS TO PERCENTILES OF THE NONCENTRAL t-DISTRIBUTION

Hardeo Sahai, Department of Biostatistics and Epidemiology, University of Puerto Rico, San Juan, Puerto Rico
Mario Miguel Ojeda, Facultad de Estadística e Informática, Universidad Veracruzana, Veracruz, México

ABSTRACT
In this paper we review the main proposed approximations to percentiles of the noncentral t-distribution. The approximations are examined for their acuracy over a wide range of values of the parameters of the distribution and for several percentil values. Tables summarizing the approximations are included.

Key words: Cornish-Fishex expansions, Taylor Series, Percentiles

RESUMEN
En este artículo revisamos las principales aproximaciones propuestas para calcular percentiles de la distribución t nocentral. La precisión de las aproximaciones se examina para un amplio rango de valores de los parámetros de la distribución y para varios valores del percentil. Se incluyen tablas que resumen las aproximaciones.

Palabras clave: Expansiones Cornish-Fishex, Series de Taylor, percentiles

1. INTRODUCTION AND SUMMARY

It is widely recognized that the noncentral t-distribution is of considerable theoretical and practical importance in many mathematical and statistical applications. For instance, noncentral t-distribution is useful in evaluating power function of the Student´s t-test (Owen, 1968, 1985; Jonhson **et al.**, 1995, pp. 509-510), calculating confidence interval of the coefficient of variation (Lehmann, 1986, p. 352, Johnson **et al.**, 1995, pp. 510-511; Vangel, 1996), approximating the distribution of sample coefficient of variation and calculation of its percentage points (McKay, 1932; Iglewicz **et al.**, 1968; Vangel, 1996), calculating confidence limits on the proportions in the tail of a normal distribution (Durrant, 1978; Odeh and Owen, 1980), constructing confidence limits on one-sided quantiles and ´tolerance limit´ for the normal distribution (Wolfowitz, 1946; Johnson **et al.**, 1995, pp. 511-512), one-sided tolerance limits for the linear regression (Kabe, 1976), and in the study of acceptance sampling plans involving proportion of defective items (Owe, 1968; 1985). Guenther (1975) describes the use of noncentral t-distribution in testing hypotheses involving the quantiles of two normal populations.

In many applications involving the noncentral t-distribution, one has to compute its percentiles involving the evaluation of the inverse probability functions (see, e.g. Bagui, 1993, 1996). However, the evaluation of such inverse functions is extremely tedious involving slow and expensive techniques of numerical iteration such as the Newton-Raphson procedure (see, e.g., Ralston and Wilf, 1967; Carnahan **et al.**,1969). There are a number of approximations for computing the percentage points of these distributions, at arbitrary probability levels, available in the literature. The applicability of several of these approximations is further enhanced by ease of their computational simplicity. The purpose of this paper is to compare these approximations to determine their accuracy. Some of these approximations were previously investigated by Johnson and Welch (1940), van Eeden (1961), Kramer (1973), Akahira (1995) and Akahira **et al**. (1995). A brief description of each procedure is given and appropriate tables comparing their accuracy, calculated for each procedure, are presented. A more comprenhensive set of tables is given in Sahai and Ojeda (1998).

2. APPROXIMATIONS

The noncentral t-distribution was first derived by Fisher (1931) who also showed how the tables of the standard normal distribution could be used to approximate this distribution. There are several approximations

to percentiles of the noncentral t-distribution available in the literature. Some of the important ones are considered here.

In this paper, $t'_{\nu}(\delta)$ will be used to denote a noncentral t-variate with ν degrees of freedom and the noncentrality parameter δ. In addition, $t'_{\nu,\alpha}(\delta)$ will denote its 100α-th percentile defined by:

$$\Pr\left[t'_{\nu}(\delta) \le t'_{\nu,\alpha}(\delta)\right] = \alpha \ .$$

Jennett and Welch (1939), that $X + K\sqrt{X_{\nu}^2}$ is approximately normally distributed, where X has a standard normal distribution, X_{ν}^2 has a chi-square distribution with ν degrees of freedom (X and X_{ν}^2 are distributed independently) and K is a constant, gave the approximation

$$t'_{\nu,\alpha}(\delta) \cong \frac{\delta b_{\nu} + z_{\alpha}\sqrt{b_{\nu}^2 + \left(1 - b_{\nu}^2\right)\left(\delta^2 - z_{\alpha}^2\right)}}{b_{\nu}^2 - z_{\alpha}^2\left(1 - b_{\nu}^2\right)}, \tag{2.1}$$

where

$$b_{\nu} = \frac{\sqrt{2}\ \Gamma\{(\nu+1)/2\}}{\sqrt{\nu}\ \Gamma(\nu-2)}$$

and z_{α} is determined by

$$\Pr\left\{Z < z_{\alpha}\right\} = \int_{-\infty}^{z_{\alpha}} (2\pi)^{-1/2} \exp\left(-\frac{1}{2}z^2\right) dz = \alpha \ .$$

Akira (1995) obtained the approximation (2.1) as a special case of the Cornish-Fisher expansion by ignoring terms of higher order than $o(\nu^{-1})$.

Johnson and Welch (1940) simplified the approximation (2.1) leading to the approximation

$$t'_{\nu,\alpha}(\delta) \cong \frac{\delta + z_{\alpha}\sqrt{1 + \frac{1}{2\nu}\left(\delta^2 - z_{\alpha}^2\right)}}{1 - z_{\alpha}^2/2\nu} \tag{2.2}$$

Masuyama (1951) obtained values of this approximations using an improved binomial paper. Akahira (1995) obtained the approximation (2.2) as a special case of (2.1) by letting $b_{\nu} \approx 1$ and $1 - b_{\nu}^2 \approx 1/(2\nu)$.

An approximation intermediate between (2.1) and (2.2) was given by van Eeden (1961) as

$$t'_{\nu,\alpha}(\delta) \cong \frac{\delta b_{\nu} + z_{\alpha}\sqrt{b_{\nu}^2 + \frac{1}{2\nu}\left(\delta^2 - z_{\alpha}^2\right)}}{b_{\nu}^2 - \frac{1}{2\nu}Z_{\alpha}^2} \ . \tag{2.3}$$

Approximations (2.1), (2.2) and (2.3), however, give real values for $t'_{\nu,\alpha}(\delta)$ only for limited ranges of values of δ and z_{α} (see, e.g., Johnson **et al.**, 1995, p. 521).

For small values of δ and large values of ν(> 20), the simple approximation of the standardized $t'_\nu(\delta)$ variate by a standard normal variate yields the result (Johnson **et al**., 1995, p. 523):

$$t'_{\nu,\alpha}(\delta) \cong \frac{\nu}{\nu-1}\delta b_\nu + z_\alpha \sqrt{\frac{\nu}{\nu-2}\left(1+\delta^2\right) - \frac{\nu^2}{(\nu-1)^2}\delta^2 b_\nu^2} \qquad (2.4)$$

The normal approximation (2.4) is, of course, applicable for very small values of δ and large values of ν and is included here for the sake of completeness.

Cornish and Fisher (1937) (see also Fisher and Cornish, 1960) expansion applied to the distribution of $t'_\nu(\delta)$ (expansion up to and including terms in ν^{-2}) yields the following approximation (van Eeden, 1961; Johnson **et al.,** . 1995, p. 524):

$$t'_{\nu,\alpha}(\delta) \cong z_\alpha + \delta + \frac{1}{4\nu}\left[z_\alpha^3 + z_\alpha + \left(2z_\alpha^2+1\right)\delta + z_\alpha\delta^2\right] + \frac{1}{96\nu^2}\left[5z_\alpha^5 + 16z_\alpha^3 + 3z_\alpha\right.$$

$$\left. + 3\left(4z_\alpha^4 + 12z_\alpha^2 + 1\right)\delta + 6\left(z_\alpha^3 + 4z_\alpha\right)\delta^2 - 4\left(z_\alpha^2 - 1\right)\delta^3 - 3z_\alpha\delta^4\right]. \qquad (2.5)$$

Shibata (1981) derived approximation (2.5) from the taylor series expansion of the characteristic function of $t'_\nu(\delta) - \delta$ with a chi-square variate having ν degrees of freedom. Akahira (1995) derived it by applying the Cornish-Fisher expansion and using the characteristic function of a chi-square variate with ν degrees of fredom.

The corresponding Cornish-Fisher expansion applied to the central t-distirbution (δ = 0) gives the result (see, e.g., Sahai and Thompson, 1974):

$$t_{\nu,\alpha} \cong z_\alpha + \frac{z_\alpha^3 z_\alpha}{4\nu} + \frac{5z_\alpha^5 + 16z_\alpha^3 + 3z_\alpha}{96\nu^2}.$$

If these terms in (2.5) are replaced by $t_{\nu,\alpha}$, (2.5) becomes (van Eeden, 1961; Johnson **et al.,** . 1995, p. 524).

$$t'_{\nu,\alpha}(\delta) \cong t_{\nu,\alpha} + \delta + \frac{\delta}{4\nu}\left(1 + 2z_\alpha^2 + \delta z_\alpha\right) + \frac{1}{96\nu^2}\delta\left[3\left(4z_\alpha^4 + 12z_\alpha^2 + 1\right) + 6\left(z_\alpha^3 + 4z_\alpha\right)\delta - 4\left(z_\alpha^2 - 1\right)\delta^2 - 3z_\alpha\delta^3\right] \qquad (2.6)$$

We will call the approximations (2.5) and (2.6) as Cornish and Fisher´s 1st and 2nd approximations respectively:

Azorin (1953), starting from the relationship

$$\text{var}\left(t'_\nu(\delta)\right) = a^2 + b^2\left[E\left(t'_\nu(\delta)\right)\right]^2 ,$$

with

$$a = \sqrt{\frac{\nu}{(\nu-2)}}, \quad b = \Gamma\left(\frac{1}{2}\nu\right)\sqrt{2\left\{(\nu-2)\left[\left(\tfrac{1}{2}(\nu-1)\right)\right]^2 - 1\right\}^{-1}}$$

and (2.7)

$$E\left(t'_\nu(\delta)\right) = \left(\tfrac{1}{2}\nu\right)^{1/2}\frac{\Gamma\left(\tfrac{1}{2}(\nu-1)\right)\delta}{\Gamma\left(\tfrac{1}{2}\nu\right)}$$

obtained the transformation

$$\frac{1}{b}\sinh^{-1}\left(\frac{b}{a}t'_\nu(\delta)\right)-\frac{1}{b}\sinh^{-1}\left(\frac{b}{a}E\left(t'_\nu(\delta)\right)\right), \tag{2.8}$$

which is to be approximated as a standard normal variate. In addition, Azorin (1953) suggested two similar transformations of simpler forms:

$$\sqrt{\nu}\,\sinh^{-1}\left(t'_\nu(\delta)/\sqrt{\nu}\right) \tag{2.9}$$

and

$$\sqrt{\tfrac{2}{3}\nu}\,\sinh^{-1}\left(t'_\nu(\delta)/\sqrt{\tfrac{2}{3}\nu}\right) \tag{2.10}$$

The standarized versions of (2.9) and (2.10), correcting for mean and standard deviation of the transformed variable to terms of order ν^{-1}, are

$$\frac{\sqrt{\nu}\,\sinh^{-1}\left(t'_\nu(\delta)/\sqrt{\nu}\right)-\delta-\frac{1}{2}\delta^2\nu^{-1/2}-\frac{1}{4}\delta\nu^{-1}}{\sqrt{1+\frac{1}{2}\left(2-\delta^2\right)\nu^{-1}}} \tag{2.11}$$

and

$$\frac{\sqrt{\tfrac{2}{3}\nu}\,\sinh^{-1}\left(t'_\nu(\delta)/\sqrt{\tfrac{2}{3}\nu}\right)-\delta-\left(\frac{1}{2}\delta^2/\sqrt{\tfrac{2}{3}\nu}\right)}{\sqrt{1+\frac{1}{2}\left(1-\delta^2\right)\nu^{-1}}} \tag{2.12}$$

We will call approximations (2.8), (2.11) and (2.12) as Azorin´s 1st, 2nd and 3rd approximations respectively.

There is a considerable degree of skewness in the noncentral t-distribution for large values of δ and small values of ν (Johnson and Welch, 1940). Thus, approximations (2.8), (2.11) and (2.12) are expected to be rather very poor for simultaneously small values of ν and large values of δ. The results of numerical computations show that these approximations are not at all satisfactory; however, they have been included here for the sake completeness. Also, for small values of ν, the quality of approximations deteriorates rather rapidly as δ increases. For very large values of ν, the approximations improve somewhat, bull still are not to be recommended. Only Azorin´s 1st approximation is included in our comparative study.

Laubscher (1960) also considered the transformation (2.8). Furthermore, he proposed two modifications of (2.8) of the form:

$$L_1=\frac{1}{b}\sinh^{-1}\left(\frac{b}{a}t'_\nu(\delta)\right)-\frac{1}{b}\sinh^{-1}\left(\frac{b}{a}\mu\right)+\frac{1}{2}b^2\mu\,\mu_2^{(-1/2)} \tag{2.13}$$

and

$$L_2=L_1-\frac{1}{6}b^4\mu_2^{(-5/2)}\mu_3\left[2\mu^2-\left(a^2/b^2\right)\right], \tag{2.14}$$

where a, b and $\mu=E\left(t'_\nu(\delta)\right)$ are given as in (2.7) and μ_2 and μ_3 are the second and thirs central moments of $t'_\nu(\delta)$ given by

$$\mu_2 = \frac{\nu\left(1+\delta^2\right)}{(\nu-2)} - \mu^2 \text{ and } \mu_3 = \mu\left[\frac{\nu\left(\delta^2+2\nu-3\right)}{(\nu-2)(\nu-3)} - 2\mu_2\right].$$

The approximations (2.13) and (2.14) are expected to eliminate more bias than (2.8).

In addition, following Laubcher´s (1960) conjecture, we consider the approximations

$$\frac{(2\nu-1)^{1/2}\left[(1/\nu)\,t'^{2}_{\nu}(\delta)\right]^{1/2} - \left[2\left(1+\delta^{-2}\right) - \left(1+2\delta^2\right)/\left(1+\delta^2\right)\right]^{1/2}}{\left[(1/\nu)t'^{2}_{\nu}(\delta) + \left(1+2\delta^2\right)/\left(1+\delta^2\right)\right]^{1/2}} \tag{2.15}$$

and

$$\frac{(1-2/9\,\nu)\left[t'^{2}_{\nu}(\delta)/(1+\delta)^2\right]^{1/3} - \left[1-2\left(1+2\delta^2\right)/9\left(1+\delta^2\right)^2\right]}{\left\{\left[2\,(1+2\delta^2)/9\,(1+\delta^2)^2\right] + (2/9\nu)\left[t'^{2}_{\nu}(\delta)/\,(1+\delta^2)\right]^{2/3}\right\}^{1/2}}, \tag{2.16}$$

where each is to be approximated as a standard normal variate.

We will call approximations (2.13), (2.14), (2.15) and (2.16) as Laubscher´s 1st, 2nd, 3rd, and 4th approximations respectively.

Harley (1957) suggested an approximation of $t'_{\nu}(\delta)$ in terms of a function of the sample correlation coefficient (r), in a random sample of size n = ν + 2 from a bivariate population with correlation coefficient ρ, by the relationship

$$t'_{\nu}(\delta) = \frac{r}{\sqrt{\left(1-r^2\right)}}\sqrt{\frac{\nu(2\nu+1)}{\left(2\nu+1+\delta^2\right)}} \tag{2.17}$$

where

$$\rho = \delta\sqrt{\frac{2}{\left(2\nu+1+\delta^2\right)}}$$

Approximation (2.17) is of course valid for $\delta \leq (2\nu + 1)^{1/2}$ and is thus appplicable for only small values of δ. The percentiles of $t'_{\nu}(\delta)$ can be approximated from the percentiles of r by using relationship (2.17).

One can obtain the percentiles of r by using the Fisher´s Z-transformation

$$Z = \frac{1}{2}\log_e\left(\frac{1+r}{1-r}\right) \tag{2.18}$$

which is considered as approximately normally distributed with mean $\frac{1}{2}\log_e\left(\frac{1+\rho}{1-\rho}\right)$ and variance 1/(n-3). However, following the recommendation of David (1938), we approximate (2.18) by a normal random variable with mean μ and variance σ^2 given by

$$\mu = \frac{1}{2}\log_e\left(\frac{1+\rho}{1-\rho}\right) + \frac{\rho}{2(n-1)}\left\{1+\frac{5+\rho^2}{4(n-1)}\right\}$$

and

$$\sigma^2 = \frac{1}{n-1}\left[1+\frac{4-\rho^2}{2(n-1)}+\frac{22-6\rho^2-3\rho^4}{6(n-1)^2}\right].$$

This approximation is considered to be the most accurate one of all the existing normal approximations of r (see, e.g., Kraemer, 1973).

Another kind of approximation of r was considered by Ruben (1966) who showed that $r\Big/\sqrt{1-r^2}$ is distributed as

$$\left[Z+\chi_{n-1}\,\rho\Big/\sqrt{(1-\rho^2)}\right]\Big/\chi_{n-2}\,, \tag{2.19}$$

where Z is a unit normal variate, χ_ν is a chi-variate with ν degrees of freedom, and Z, χ_{n-1} and χ_{n-2} are mutually independent. For large values of n, χ_{n-1} and χ_{n-2} may be approximated by normal variates using Fisher´s approximation that $\sqrt{2\chi_\nu^2}-2\sqrt{2\nu-1}$ is approximately distributed as a unit normal variate. Using these results, it can be shown that the transformed variate

$$\frac{r\left(1-r^2\right)^{-1/2}\left(n-\frac{3}{2}\right)^{-1/2}-\rho\left(1-\rho^2\right)^{-1/2}\left(n-\frac{5}{2}\right)^{1/2}}{\left[1+\frac{1}{2}r^2\left(1-r^2\right)^{-1}+\frac{1}{2}\rho^2\left(1-\rho^2\right)^{-1}\right]^{1/2}} \tag{2.20}$$

has a standard normal distribution.

We will call the approximations of the type (2.17), based on percentiles of r via (2.18), its improved version due to David, and (2.20), as Harley´s, 1st, 2nd, and 3rd approximations respectively.

Merrington and Pearson (1958) gave an approximation to the percentiles of $t'_\nu(\delta)$ based on an approximations by a Pearson Type IV distribution. Let β_1 and β_2 be the moment rations, i.e.,

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \text{ and } \beta_2 = \frac{\mu_4}{\mu_2^2},$$

where μ_2, μ_3 and μ_4 denote the second, third and fourth central moments respectively of the noncentral t-distribution. Then, we have

$$t'_{\nu,\alpha}(\delta) \approx \mu + \sigma\, U(\beta_1, \beta_2, \alpha), \tag{2.21}$$

where $U(\beta_1, \beta_2, \alpha)$ is the 100α-th percentiles of the standardized Pearson Type IV distribution and μ and σ are the mean and standard deviation respectively of the noncentral t-distribution. The approximation (2.21) is of course applicable for $\nu > 4$ since μ_4 does not exist for $\nu \le 4$. This approximation, however, is not included in our comparative study because of complexity in computing the percentiles of the Pearson Type IV distribution.

Halperin (1963) developed bounds for the percentiles of $t'_{\nu,\alpha}(\delta)$ given by

$$t'_{\nu,\alpha}(\delta) \le \frac{\delta\sqrt{\nu}}{\chi_{\nu,1-\alpha}} + t_{\nu,\alpha} \qquad (\alpha \ge 0.5)$$

and (2.22)

$$t'_{\nu,\alpha}(\delta) \ge \frac{\delta\sqrt{\nu}}{\chi_{\nu,1-\alpha}} + t_{\nu,\alpha} \qquad (\alpha \le 0.43)\ ,$$

where $t_{\nu,\alpha}$ and $\chi^2_{\nu,\alpha}$ denote 100α - th percentiles of central t and χ^2 distributions respectively. Although approximations (2.22) in not of great accuracy, it is included here to investigate the sharpness of the bounds.

Kraemer and Paik (1979) proposed a central t approximation to the noncentral t-distribution by the relationship

$$\Pr\left[t'_\nu(\delta) \le t\right] \approx \Pr\left[t_\nu \le t\left(1+\delta^2/\nu\right)^{1/2} - \delta\left(1+t^2/\nu\right)^{1/2}\right].$$

Akahira (1995) and Akahira **et al**., (1995) derived an higher order approximation formula from the Cornish-Fisher expansion for the statistic based on a linear combination of a normal random variable and a chi-square random variable. The approximate percentile $t'_{\nu,\alpha}(\delta)$ derived from the formula is determined by the relationship:

$$\frac{b_\nu t'_{\nu,\alpha}(\delta) - \delta}{\sqrt{1+t^{2'}_{\nu,\alpha}(\delta)\left(1-b_\nu^2\right)}} \approx z_\alpha - \frac{t^{3'}_{\nu,\alpha}(\delta)\left(z^2_{\alpha-1}\right)}{24\left\{1+t^{2'}_{\nu,\alpha}(\delta)\left(1-b_\nu^2\right)\right\}^{3/2}}\left\{\frac{1}{\nu^2}+\frac{1}{4\nu^3}\right\},$$

(2.23)

where b_ν is defined as in (2.1).

Approximation (2.23) gives only an implicit expression and will require an iterative procedure for its solution. Akahira **et al**., (1995) showed that for a fixed α such that $|z_\alpha| \ge 1$ and for sufficiently large ν which is independent of δ, the solution to (2.23) exists uniquely. The existence of solution is guaranteed when $0.1 \le \alpha \le 0.15$ for $\nu = 1$, $0.03 \le \alpha \le 0.15$ for $\nu = 2$, $0.01 \le \alpha \le 0.15$ for $\nu = 3$, and $0.003 \le \alpha \le 0.15$ for $\nu \ge 4$. Approximation (2.23) is not included in our comparative study. However, Akahira **et al**., (1995) made a detailed numerical comparison of this along with Jennett-Welch, Johnson-Welch and van Eeden (1st Cornish-Fisher) approximations and found that approximation (2.23) had better numerical precision than others included in the study.

3. RESULTS

The percentiles $t'_{\nu,\alpha}(\delta)$ calculated for various approximations as well as the exact values, for selected values of α, ν and δ, are given in Table 1. On comparing the approximate values of $t'_{\nu,\alpha}(\delta)$ with the exact values one notices some very interesting results. For higher percentiles and small values of ν, Johnson-Welch and normal approximations perform best; however, for smaller values of δ, the Johnson-Welch approximation in less accurate than the normal. As δ increases, the accuracy of Johnson-Welch approximation improves. For moderate to large values of ν, the Jennett-Welch and van Eeden approximations are superior to others; the former being better than the latter. For 50th percentile ($\alpha = 0.5$), the Jennett-Welch and van Eeden approximation are equivalent and perform better than others. In this case, the Johnson-Welch approximation reduces to δ. This fact partially confirms the validity of the computations since they were obtained using the general formulae for all the approximations. For the lower percentiles, the normal approximation performs very poorly. In this case, the van Eeden approximation perfomrs better except

when both ν and δ are small and the Jennett-Welch approximation is superior. For all the cases, when ν is sufficiently large, all the approximations compare favorably. Both the 1st and 2nd Cornish-Fisher approximations provide excellent results for moderate to large values of ν and small values of δ. However, both approximations progressively degenerate as λ increases, especially for small values of ν and extreme lower and upper percentiles. In this regard, the performance of the 1st Cornish-Fisher is much worse than the 2nd one. The three approximations due to Azorín perform poorly, especially for small values of ν. Azorín´s 2nd and 3rd approximations show even much poorer performance than the 1st approximation and their results are not included in Table 1.

Similarly, Laubscher´s approximations, especially 3rd and 4th, show a rather poor performance, particularly for higher percentiles. For large values of ν, both Azorín and Laubscher type approximations show a spectacular improvement, but are still not to be recommended. There is not much difference among three approximations of Harley, based on distinct approximations of percentiles of the sample correlation coefficient and the approximations deteriorate rather rapidly for large values of δ. As expected, Halperin´s bounds are not too sharp, and are to be recommended only as a crude approximation. Finally, Kraemer-Pike approximation gives a uniformly poor performance, especially for small values of ν and large values of δ. For lower percentiles, the approximation gets better as ν increases, but is still not to be recommended.

ACKNOWLEDGMENTS

The exact percentiles of the noncentral t-distribution were calculated using AMOSLIB, a special function library prepared at Sandia Laboratories, Alburquerque, New México. We are indebted to Dr. Donald E. Amos for his courtesy in providing the pertinent information about the routiness including the necessary software to perform the requisite computations. We also thank Professor Constance van Eeden for reading a preliminary draft of the manuscript and making some helpful comments and suggestions. This work would not have been possible without the generous dedication of time and efforts by Rafael Guajardo Panes and Lorena López whose contributions are gratefully acknowledged.

Table 1. Approximate and exact percentiles of the noncentral t-distribution

α = 0.05

ν	APPROXIMATIONS	δ					
		1	**4 -**	**7**	**12**	**25**	**32**
4	Jennet-Welch	-0.7411	2.1020	4.3267	7.7942	16.5568	19.9033
	Johnson-Welch	-0.6936	1.9891	4.1023	7.3954	15.7145	18.8914
	van Eeden	-0.7458	2.0824	4.2754	7.6944	16.3380	19.6396
	Normal Approximation	-1.3103	0.1152	0.8794	1.9010	4.2925	5.1877
	1st Cornish-Fisher	-0.7384	2.1824	7.5897	54.0981	1130.31	2394.57
	2nd Cornish-Fisher	-0.7384	1.9281	3.3582	4.4074	15.1171	28.9562
	1st Azorín	-1.1456	1.4591	3.3179	6.1314	13.1484	15.8195
	1st Laubscher	-1.3163	1.0992	2.7857	5.2813	11.4376	13.7729
	2nd Laubscher	-1.3379	1.1301	2.8193	5.3169	11.4713	13.8066
	3rd Laubscher	0.0974	2.1492	4.2658	7.6311	16.1752	19.4412
	4th Laubscher	-0.3424	2.1079	4.2929	7.7189	14.1008	17.3473
	1st Harley	-0.9217	---------	---------	---------	---------	---------
	2nd Harley	-0.7155	---------	---------	---------	---------	---------
	3rd Harley	-0.7816	---------	---------	---------	---------	---------
	Halperin	-1.4825	0.4654	2.4133	5.6598	14.1008	17.3473
	Kraemer-Paik	0.9219	1.0793	2.4710	4.5713	9.8061	11.7985
	Exact	-0.7389	2.0801	4.2453	7.6163	16.1484	19.4090

'---------' designates undefined values. **Table 1.** (Continued)

10	Jennet-Welch	-0.6800	2.2227	4.7373	8.6645	18.5322	22.2927
	Johnson-Welch	-0.6628	2.1715	4.6319	8.4754	18.1314	21.8110
	van Eeden	-0.6806	2.2186	4.7239	8.6359	18.4668	22.2135
	Normal Approximation	-0.8106	1.7557	3.9256	7.2763	15.0420	18.8246
	1st Cornish-Fisher	-0.6797	2.2260	5.0854	14.8637	187.36	387.538
	2nd Cornish-Fisher	-0.6798	2.1853	4.4084	6.9132	8.9293	9.0394
	1st Azorín	-0.7698	2.0629	4.4885	8.2568	17.6988	21.2945
	1st Laubscher	-0.8042	1.9537	4.3153	7.9755	17.1287	20.6121
	2nd Laubscher	-0.8129	1.9577	4.3246	7.9875	17.1419	20.6255
	3rd Laubscher	0.0942	2.3082	4.7337	8.5964	18.3480	20.0674
	4th Laubscher	-0.3194	2.4223	4.7316	8.6415	18.4762	22.2247
	1st Harley	-0.7394	3.2218	---------	---------	---------	---------
	2nd Harley	-0.6802	3.4743	---------	---------	---------	---------
	3rd Harley	-0.6971	3.0268	---------	---------	---------	---------
	Halperin	-1.0734	1.1439	3.3611	7.0565	16.6645	20.3599
	Kraemer-Paik	0.7483	1.6879	3.6658	6.7187	14.3722	17.2884
	Exact	-0.6798	2.2171	4.7078	8.5876	18.3400	22.0581
16	Jennet-Welch	-0.6664	2.2652	4.9062	9.0540	19.4542	23.4126
	Johnson-Welch	-0.6559	2.2318	4.8360	8.9270	19.1843	23.0881
	van Eeden	-0.6666	2.2634	4.8998	9.0394	19.4195	32.3704
	Normal Approximation	-0.7388	2.0037	4.4521	8.2770	17.8370	21.4720
	1st Cornish-Fisher	-0.6663	2.2650	5.0048	11.1435	82.2995	161.078
	2nd Cornish-Fisher	-0.6663	2.2491	4.7403	8.0379	12.5998	13.2271
	1st Azorín	-0.7163	2.1756	4.7687	8.8305	18.9993	22.8679
	1st Laubscher	-0.7352	2.1103	4.6628	8.6581	18.6504	22.4504
	2nd Laubscher	-0.7406	2.1107	4.6671	8.6649	18.6586	22.4588
	3rd Laubscher	0.0934	2.3633	4.9213	9.0154	19.3307	23.2600
	4th Laubscher	-0.3143	2.2880	4.9068	9.0426	19.4236	23.3752
	1st Harley	-0.7021	2.4773	---------	---------	---------	---------
	2nd Harley	-0.6677	2.6039	---------	---------	---------	---------
	3rd Harley	-0.6772	2.3907	---------	---------	---------	---------
	Halperin	-0.9659	1.3743	3.7144	7.6145	17.7550	21.6551
	Kraemer-Paik	0.7085	1.8954	4.1188	7.5723	16.2270	19.5231
	Exact	-0.6663	2.2627	4.8900	9.0062	19.3261	23.2550
30	Jennet-Welch	-0.6561	2.3037	5.0779	9.4849	20.5284	24.7245
	Johnson-Welch	-0.6506	2.2851	5.0379	9.4116	20.3719	24.5363
	van Eeden	-0.6562	2.3031	5.0756	9.4792	20.5139	24.7065
	Normal Approximation	-0.6917	2.1750	4.8549	9.1037	19.7350	23.7725
	1st Cornish-Fisher	-0.6561	2.3032	5.0929	9.9338	36.7232	61.0747
	2nd Cornish-Fisher	-0.6561	2.2987	5.0176	9.0504	16.8975	19.0193
	1st Azorín	-0.6806	2.2597	5.0116	9.3789	20.3144	24.4685
	1st Laubscher	-0.6898	2.2257	4.9553	9.2864	20.1277	24.2452
	2nd Laubscher	-0.6929	2.2248	4.9563	9.2893	20.1312	24.2496
	3rd Laubscher	0.0928	2.4132	5.1102	9.4732	20.4593	24.6372
	4th Laubscher	-0.3106	2.3289	5.0828	9.4184	20.5153	24.7083
	1st Harley	-0.6747	2.3260	8.3732	---------	---------	---------
	2nd Harley	-0.6572	2.3909	8.5449	---------	---------	---------
	3rd Harley	-0.6620	2.2854	8.2229	---------	---------	---------
	Halperin	-0.8691	1.6142	4.0978	8.2371	18.9993	23.1386
	Kraemer-Paik	0.6784	2.0860	4.5741	8.4754	18.2421	21.9574
	Exact	-0.6561	2.3028	5.0713	9.4614	20.4564	24.6346

'---------' designates undefined values.

Table 1. (Continued)

50	Jennet-Welch	-0.6516	2.3231	5.1748	9.7538	21.2508	25.6147
	Johnson-Welch	-0.6483	2.3118	5.1498	9.7076	21.1516	25.4953
	van Eeden	-0.6516	2.3229	5.1738	9.7512	21.2489	25.6061
	Normal Approximation	-0.6721	2.2487	5.0459	9.5337	20.7928	25.0651
	1st Cornish-Fisher	-0.6516	2.3229	5.1771	9.8714	26.4200	37.5459
	2nd Cornish-Fisher	-0.6516	2.3213	5.1500	9.5534	19.2827	22.4060
	1st Azorín	-0.6657	2.2978	5.1370	9.6941	21.1315	25.4720
	1st Laubscher	-0.6710	2.2776	5.1030	9.6378	21.0179	25.3362
	2nd Laubscher	-0.6727	2.2766	5.1030	9.6391	21.0202	25.3388
	3rd Laubscher	0.0925	2.4385	5.2164	9.7564	21.2103	25.5616
	4th Laubscher	-0.3089	2.3495	5.1815	9.7534	21.2446	25.6068
	1st Harley	-0.6626	2.3149	5.8494	---------	---------	---------
	2nd Harley	-0.6523	2.3540	5.8494	---------	---------	---------
	3rd Harley	0.6551	2.2914	5.7884	---------	---------	---------
	Halperin	-0.8153	1.7666	4.3485	8.6517	19.8399	24.1431
	Kraemer-Paik	0.6649	2.1853	4.8357	9.0313	19.5349	23.5260
	Exact	-0.6516	2.3228	5.1718	9.7415	21.2075	25.5595
70	Jennet-Welch	-0.6496	2.3319	5.2213	9.8928	21.6519	26.1136
	Johnson-Welch	-0.6473	2.3237	5.2031	9.8590	21.5788	26.0257
	van Eeden	-0.6496	2.3318	5.2208	9.8914	21.6476	26.1083
	Normal Approximation	-0.6641	2.2795	5.1307	9.7382	21.3302	25.7276
	1st Cornish-Fisher	-0.6496	2.3318	5.2216	9.9389	24.0271	31.7238
	2nd Cornish-Fisher	-0.6496	2.3310	5.2078	9.7767	20.3057	23.9993
	1st Azorín	-0.6596	2.3141	5.1949	9.8515	21.5698	26.0155
	1st Laubscher	-0.6633	2.2997	5.1705	9.8109	21.4877	25.9175
	2nd Laubscher	-0.6645	2.2989	5.1702	9.8115	21.4893	25.9192
	3rd Laubscher	0.0924	2.4500	5.2673	9.9023	21.6248	26.0765
	4th Laubscher	-0.3082	2.3588	5.2287	9.8936	21.6482	26.1088
	1st Harley	-0.6575	2.3195	5.5206	---------	---------	---------
	2nd Harley	-0.6502	2.3475	5.5709	---------	---------	---------
	3rd Harley	-0.6522	2.3029	5.4801	--------	---------	---------
	Halperin	-0.7876	1.8504	4.4884	8.8850	20.3162	24.7129
	Kraemer-Paik	0.6591	2.2310	4.9642	9.3213	20.2390	24.3847
	Exact	0.6498	2.3318	5.2196	9.8852	21.6216	26.0744
α = 0.10							
4	Jennet-Welch	-0.3070	2.4718	4.8219	8.5580	18.0769	21.7197
	Johnson-Welch	-0.2882	2.3375	4.5663	8.1088	17.1323	20.5853
	van Eeden	-0.3076	2.4513	4.7727	8.4644	17.8735	20.4747
	Normal Approximation	-0.7441	1.1971	2.6230	4.8030	10.2650	12.3466
	1st Cornish-Fisher	-0.3071	2.5886	7.9035	47.8337	915.598	1923.13
	2nd Cornish-Fisher	-0.3071	2.3427	4.0613	5.9576	17.5251	29.7081
	1st Azorín	-0.6050	2.0727	4.1958	7.5227	15.9509	19.1718
	1st Laubscher	-0.7653	1.6770	3.5780	6.5126	13.8899	16.7036
	2nd Laubscher	-0.7885	1.7108	3.6168	6.5528	13.9305	16.7442
	3rd Laubscher	0.2549	2.5025	4.7943	8.4801	17.8943	21.4986
	4th Laubscher	0.0087	2.4749	4.7922	8.4933	17.9326	21.5456
	1st Harley	-0.4963	---------	---------	---------	---------	---------
	2nd Harley	-0.3213	---------	---------	---------	---------	---------
	3rd Harley	-0.3905	---------	---------	---------	---------	---------
	Halperin	-0.8161	1.3350	3.4862	7.0715	16.3933	19.9786
	Kraemer-Paik	0.4541	1.6188	3.3293	5.7043	12.2745	14.7518
	Exact	-0.3069	2.4589	4.7779	8.4626	17.8580	21.4547

'---------' designates undefined values.

Table 1. (Continued)

10	Jennet-Welch	-0.2914	2.5880	5.1811	9.3030	19.7552	23.7484
	Johnson-Welch	-0.2841	2.5279	5.0640	9.0955	19.3178	23.2229
	van Eeden	-0.2915	2.5836	5.1688	9.2774	19.6973	23.6784
	Normal Approximation	-0.3918	2.3254	4.7343	8.5415	18.1712	21.8477
	1st Cornish-Fisher	0.2914	2.5964	5.5252	14.6092	155.961	316.401
	2nd Cornish-Fisher	-0.2914	2.5570	4.9105	7.9090	12.2690	13.4532
	1st Azorín	-0.3601	2.5208	5.0881	9.1555	19.4570	23.3915
	1st Laubscher	0.3941	2.4069	4.9028	8.8502	18.8334	22.6446
	2nd Laubscher	-0.4028	2.4111	4.9128	8.8633	18.8479	22.6593
	3rd Laubscher	0.2485	2.6381	5.1823	9.2734	19.6739	23.6489
	4th Laubscher	0.0084	2.6031	5.1758	9.2833	19.7080	23.6911
	1st Harley	-0.3592	3.7319	---------	---------	---------	---------
	2nd Harley	-0.3035	3.9972	---------	---------	---------	---------
	3rd Harley	0.3261	3.5602	---------	---------	---------	---------
	Halperin	-0.5813	1.7914	4.1640	8.1185	18.4000	22.3544
	Kraemer-Paik	0.3491	2.1478	4.2976	7.6962	16.3177	19.6128
	Exact	-0.2914	2.5847	5.1665	9.2657	19.6622	23.6348
16	Jennet-Welch	-0.2876	2.6295	5.3286	9.6335	20.5299	24.6885
	Johnson-Welch	-0.2832	2.5905	5.2514	9.4559	20.2391	24.3391
	van Eeden	-0.2877	2.6276	5.3228	9.6207	20.4998	24.6519
	Normal Approximation	-0.3437	2.4889	5.0924	9.2324	19.6960	23.6879
	1st Cornish-Fisher	-0.2876	2.6312	5.4309	11.4384	71.2901	135.069
	2nd Cornish-Fisher	-0.2876	2.6158	5.1908	8.8211	15.1606	16.7306
	1st Azorín	-0.3261	2.5978	5.2903	9.5766	20.4177	24.5544
	1st Laubscher	-0.3449	2.5306	5.1795	9.3938	20.0448	24.1079
	2nd Laubscher	-0.3503	2.5310	5.1840	9.4010	20.0536	24.1169
	3rd Laubscher	-0.2469	2.6860	5.3287	9.6238	20.5043	24.6572
	4th Laubscher	-0.5124	1.9609	4.4341	9.5561	19.2733	23.3954
	1st Harley	-0.3292	2.8746	---------	---------	---------	---------
	2nd Harley	-0.2960	3.0043	---------	---------	---------	---------
	3rd Harley	-0.3096	2.7933	---------	---------	---------	---------
	Halperin	-0.5124	1.9609	4.4341	8.5561	19.2733	23.3954
	Kraemer-Paik	0.3235	2.3270	4.6862	8.4252	17.8981	21.5165
	Exact	-0.3876	2.6280	5.3209	5.3209	20.4706	24.6155
30	Jennet-Welch	-0.2848	2.6673	5.4780	9.9962	21.4229	25.7779
	Johnson-Welch	-0.2824	2.6458	5.4345	9.9180	21.2569	25.5783
	van Eeden	-0.2848	2.6668	5.4760	9.9913	21.4106	25.7629
	Normal Approximation	-0.3123	2.6010	5.3687	9.8121	21.0409	25.3196
	1st Cornish-Fisher	-0.2848	2.6674	5.4959	10.3923	34.5385	54.9707
	2nd Cornish-Fisher	-0.2848	2.6630	5.4276	9.6479	18.5725	21.3099
	1st Azorín	-0.3037	2.6544	5.4663	9.9823	21.3985	25.7490
	1st Laubscher	-0.3129	2.6199	5.4083	9.8863	21.2031	25.5152
	2nd Laubscher	-0.3157	2.6190	5.4094	9.8893	21.2075	25.5197
	3rd Laubscher	0.2457	2.7295	5.4956	9.9923	21.3944	25.7418
	4th Laubscher	0.0082	2.6876	5.4818	9.9932	21.4121	25.7646
	1st Harley	-0.3066	2.6960	9.0210	---------	---------	---------
	2nd Harley	-0.3157	2.6190	9.1984	---------	---------	---------
	3rd Harley	0.2457	2.7295	8.8795	---------	---------	---------
	Halperin	-0.4471	2.1427	4.7325	9.0488	20.2713	24.5876
	Kraemer-Paik	-0.3039	2.4902	5.0711	9.1828	19.5825	23.5506
	Exact	-0.2848	2.6668	5.4749	9.9858	21.3912	25.7383

'---------' designates undefined values.

Table 1. (Continued)

50	Jennet-Welch	-0.2835	2.6866	5.5622	10.2208	22.0173	26.5093
	Johnson-Welch	-0.2821	2.6734	5.5352	10.1720	21.9131	26.3840
	van Eeden	-0.2835	2.6864	5.5613	10.2187	22.0114	26.5021
	Normal Approximation	-0.2994	2.6490	5.5012	10.1190	21.8066	26.2566
	1st Cornish-Fisher	-0.2835	2.6866	5.5660	10.328	26.2192	36.1128
	2nd Cornish-Fisher	-0.2835	2.6850	5.5414	10.0600	20.4715	23.9949
	1st Azorín	-0.2944	2.6799	5.5578	10.2180	22.0149	26.5068
	1st Laubscher	-0.2997	2.6596	5.5231	10.1602	21.8974	26.3662
	2nd Laubscher	-0.3014	2.6586	5.5231	10.1615	21.8999	26.3688
	3rd Laubscher	0.2451	2.7518	5.5839	10.2231	21.0009	26.4878
	4th Laubscher	0.0082	2.7079	5.5674	10.2204	22.0121	26.5028
	1st Harley	-0.2965	2.6797	6.2851	---------	---------	---------
	2nd Harley	-0.2863	2.7190	6.3602	---------	---------	---------
	3rd Harley	-0.2906	2.6558	5.2258	---------	---------	---------
	Halperin	-0.4090	2.2601	4.9291	9.3776	20.9436	25.3920
	Kraemer-Paik	0.2949	2.5748	5.2896	9.6426	20.6464	26.8408
	Exact	-0.2835	2.6864	5.5608	10.2155	21.9988	26.4857
70	Jennet-Welch	-0.2829	2.6953	5.6025	10.3365	22.3449	26.9162
	Johnson-Welch	-0.2819	2.6858	5.5830	10.3008	22.2687	26.8245
	van Eeden	-0.2929	2.6952	5.6020	10.3352	22.3414	26.9118
	Normal Approximation	-0.2941	2.6692	5.5604	10.2666	22.2008	26.7433
	1st Cornish-Fisher	-0.2829	2.6953	5.6037	10.3800	24.2829	31.4418
	2nd Cornish-Fisher	-0.2829	2.6945	5.5912	5.5912	21.3504	25.2592
	1st Azorín	-0.2906	2.6909	5.6002	10.3364	22.3479	26.9201
	1st Laubscher	-0.2944	2.6764	5.5754	10.2950	22.2635	26.8191
	2nd Laubscher	-0.2956	2.6756	5.5752	10.2956	22.2652	26.8209
	3rd Laubscher	0.2449	2.7618	5.6262	10.3416	22.3342	26.9014
	4th Laubscher	0.0082	2.7170	5.6084	10.3371	22.3419	26.9123
	1st Harley	-0.2922	2.6832	5.9231	---------	---------	---------
	2nd Harley	-0.2850	2.7113	5.9740	---------	---------	---------
	3rd Harley	-0.2880	2.6660	5.8832	---------	---------	---------
	Halperin	-0.3891	2.3250	5.0390	9.5625	21.3234	25.8468
	Kraemer-Paik	0.2911	2.6135	5.3963	9.8805	21.2205	25.5405
	Exact	-0.2829	2.6952	5.6017	10.3332	22.3322	26.8998
$\alpha = 0.25$							
4	Jennet-Welch	0.3414	3.1940	5.8458	10.1739	21.3257	25.6053
	Johnson-Welch	0.3212	3.0143	5.5208	9.6109	20.1482	24.1917
	van Eeden	0.3411	3.1764	5.8080	10.1042	21.1768	25.4253
	Normal Approximation	0.2021	3.0048	5.5363	9.6521	20.2447	24.3087
	1st Cornish-Fisher	0.3416	3.3594	8.2035	34.8714	530.69	1090.15
	2nd Cornish-Fisher	0.3417	3.1365	5.4826	8.9754	22.0879	31.4084
	1st Azorín	0.2589	3.2777	6.0198	10.4842	21.9812	26.3928
	1st Laubscher	0.1031	2.7892	5.2071	9.1189	19.1613	23.0117
	2nd Laubscher	0.0837	2.8305	5.2579	9.1730	19.2168	23.0673
	3rd Laubscher	0.6168	3.2694	5.8971	10.2671	21.5270	25.8477
	4th Laubscher	0.5549	3.1986	5.8330	10.1443	21.2588	25.5245
	1st Harley	0.1744	---------	---------	---------	---------	---------
	2nd Harley	0.3163	---------	---------	---------	---------	---------
	3rd Harley	0.1953	---------	---------	---------	---------	---------
	Halperin	0.1211	2.7067	5.2922	9.6014	20.8053	25.1145
	Kraemer-Paik	0.2383	2.6093	4.7685	8.2910	17.3717	20.8562
	Exact	0.3415	3.2130	5.8905	10.2573	21.5050	25.8210

'---------' designates undefined values.

Table 1. (Continued)

10	Jennet-Welch	0.3319	3.2501	6.0263	10.5510	22.1777	26.6353
	Johnson-Welch	0.3237	3.1730	5.8852	10.3057	21.6638	26.0184
	van Eeden	0.3318	3.2465	6.0176	10.5337	22.1392	26.5889
	Normal Approximation	0.3072	3.2773	6.0851	10.6557	22.3974	26.8991
	1st Cornish-Fisher	0.3319	3.2689	6.3160	13.965	99.7219	190.797
	2nd Cornish-Fisher	0.3319	3.2333	5.8807	9.8217	18.3456	21.3979
	1st Azorín	0.3193	3.3357	6.1891	10.8350	22.7719	27.3486
	1st Laubscher	0.2853	3.2119	5.9801	10.4836	22.4795	26.4795
	2nd Laubscher	0.2767	3.2164	5.9914	10.4986	22.0638	26.4965
	3rd Laubscher	0.6002	3.2653	6.0448	10.5880	22.2628	26.7384
	4th Laubscher	0.5419	3.2596	6.0237	10.5406	22.1525	26.6048
	1st Harley	0.2671	4.7106	---------	---------	---------	---------
	2nd Harley	0.3189	4.9996	---------	---------	---------	---------
	3rd Harley	0.2691	4.5183	---------	--------	---------	---------
	Halperin	0.1929	2.8709	5.5490	10.0124	21.6173	26.0807
	Kraemer-Paik	0.2902	2.9684	5.4695	9.5441	20.0283	24.0500
	Exact	0.3319	3.2550	6.0418	10.5844	22.2542	26.7280
16	Jennet-Welch	0.3295	3.2730	6.1064	10.7292	22.5940	27.1405
	Johnson-Welch	0.3244	3.2237	6.0154	10.5705	22.2613	26.7410
	van Eeden	0.3295	3.2715	6.1024	10.7208	22.5749	27.1172
	Normal Approximation	0.3166	3.2997	6.1623	10.8284	22.8022	27.3904
	1st Cornish-Fisher	0.3295	3.2792	6.2007	11.916	51.5812	89.1733
	2nd Cornish-Fisher	0.3295	3.2653	6.0307	10.2975	19.7936	23.0019
	1st Azorín	0.3232	3.3320	6.2198	10.9278	23.0100	27.6398
	1st Laubscher	0.3044	3.2611	6.0995	10.7255	22.5929	27.1398
	2nd Laubscher	0.2991	3.2616	6.1044	10.7335	22.6027	27.1498
	3rd Laubscher	0.5961	3.2840	6.1164	10.7514	22.6484	27.2098
	4th Laubscher	0.5389	3.2841	6.1067	10.7239	22.5802	27.1236
	1st Harley	0.2892	3.5967	---------	---------	---------	---------
	2nd Harley	0.3210	3.7329	---------	---------	---------	---------
	3rd Harley	0.2895	3.5015	---------	---------	---------	---------
	Halperin	0.2188	2.9454	5.6720	10.2165	22.0319	26.5764
	Kraemer-Paik	0.3034	3.0831	5.7124	9.9949	21.0012	25.2214
	Exact	0.3295	3.2753	6.1149	10.7496	22.6431	27.2009
30	Jennet-Welch	0.3276	3.2948	6.1894	10.9276	23.0797	27.7327
	Johnson-Welch	0.3249	3.2679	6.1393	10.8399	22.8954	27.5113
	van Eeden	0.3276	3.2944	6.1880	10.9245	23.0722	27.7235
	Normal Approximation	0.3216	3.3128	6.2273	10.9957	23.2230	27.9047
	1st Cornish-Fisher	0.3276	3.2963	6.2097	11.2011	30.6207	44.2072
	2nd Cornish-Fisher	0.3276	3.2923	6.1613	10.7407	21.5789	25.3851
	1st Azorín	0.3248	3.3285	6.2555	11.0444	24.3249	28.0270
	1st Laubscher	0.3157	3.2930	6.1946	10.9419	23.1138	27.7741
	2nd Laubscher	0.3128	3.2920	6.1957	10.9451	23.1186	27.7790
	3rd Laubscher	0.5961	3.2840	6.1164	10.7514	23.0738	27.2068
	4th Laubscher	0.5365	3.3071	6.1917	10.9260	23.0738	27.7254
	1st Harley	0.3062	3.3425	10.1968	---------	---------	---------
	2nd Harley	0.3230	3.4096	10.3841	---------	---------	---------
	3rd Harley	0.3060	3.2938	10.0443	---------	---------	---------
	Halperin	0.2457	3.0312	5.8166	10.4590	22.5292	27.1716
	Kraemer-Paik	0.2137	3.1855	5.9462	10.4486	22.0032	26.4308
	Exact	0.3276	3.2956	6.1929	10.9376	23.1069	27.7662

'---------' designates undefined values

Table 1. (Continued)

50	Jennet-Welch	0.3260	3.3063	6.2367	11.0512	23.4038	28.1311
	Johnson-Welch	0.3252	3.2899	6.2062	10.9974	23.2904	27.9949
	van Eeden	0.3268	3.3061	6.2362	11.0499	23.4003	28.1268
	Normal Approximation	0.3234	3.3180	6.2620	11.0971	23.5007	28.2475
	1st Cornish-Fisher	0.3268	3.3067	6.2425	11.1303	25.8422	33.5815
	2nd Cornish-Fisher	0.3268	3.3053	6.2251	10.9646	22.5872	26.8055
	1st Azorín	0.3252	3.3271	6.2782	11.1253	23.5597	28.3183
	1st Laubscher	0.3199	3.3063	6.2424	11.0648	23.4353	28.1693
	2nd Laubscher	0.3183	3.3054	6.2425	11.0661	23.4379	28.1721
	3rd Laubscher	0.5916	3.3125	6.2367	11.0554	23.4206	28.1523
	4th Laubscher	0.5355	3.3191	6.2398	11.0511	23.4010	28.1276
	1st Harley	0.3139	3.2059	7.0514	---------	---------	---------
	2nd Harley	0.3239	3.3457	7.1290	---------	---------	---------
	3rd Harley	0.3137	3.2765	6.9868	---------	---------	---------
	Halperin	0.2627	3.0890	5.9153	10.6259	22.8733	27.5838
	Kraemer-Paik	0.3184	3.2378	6.0762	10.7169	22.6188	27.1766
	Exact	0.3268	3.3066	6.2383	11.0564	23.4199	28.1513
70	Jennet-Welch	0.3264	3.3115	6.2596	11.1150	23.5822	28.3524
	Johnson-Welch	0.3253	3.2998	6.2375	11.0761	23.5002	28.2539
	van Eeden	0.3264	3.3114	6.2593	11.1142	23.5801	28.3499
	Normal Approximation	0.3240	3.3202	6.2784	11.1495	23.6556	28.4405
	1st Cornish-Fisher	0.3264	3.3117	6.2621	11.1492	24.7169	30.9349
	2nd Cornish-Fisher	0.3264	3.3110	6.2532	11.0646	23.0562	27.4778
	1st Azorín	0.3254	3.3266	6.2898	11.1693	23.6971	28.4904
	1st Laubscher	0.3216	3.3119	6.2644	11.1264	23.6087	28.3845
	2nd Laubscher	0.3204	3.3111	6.2642	11.1270	23.6104	28.3864
	3rd Laubscher	0.5910	3. 3171	6.2581	11.1166	23.5936	28.3672
	4th Laubscher	0.5350	3.3246	6.2631	11.1153	23.5806	28.3503
	1st Harley	0.3172	3.3026	6.6213	---------	---------	---------
	2nd Harley	0.3244	3.3310	6.6734	---------	---------	---------
	3rd Harley	0.3170	3.2815	6.5775	---------	---------	---------
	Halperin	0.2719	3.1216	5.9714	10.7209	23.0698	27.8194
	Kraemer-Paik	0.3204	3.2616	6.1389	10.8539	22.9456	27.5744
	Exact	0.3254	3.3116	6.2604	11.1182	23.5934	28.3667
$\alpha = 0.50$							
4	Jennet-Welch	1.0638	4.2554	7.4469	12.7662	26.5962	31.9154
	Johnson-Welch	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	van Eeden	1.0638	4.2554	7.4469	12.7662	26.5962	31.9154
	Normal Approximation	1.2533	5.0133	8.7732	15.0398	31.3329	37.5994
	1st Cornish-Fisher	1.0671	4.4245	8.3444	17.2734	67.3014	102.246
	2nd Cornish-Fisher	1.0671	4.2936	7.5671	13.1270	28.1924	34.2207
	1st Azorín	1.2533	5.0133	8.7732	15.0398	31.3329	37.5994
	1st Laubscher	1.0841	4.3630	7.6461	13.1150	27.3296	32.7963
	2nd Laubscher	1.0632	4.4175	7.7163	13.1913	27.4084	32.8753
	3rd Laubscher	1.1721	4.3223	7.6026	13.0595	27.2308	32.6796
	4th Laubscher	1.1952	4.2801	7.4841	12.8287	26.7261	32.0714
	1st Harley	0.9487	---------	---------	---------	---------	---------
	2nd Harley	1.0698	---------	---------	---------	---------	---------
	3rd Harley	0.8367	---------	---------	---------	---------	---------
	Halperin	1.0916	4.3665	7.6414	13.0995	27.2907	32.6488
	Kraemer-Paik	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	Exact	1.0669	4.3312	7.6165	13.0839	27.2829	32.7423

'---------' designates undefined values

Table 1. (Continued)

10	Jennet-Welch	1.0253	4.1011	7.1769	12.3033	25.6318	30.7582
	Johnson-Welch	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	van Eeden	1.0253	4.1011	7.1769	12.3033	25.6318	30.7582
	Normal Approximation	1.0837	4.3349	7.5861	13.0047	27.0931	32.5117
	1st Cornish-Fisher	1.0257	4.1279	7.3201	13.0238	32.1432	42.0094
	2nd Cornish-Fisher	1.0257	4.1070	7.1957	12.3603	25.8857	31.1253
	1st Azorín	1.0837	4.3349	7.5861	13.0047	27.0931	32.5117
	1st Laubscher	1.0489	4.1966	7.3450	12.5924	26.2352	31.4824
	2nd Laubscher	1.0400	4.2016	7.3580	12.6100	26.2552	31.5026
	3rd Laubscher	1.1127	4.1032	7.2172	12.3974	25.8502	31.0228
	4th Laubscher	1.1471	4.1077	7.1826	12.3119	25.6495	30.7794
	1st Harley	0.9770	6.0270	---------	---------	---------	---------
	2nd Harley	1.0267	6.3445	---------	---------	---------	---------
	3rd Harley	0.9293	5.7328	---------	---------	---------	---------
	Halperin	1.0346	4.1385	7.2424	12.4155	25.8657	31.0388
	Kraemer-Paik	1.0000	4.0000	7.0000	12. 0000	25.0000	30.0000
	Exact	1.0257	4.1179	7.2239	14.4024	25.8588	31.0331
16	Jennet-Welch	1.0157	4.0629	7.1102	12.1888	25.3934	30.4721
	Johnson-Welch	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	van Eeden	1.0157	4.0629	7.1102	12.1888	25.3934	30.4721
	Normal Approximation	1.0501	4.2006	7.3510	12.6017	26.2535	31.5042
	1st Cornish-Fisher	1.0159	4.0734	7.1661	12.4202	27.9368	34.8669
	2nd Cornish-Fisher	1.0159	4.0652	7.1175	12.2111	25.4925	30.6154
	1st Azorín	1.0501	4.2006	7.3510	12.6017	26.2535	31.5042
	1st Laubscher	1.0311	4.1245	7.2182	12.3745	25.7807	30.9370
	2nd Laubscher	1.0256	4.1250	7.2236	12.3835	25.7917	30.9483
	3rd Laubscher	1.0986	4.0513	7.1259	12.2406	25.5233	30.6304
	4th Laubscher	1.1359	4.0677	7.1127	12.1922	25.4000	30.4800
	1st Harley	0.9852	4.5037	---------	---------	---------	---------
	2nd Harley	1.0163	4.6487	---------	---------	---------	---------
	3rd Harley	0.9549	4.3651	---------	---------	---------	---------
	Halperin	1.0213	4.0853	7.1494	12.2560	25.5334	30.6401
	Kraemer-Paik	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	Exact	1.0152	4.0705	7.1341	12.2441	25.5268	30.6345
30	Jennet-Welch	1.0084	4.0335	7.0586	12.1004	25.2092	30.2510
	Johnson-Welch	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	van Eeden	1.0084	4.0335	7.0586	12.1004	25.2092	30.2510
	Normal Approximation	1.0259	4.1036	7.1813	12.3108	25.6475	30.7770
	1st Cornish-Fisher	1.0084	4.0364	7.0745	12.1804	25.9326	31.5010
	2nd Cornish-Fisher	1.0084	4.0341	7.0606	12.1067	25.2373	30.2917
	1st Azorín	1.0259	4.1036	7.1813	12.3108	25.6475	30.7770
	1st Laubscher	1.0167	4.0667	7.1168	12.2003	25.4174	30.5010
	2nd Laubscher	1.0138	4.0657	7.1179	12.2037	25.4227	30.5064
	3rd Laubscher	1.0879	4.0117	7.0562	12.1209	25.2737	30.3309
	4th Laubscher	1.1275	4.0375	7.0598	12.1014	25.2110	30.2532
	1st Harley	0.9919	4.1110	11.6561	---------	---------	---------
	2nd Harley	1.0085	4.1805	11.8545	---------	---------	---------
	3rd Harley	0.9755	4.0431	11.4634	---------	---------	---------
	Halperin	1.0113	4.0450	7.0788	12.1350	25.2813	30.3376
	Kraemer-Paik	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	Exact	1.0084	4.0359	7.0677	12.1249	25.2751	30.3323

'---------' designates undefined values

Table 1. (Continued)

50	Jennet-Welch	1.0050	4.0200	7.0351	12.0601	25.1253	30.1504
	Johnson-Welch	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	van Eeden	1.0050	4.0200	7.0371	12.0601	25.1253	30.1504
	Normal Approximation	1.0153	4.0613	7.1072	12.1838	25.3830	30.4596
	1st Cornish-Fisher	1.0050	4.0211	7.0408	12.0890	25.3857	30.6004
	2nd Cornish-Fisher	1.0050	4.0203	7.0358	12.0624	25.1354	30.1650
	1st Azorín	1.0050	4.0200	7.0351	12.0601	25.1253	30.1504
	1st Laubscher	1.0100	4.0400	7.0701	12.1201	25.2504	30.3005
	2nd Laubscher	1.0083	4.0390	7.0701	12.1215	25.2532	30.3035
	3rd Laubscher	1.0830	3.9938	7.0247	12.0668	25.1609	30.1955
	4th Laubscher	1.1237	4.0238	7.0360	12.0606	25.1260	30.1511
	1st Harley	0.9951	4.0311	---------	---------	---------	---------
	2nd Harley	1.0051	4.0716	---------	---------	---------	---------
	3rd Harley	0.9852	3.9909	---------	---------	---------	---------
	Halperin	1.0067	4.0267	7.0470	12.0806	25.1679	30.2015
	Kraemer-Paik	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	Exact	1.0050	4.0210	7.0390	12.0723	25.1621	30.1965
70	Jennet-Welch	1.0036	4.0143	7.0250	12.0429	25.0894	30.1073
	Johnson-Welch	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	van Eeden	1.0036	4.0143	7.0250	12.0429	25.0894	30.1073
	Normal Approximation	1.0109	4.0435	7.0761	12.1305	25.2719	30.3263
	1st Cornish-Fisher	1.0036	4.0149	7.0280	12.0576	25.2223	30.3369
	2nd Cornish-Fisher	1.0036	4.0149	7.0254	12.0441	25.0946	30.1148
	1st Azorín	1.0109	4.0435	7.0761	12.1305	25.2719	30.3263
	1st Laubscher	1.0071	4.0286	7.0500	12.0858	25.1788	30.2146
	2nd Laubscher	1.0059	4.0277	7.0498	12.0865	25.1806	30.2165
	3rd Laubscher	1.0810	3.9861	7.0113	12.0438	25.1129	30.1379
	4th Laubscher	1.1220	4.0181	7.0259	12.0432	25.0898	30.1077
	1st Harley	0.9965	4.0117	7.4388	---------	---------	--------
	2nd Harley	1.0036	4.0405	7.4923	---------	---------	---------
	3rd Harley	---------	3.9831	---------	---------	---------	---------
	Halperin	1.0048	4.0192	7.0335	12.0575	25.1197	30.1436
	Kraemer-Paik	1.0000	4.0000	7.0000	12.0000	25.0000	30.0000
	Exact	1.0036	4.0148	7.0272	12.0503	25.1142	30.1386
α = 0.75							
4	Jennet-Welch	1.9219	5.8595	9.9978	16.9866	35.2586	42.2959
	Johnson-Welch	1.7994	5.4681	9.3234	15.8362	32.8666	39.4260
	van Eeden	1.9330	5.9198	10.1103	17.1844	35.6751	42.7962
	Normal Approximation	2.3046	7.0217	12.0101	20.4275	42.4210	50.8902
	1st Cornish-Fisher	1.9284	5.8911	8.6406	- 2.7594	- 429.652	- 945.486
	2nd Cornish-Fisher	1.9284	5.8923	10.3561	18.3267	35.6832	38.3369
	1st Azorín	2.4052	7.3785	12.6286	21.4843	44.6199	53.5285
	1st Laubscher	2.2012	6.4847	11.0454	18.7584	38.9305	46.7001
	2nd Laubscher	2.1762	6.5594	11.1440	18.8663	39.0425	46.8124
	3rd Laubscher	0.6168	3.2269	5.8971	10.2671	21.5270	25.8477
	4th Laubscher	0.5549	3.1986	5.8330	10.1443	21.2589	25.5245
	1st Harley	1.8686	---------	---------	---------	---------	---------
	2nd Harley	1.9727	---------	---------	---------	---------	---------
	3rd Harley	0.1953	---------	---------	---------	---------	---------
	Halperin	2.1831	6.5104	10.8376	18.0497	36.8891	44.0132
	Kraemer-Paik	0.2383	2.6093	4.7685	8.2910	17.3711	20.8562
	Exact	1.9309	5.9577	10.2069	17.3736	36.0914	43.2984

'---------' designates undefined values

Table 1. (Continued)

10	Jennet-Welch	1.7675	5.1476	8.6697	14.6422	30.3082	36.3477
	Johnson-Welch	1.7228	5.0132	8.4407	14.2530	29.5000	35.3782
	van Eeden	1.7689	5.1566	8.6879	14.6756	30.3802	36.4344
	Normal Approximation	1.8603	5.3925	9.0870	15.3537	31.7887	38.1242
	1st Cornish-Fisher	1.7686	5.1603	8.5401	12.0205	- 40.1235	- 115.531
	2nd Cornish-Fisher	1.7686	5.1605	8.8146	15.3942	34.3302	41.8803
	1st Azorín	1.8800	5.4613	9.2056	15.5559	32.2090	38.6285
	1st Laubscher	1.8432	5.3044	8.9253	15.0705	31.1931	37.4088
	2nd Laubscher	1.8339	5.3100	8.9404	15.0913	31.2167	37.4327
	3rd Laubscher	0.6002	3.2653	6.0448	10.5880	22.2628	26.7384
	4th Laubscher	0.5419	3.2596	6.0237	10.5406	22.1525	26.6048
	1st Harley	1.7365	7.6493	---------	---------	---------	---------
	2nd Harley	1.7859	7.9956	---------	---------	---------	---------
	3rd Harley	0.2691	4.5183	---------	---------	---------	---------
	Halperin	1.9181	5.5731	9.2280	15.3196	31.1577	37.2493
	Kraemer-Paik	0.2902	2.9684	5.4695	9.5441	20.0283	24.0500
	Exact	0.7688	5.1649	8.7135	14.7323	30.5128	36.5953
16	Jennet-Welch	1.7317	4.9719	8.3221	14.0055	28.9365	34.6962
	Johnson-Welch	1.7044	4.8917	8.1865	13.7756	28.4598	34.1243
	van Eeden	1.7322	4.9753	8.3296	14.0198	28.9680	34.7342
	Normal Approximation	1.7837	5.1014	8.5397	14.3749	29.7048	35.6180
	1st Cornish-Fisher	1.7321	4.9780	8.2903	13.1281	2.7277	- 22.5389
	2nd Cornish-Fisher	1.7321	4.9781	8.3976	14.4460	31.8112	38.9501
	1st Azorín	1.7944	5.1387	8.6038	14.4841	29.9315	35.8900
	1st Laubscher	1.7748	5.0561	8.4564	14.2283	29.3953	35.2461
	2nd Laubscher	1.7692	5.0566	8.4623	14.2384	29.4079	35.2590
	3rd Laubscher	0.5961	3.2840	6.1164	10.7514	22.6486	27.2068
	4th Laubscher	0.5389	3.2841	6.1067	10.7239	22.5802	27.1236
	1st Harley	1.7112	5.5476	---------	---------	---------	---------
	2nd Harley	1.7425	5.7026	---------	---------	---------	---------
	3rd Harley	0.2895	3.5015	---------	---------	---------	---------
	Halperin	1.8419	5.3259	8.8028	14.5975	29.6638	35.4585
	Kraemer-Paik	0.3034	3.0831	5.7124	9.9949	21.0012	25.2214
	Exact	1.7322	4.9791	8.3425	14.0508	29.0446	34.8277
30	Jennet-Welch	1.7046	4.8342	8.0365	13.4596	27.7270	33.2351
	Johnson-Welch	1.6904	4.7932	7.9676	13.3435	27.4867	32.9471
	van Eeden	1.7048	4.8352	8.0388	13.4643	27.7379	33.2486
	Normal Approximation	1.7302	4.8944	8.1353	13.6259	28.0720	33.6493
	1st Cornish-Fisher	1.7048	4.8363	8.0340	13.2742	20.9763	18.1256
	2nd Cornish-Fisher	1.7048	4.8363	8.0645	13.6491	29.2489	35.6158
	1st Azorín	1.7354	4.9123	8.1660	13.6782	28.1804	33.7794
	1st Laubscher	1.7260	4.8737	8.0972	13.5587	27.9295	33.4780
	2nd Laubscher	1.7231	4.8726	8.0985	13.5624	27.9352	33.4839
	3rd Laubscher	0.5930	3.3025	6.1925	10.9377	23.1088	27.7686
	4th Laubscher	0.5365	3.3071	6.1917	10.9260	23.0738	27.7254
	1st Harley	1.6932	4.9441	13.2984	-------	-------	-------
	2nd Harley	1.7100	5.0165	13.5083	-------	-------	-------
	3rd Harley	0.3060	3.2938	10.0443	-------	-------	-------
	Halperin	1.7898	5.1110	8.4323	13.9676	28.3596	33.8949
	Kraemer-Paik	0.3137	3.1855	5.9462	10.4486	22.0032	26.4308
	Exact	1.7048	4.8365	8.0436	13.4778	27.7750	33.2945

'---------' designates undefined values

Table 1. (Continued)

50	Jennet-Welch	1.6925	4.7708	7.8980	13.1798	27.7768	32.4466
	Johnson-Welch	1.6840	4.7466	7.8578	13.1123	26.9381	32.2793
	van Eeden	1.6925	4.7711	7.8989	13.1817	27.0823	32.4523
	Normal Approximation	1.7073	4.8046	7.9525	13.2706	27.2652	32.6716
	1st Cornish-Fisher	1.6925	4.7716	7.8987	13.1324	24.9237	27.4875
	2nd Cornish-Fisher	1.6925	4.7716	7.9097	13.2674	27.9019	33.7840
	1st Azorín	1.7102	4.8148	7.9701	13.3005	27.3272	32.7460
	1st Laubscher	1.7049	4.7930	7.9314	13.2332	27.1858	32.5761
	2nd Laubscher	1.7032	4.7919	7.9314	13.2347	27.1888	32.5792
	3rd Laubscher	0.5916	3.3125	6.2367	11.0554	23.4206	28.1523
	4th Laubscher	0.5355	3.3191	6.2398	11.0511	23.4010	28.1276
	1st Harley	1.6855	4.7936	8.9535	-------	-------	-------
	2nd Harley	1.6956	4.8353	9.0372	-------	-------	-------
	3rd Harley	0.3137	3.2765	6.9868	-------	-------	-------
	Halperin	1.7585	4.9956	8.2328	13.6281	27.6558	33.0511
	Kraemer-Paik	0.3184	3.2378	6.0762	10.7169	22.6188	27.1766
	Exact	1.6925	4.7716	7.9010	13.1883	27.1028	32.4780
70	Jennet-Welch	1.6873	4.7434	7.8365	13.0497	26.7608	32.0592
	Johnson-Welch	1.6813	4.7263	7.8081	13.0021	26.6628	31.9417
	van Eeden	1.6873	4.7436	7.8370	13.0507	26.7635	32.0625
	Normal Approximation	1.6977	4.7668	7.8739	13.1115	26.8882	32.2120
	1st Cornish-Fisher	1.6873	4.7438	7.8373	13.0316	25.7713	29.7274
	2nd Cornish-Fisher	1.6873	4.7438	7.8429	13.1005	27.2908	32.9399
	1st Azorín	1.6998	4.7740	7.8862	13.1324	26.9316	32.2640
	1st Laubscher	1.6960	4.7588	7.8593	13.0858	26.8334	32.1461
	2nd Laubscher	1.6948	4.4579	7.8590	13.0865	26.8353	32.1481
	3rd Laubscher	0.5910	3.3171	6.2581	11.1153	23.5936	28.3672
	4th Laubscher	0.5350	3.3246	6.2630	11.1153	23.5806	32.6326
	1st Harley	1.6823	4.7473	8.3053	-------	-------	-------
	2nd Harley	1.6952	4.7766	8.3604	-------	-------	-------
	3rd Harley	0.3170	3.2815	6.5775	-------	-------	-------
	Halperin	1.7432	4.9386	8.1341	13.4599	27.3069	32.6326
	Kraemer-Paik	0.3204	3.2616	6.1389	10.8539	22.9456	27.5744
	Exact	1.6873	4.7438	7.8381	13.0547	26.7772	32.0800
$\alpha = 0.90$							
4	Jennet-Welch	3.0223	8.3895	14.1853	24.0259	49.8062	63.7146
	Johnson-Welch	2.8048	7.7292	13.0504	22.0911	45.7842	58.5678
	van Eeden	3.0793	8.6355	14.6291	24.7959	51.4188	65.7800
	Normal Approximation	3.2507	8.8295	14.9234	25.2766	52.4007	67.0340
	1st Cornish-Fisher	2.9264	7.8318	9.5593	-21.7114	-901.408	-1933.7
	2nd Cornish-Fisher	2.9264	7.8692	13.6469	24.1107	43.8967	43.4716
	1st Azorín	3.6954	10.2889	17.4371	29.5623	61.3096	73.5407
	1st Laubscher	3.4383	9.0812	15.2757	25.8263	53.4994	64.1655
	2nd Laubscher	3.4070	9.1820	15.4101	25.9742	53.6530	64.3197
	3rd Laubscher	0.2549	2.5025	4.7943	8.4801	17.8943	21.4986
	4th Laubscher	0.0087	2.4749	4.7922	8.4933	17.9326	21.5456
	1st Harley	2.9372	---------	---------	---------	---------	---------
	2nd Harley	3.0168	---------	---------	---------	---------	---------
	3rd Harley	-0.3905	---------	---------	---------	---------	---------
	Halperin	3.4725	9.2902	15.1080	24.8043	50.0147	59.7110
	Kraemer-Paik	0.4541	1.6118	3.2393	5.7943	12.2745	14.7518
	Exact	2.9843	8.1941	13.8366	23.4263	48.5565	58.2403

"---------' designates undefined values

Table 1. (Continued)

10	Jennet-Welch	2.5301	6.3670	10.4901	17.5620	36.2133	46.2958
	Johnson-Welch	2.4631	6.1878	10.1885	17.0516	35.1555	44.9425
	van Eeden	2.5357	6.3935	10.5412	17.6539	36.4096	46.5477
	Normal Approximation	2.5592	6.3444	10.4378	17.4678	36.0149	46.0416
	1st Cornish-Fisher	2.5248	6.3050	9.9286	11.2729	-108.477	-263.836
	2nd Cornish-Fisher	2.5248	6.3110	10.5826	18.6044	42.7721	52.5118
	1st Azorín	2.6431	6.6110	10.8926	18.2394	37.6166	45.0969
	1st Laubscher	2.6037	6.4335	10.5700	17.6766	36.4331	43.6755
	2nd Laubscher	2.5937	6.4399	10.5874	17.7007	36.4606	43.7034
	3rd Laubscher	0.2485	2.6381	5.1823	9.2734	19.6739	23.6489
	4th Laubscher	0.0084	2.6031	5.1758	9.2833	19.7080	23.6911
	1st Harley	2.4942	9.4387	---------	---------	---------	---------
	2nd Harley	2.5447	9.8090	---------	---------	---------	---------
	3rd Harley	-0.3261	3.5602	---------	---------	---------	---------
	Halperin	2.8059	7.1069	11.4079	18.5763	37.2140	44.3824
	Kraemer-Paik	0.3491	2.1478	4.2976	7.6962	16.3177	19.6128
	Exact	2.5261	6.3364	10.4261	17.4439	35.9596	43.1085
16	Jennet-Welch	2.4308	5.9430	9.6733	16.0841	33.0484	42.2304
	Johnson-Welch	2.3914	5.8423	9.5060	15.8025	32.4659	41.4854
	van Eeden	2.4327	5.9526	9.6926	16.1201	33.1267	42.3312
	Normal Approximation	2.4440	5.9122	9.6095	15.9709	32.8110	41.9264
	1st Cornish-Fisher	2.4290	5.9218	9.4885	13.8993	- 21.0177	- 76.4677
	2nd Cornish-Fisher	2.4290	5.9241	9.7439	16.7632	38.0638	47.1058
	1st Azorín	2.4894	6.0552	9.8525	16.3824	33.6635	40.3430
	1st Laubscher	2.4689	5.9657	9.6898	16.0972	33.0626	39.6211
	2nd Laubscher	2.4630	5.9663	9.6963	16.1085	33.0767	39.6355
	3rd Laubscher	0.2469	2.6860	5.3384	9.6177	20.4771	24.6232
	4th Laubscher	0.0083	2.6476	5. 3287	9.6238	20.5043	24.6572
	1st Harley	2.4084	6.6304	---------	---------	---------	---------
	2nd Harley	2.4405	6.7952	---------	---------	---------	---------
	3rd Harley	-0.3096	2.7933	---------	---------	---------	---------
	Halperin	2.6475	6.5799	10.5123	17.0662	34.1065	40.6604
	Kraemer-Paik	0.3235	2.3270	4.6862	8.4252	17.8981	21.5165
	Exact	2.4293	5.9303	9.6436	16.0252		
30	Jennet-Welch	2.3587	5.6285	9.0398	14.8914	30.4263	38.8497
	Johnson-Welch	2.3387	5.5794	8.9595	14.7575	30.1503	38.4969
	van Eeden	2.3593	5.6311	9.0453	14.9024	30.4512	38.8819
	Normal Approximation	2.3641	5.6062	8.9939	14.8095	30.2541	38.6292
	1st Cornish-Fisher	2.3583	5.6232	8.9989	14.3881	16.3722	5.6314
	2nd Cornish-Fisher	2.3583	5.6239	9.0715	15.2027	33.1776	40.7812
	1st Azorín	2.3859	5.6745	9.1093	15.0044	30.6572	36.7178
	1st Laubscher	2.3764	5.6340	9.0362	14.8761	30.3856	36.3914
	2nd Laubscher	2.3734	5.6329	9.0376	14.8801	30.3918	36.3978
	3rd Laubscher	0.2457	2.7295	5.4956	9.9923	21.3944	25.7418
	4th Laubscher	0.0082	2.6876	5.4818	9.9932	21.4121	25.7646
	1st Harley	2.3468	5.7599	14.9543	---------	---------	---------
	2nd Harley	2.3639	5.8355	15.1747	---------	---------	---------
	3rd Harley	- 0.2966	2.6557	8.8794	---------	---------	---------
	Halperin	2.5172	6.1376	9.7580	15.7920	31.4804	37.5144
	Kraemer-Paik	0.3039	2.4902	5.0711	9.1828	19.5825	23.5506
	Exact	2.3584	5.6247	9..0292	14.8675		

'---------' designates undefined values

Table 1. (Continued)

50	Jennet-Welch	2.2372	5.4884	8.7441	14.3042	29.0776	37.0973
	Johnson-Welch	2.3155	5.4601	8.6986	14.2282	28.9218	36.8994
	van Eeden	2.3274	5.4893	8.7462	14.3085	29.0868	37.1106
	Normal Approximation	2.3301	5.4735	8.7133	14.2487	28.9593	36.9471
	1st Cornish-Fisher	2.3271	5.4866	8.7321	14.1586	24.5371	24.6181
	2nd Cornish-Fisher	2.3271	5.4869	8.7583	14.4519	30.5871	37.2721
	1st Azorín	2.3426	5.5125	8.7791	14.3595	29.1881	34.9369
	1st Laubscher	2.3371	5.4901	8.7388	14.2888	29.0382	34.7566
	2nd Laubscher	2.3354	5.4890	8.7388	24.2903	29.0413	34.7599
	3rd Laubscher	0.2451	2.7518	5.5839	10.2231	22.0009	26.4878
	4th Laubscher	0.0082	2.7079	5.5674	10.2204	22.0121	26.5028
	1st Harley	2.3201	5.5178	9.9136	---------	---------	---------
	2nd Harley	2.3303	5.5608	10.0004	---------	---------	---------
	3rd Harley	- 0.2906	2.6558	6.2258	---------	---------	---------
	Halperin	2.4505	5.9059	9.3614	15.1204	30.0939	35.8529
	Kraemer-Paik	0.2949	2.5748	5.2896	9.6424	20.6464	24.8408
	Exact	2.3271	5.4869	8.7397	14.2929	29.0460	34.7656
70	Jennet-Welch	2.3140	5.4289	8.6150	14.0363	28.4317	36.2530
	Johnson-Welch	2.3057	5.4091	8.5632	13.9841	28.3248	36.1164
	van Eeden	2.3141	5.4294	8.6160	14.0386	28.4374	36.2605
	Normal Approximation	2.3159	5.4179	8.5919	13.9944	28.3430	36.1393
	1st Cornish-Fisher	2.3139	5.4280	8.6096	13.9733	26.3216	29.1934
	2nd Cornish-Fisher	2.3139	5.4282	8.6229	14.1229	29.4083	35.6495
	1st Azorín	2.3247	5.4452	8.6379	14.0718	28.5025	34.1005
	1st Laubscher	2.3209	5.4296	8.6102	14.0233	28.3995	33.9766
	2nd Laubscher	2.3197	5.4287	0.6099	10.0240	28.4015	33.9788
	3rd Laubscher	0.2449	5.7618	5.6262	10.3416	22.3342	26.9014
	4th Laubscher	0.0082	5.7170	5.6084	10.3371	22.3419	26.9123
	1st Harley	2.3089	5.4359	9.1318	---------	---------	---------
	2nd Harley	2.3161	5.4659	9.1885	---------	---------	---------
	3rd Harley	- 0.2880	5.6660	5.8832	---------	---------	---------
	Halperin	2.4186	5.7929	9.1673	14.7913	29.4136	35.0376
	Kraemer-Paik	0.2911	2.6135	5.3963	9.8805	21.2205	25.5405
	Exact	2.3139	5.4282	8.6125	14.0296	28.4117	33.9907
α = 0.95							
4	Jennet-Welch	4.0476	11.1239	18.8186	31.8833	66.1049	84.5663
	Johnson-Welch	3.7156	10.0990	17.0519	28.8689	59.8362	76.5441
	van Eeden	4.1929	11.7059	19.8542	33.6706	69.8392	89.3479
	Normal Approximation	3.8170	9.9113	16.6670	28.1786	58.3732	74.6677
	1st Cornish-Fisher	3.7760	9.4351	10.6889	- 32.8902	-1192.95	-2543.02
	2nd Cornish-Fisher	3.7760	9.3804	16.0614	28.1752	49.6107	47.3358
	1st Azorín	4.6372	12.5075	21.1237	35.7685	74.1431	88.9303
	1st Laubscher	4.3364	11.0558	18.5158	31.2546	64.7011	77.5957
	2nd Laubscher	4.2998	11.1768	18.6780	31.4332	64.8869	77.7821
	3rd Laubscher	0.0974	2.1492	4.2658	7.6311	16.1752	19.4412
	4th Laubscher	-0.3424	2.1079	4.2929	7.7189	16.3875	19.6990
	1st Harley	3.7409	---------	---------	---------	---------	---------
	2nd Harley	3.7956	---------	---------	---------	---------	---------
	3rd Harley	-0.7816	---------	---------	---------	---------	---------
	Halperin	4.5042	11.6213	18.7383	30.6001	61.4407	73.3025
	Kraemer-Paik	0.9219	1.0793	2.4710	4.5713	9.8061	11.7985
	Exact	3.8385	10.1155	16.9837	28.6924		

'---------' designates undefined values

Table 1. (Continued)

10	Jennet-Welch	3.0602	7.2979	11.9238	19.8973	40.9717	52.3702
	Johnson-Welch	2.9757	7.0800	11.5583	19.2792	39.6906	50.7314
	van Eeden	3.0710	7.3433	12.0094	20.0497	41.2051	52.7851
	Normal Approximation	2.9775	6.9141	11.2463	18.7331	38.5441	49.2627
	1st Cornish-Fisher	3.0381	7.1221	10.9455	10.9976	-150.574	-355.12
	2nd Cornish-Fisher	3.0381	7.1134	11.8051	20.7680	48.2355	59.3373
	1st Azorín	3.1286	7.3722	12.0231	20.0488	41.2711	49.4695
	1st Laubscher	3.0872	7.1804	11.6716	19.4327	39.9741	47.9115
	2nd Laubscher	3.0766	7.1873	11.6906	19.4590	40.0043	47.9421
	3rd Laubscher	0.0942	2.3082	4.7337	8.5964	18.3480	22.0674
	4th Laubscher	-0.3194	2.2422	4.7316	8.6415	18.4762	22.2247
	1st Harley	2.9946	10.6898	---------	---------	---------	---------
	2nd Harley	3.0460	11.0725	---------	---------	---------	---------
	3rd Harley	-0.6971	3.0268	---------	---------	---------	---------
	Halperin	3.4055	8.1847	12.9640	20.9293	20.6393	49.6046
	Kraemer-Paik	0.7483	1.6879	3.6658	6.7187	14.3722	17.2884
	Exact	3.0417	7.1800	11.6789	19.4420		
16	Jennet-Welch	2.8886	6.6236	10.6492	17.6123	36.1007	46.1170
	Johnson-Welch	2.8406	6.5071	10.4570	17.2896	35.4335	45.2638
	van Eeden	2.8922	6.6390	10.6795	17.6680	36.2209	46.2715
	Normal Approximation	2.8391	6.3974	10.2498	16.9261	34.6700	44.2855
	1st Cornish-Fisher	2.8814	6.5621	10.3144	14.4851	- 35.5827	-109.598
	2nd Cornish-Fisher	2.8814	6.5587	10.6502	18.3017	42.0772	52.2989
	1st Azorín	2.9206	6.6417	10.6618	17.6217	36.1093	43.2625
	1st Laubscher	2.8995	6.5475	10.4889	17.3172	35.4659	42.4892
	2nd Laubscher	2.8935	6.5481	10.4959	17.3292	35.4810	42.5047
	3rd Laubscher	0.0934	2.3633	4.9213	9.0154	19.3307	23.2600
	4th Laubscher	-0.3143	2.2880	4.9068	9.0426	19.4236	23.3752
	1st Harley	2.8528	7.3547	---------	---------	---------	---------
	2nd Harley	2.8857	7.5254	---------	---------	---------	---------
	3rd Harley	-0-6772	2.3906	---------	---------	---------	---------
	Halperin	3.1635	7.4163	11.6692	18.7573	37.1863	44.2744
	Kraemer-Paik	0.7085	1.8954	4.1188	7.5723	16.2270	19.5231
	Exact	2.8823	6.5780	10.5450	17.4058		
30	Jennet-Welch	2.7689	6.1475	9.7116	15.8685	32.2912	41.2095
	Johnson-Welch	2.7451	6.0927	9.6232	15.7217	31.9892	40.8235
	van Eeden	2.7698	6.1515	9.7199	15.8846	32.3272	41.2561
	Normal Approximation	2.7435	6.0322	9.5077	15.5178	31.5600	40.2735
	1st Cornish-Fisher	2.7672	6.1315	9.6314	15.1279	13.5895	- 2.0023
	2nd Cornish-Fisher	2.7672	6.1306	9.7269	15.2135	35.6794	44.0484
	1st Azorín	2.7826	6.1483	9.7024	15.8452	32.2357	38.5916
	1st Laubscher	2.7728	6.1066	9.6265	15.7111	31.9510	38.2492
	2nd Laubscher	2.7698	6.1055	9.6279	15.7153	31.9574	38.2560
	3rd Laubscher	0.0928	2.4132	5.1102	9.4732	20.4593	24.6372
	4th Laubscher	- 0.3106	2.3289	5.0828	9.4814	20.5153	24.7083
	1st Harley	2.7517	6.2825	16.0349	---------	---------	---------
	2nd Harley	2.7691	6.3602	16.2614	---------	---------	---------
	3rd Harley	- 0.6619	2.2854	8.2229	---------	---------	---------
	Halperin	2.9709	6.7920	10.6130	16.9814	33 5393	39.9077
	Kraemer-Paik	0.6748	2.0860	4.5741	8.4754	18.2421	21.9574
	Exact	2.7673	6.1343	9.6771	15.7914		

'---------' designates undefined values

Table 1. (Continued)

50	Jennet-Welch	2.7178	5.9417	9.2887	15.0407	30.4042	38.7623
	Johnson-Welch	2.7039	5.9107	9.2395	14.9598	30.2388	38.5509
	van Eeden	2.7181	5.9430	9.2916	15.0467	30.4185	38.7810
	Normal Approximation	2.7028	5.8739	9.1686	14.8340	29.9732	38.2105
	1st Cornish-Fisher	2.7172	5.9362	9.2632	14.8185	24.3337	22.8895
	2nd Cornish-Fisher	2.7172	5.9358	9.2975	15.2093	32.2861	39.4677
	1st Azorín	2.7252	5.9401	9.2792	15.0195	30.3552	36.3120
	1st Laubscher	2.7197	5.9172	9.2379	14.9466	30.1999	36.1251
	2nd Laubscher	2.7179	5.9161	9.2379	14.9482	30.2032	36.1286
	3rd Laubscher	0.0925	2.4385	5.2164	9.7564	21.2103	25.5616
	4th Laubscher	- 0.3089	2.3495	5.1815	9.7534	21.2446	25.6068
	1st Harley	2.7079	5.9708	10.5234	---------	---------	---------
	2nd Harley	2.7182	6.0145	10.6121	---------	---------	---------
	3rd Harley	- 0.6551	2.2914	5.7884	---------	---------	---------
	Halperin	2.8752	6.4730	10.0708	16.0672	31.6578	37.6541
	Kraemer-Paik	0.6649	2.1853	4.8357	9.0313	19.5347	23.5260
	Exact	2.7172	5.9368	9.2747	15.0058		
70	Jennet-Welch	2.6965	5.8555	9.1067	14.6694	29.5194	37.6055
	Johnson-Welch	2.6867	5.8339	9.0728	14.6140	29.4065	37.4613
	van Eeden	2.6966	5.8562	9.1082	14.6726	29.5273	37.6159
	Normal Approximation	2.6858	5.8075	9.0216	14.5228	29.2136	37.2140
	1st Cornish-Fisher	2.6962	5.8527	9.0946	14.5701	26.6845	28.8909
	2nd Cornish-Fisher	2.6962	5.8525	9.1121	14.7695	30.7418	37.3492
	1st Azorín	2.7015	5.8538	9.0988	14.6521	29.4794	35.2435
	1st Laubscher	2.6977	5.8380	9.0706	14.6024	29.3734	35.1159
	2nd Laubscher	2.6965	5.8371	9.0703	14.6031	29.3755	35.1182
	3rd Laubscher	0.0924	2.4500	5.2673	9.9023	21.6248	26.0765
	4th Laubscher	- 0.3082	2.3588	5.2287	9.8936	21.6482	26.1088
	1st Harley	2.6896	5.8617	9.6495	---------	---------	---------
	2nd Harley	2.6969	6.8922	9.7073	---------	---------	---------
	3rd Harley	- 0.6522	2.3029	5.4801	---------	---------	---------
	Halperin	2.8301	6.3195	9.8090	15.6248	30.7459	36.5617
	Kraemer-Paik	0.6591	2.2310	4.9642	9.3213	20.2390	24.3845
	Exact	2.6962	5.8530	9.0991	14.6489		

'---------' designates undefined values'

REFERENCES

AKAHIRA, M. (1995): "A higher Order Approximation to a Percentage Point of the Non-Central t-Distribution", **Communications in Statistics, Part B: Simulation and Computation**, 24, 595-605.

AKAHIRA, M.; M. SATO and N. TORIGOE (1995): "On the New Approximation to Non-Central t-Disribution", **Journal of Japan Statistical Society**, 25, 1-18.

AZORIN, P.F. (1953): "On the Noncentral t-Distribution", I. II, **Trabajos de Estadística**, 4, 173-198 and 307-337 (In Spanish).

BAGUI, S.C. (1993): **CRC Handbook of Percentiles of Noncentral t-Distributions**. CRC Press, Boca Ratón, Florida.

_____ (1996):): **CRC Handbook of Percentiles of Noncentral t-Distributions**. CRC Press, Boca Ratón, Florida.

CARNAHAN, B.; H.A. LUTHER and J.O. WILKES (1969): **Applied Numerical Methods**. John Wiley and Sons, New York.

CORNISH, E.A. and R.A. FISHER (1937): "Moments and Cumulants in the Specification of Distribution", **Revue de Institute International de Statistique**, 4, 1-14.

DURRANT, N.F. (1978): "A Nomogram for Confidence Limits on Quantiles of the Normal Distribution with Application to Extreme Value Distribution", **Journal of Quality Technology**, 10, 155-158.

FISHER, R.A: (1931): "Introduction to Table of Hh Functions", In: **Table of Hh Function**, pp. xxvi-xxxv. (Ed. J.R. Airey). British Association Mathematical Tables, London (2nd ed. 1946, 3rd ed. 1951).

GUENTHER, W.C. (1975): "A Sample Size Formula for a Non-Central t test", **The American Statistician**, 29, 120-121.

HALPERIN, M. (1963): "Approximations to the Noncentral t, with Applications", **Technometrics**, 5, 295-305.

HARLEY, B.I. (1957): Relation Between the Distribution of Noncentral t and a Transformed Correlation Coefficient", **Biometrika**, 44, 219-224.

IGLEWICZ, B.; R.H. MYERS and R.B. HOWE (1968): "On the Percentage Points of the Sample Coefficient of Variation", **Biometrika**, 55, 580-581.

JOHNSON, N.L; S. KOTZ and N. BALKRISHNAN (1995): **Continuous Univariate Distributions**, 2, 2nd. ed. John Wiley and Sons, New York.

_____ and B.L. WELCH (1940): "Application of the Noncentral t-Distribution", **Biometrika**, 31, 362-389.

KABE, G. (1976): "On Confidence Bands for Quantiles of a Normal Population", **Journal of the American Statistical Association**, 71, 417-419.

KRAEMER, H.C. (1973): "Improved Approximation to the Non-null Distribution of the Correlation Coefficient", **Journal of the American Statistical Association**, 68, 1004-1008.

_____ and M. PAIK (1979): "A Central t-Approximation to the Noncentral t-Distribution", **Technometrics**, 21, 357-360.

LAUBSCHER, N.F. (1960): "Normalizing the Noncentral t and F Distributions", **Annals of Mathematical Statistics**, 31, 1105-1112.

LEHMANN, E.L. (1986): **Testing Statistical Hypotheses**, 2nd. ed. John Wiley and Sons, New York.

MERINGTON, M. and E.S. PEARSON (1958): "An Approximation to the Distribution of Noncentral t". **Biometrika**, 45, 484-491.

ODEH, R.E. and D.B. OWEN (1980): **Tables for Normal Tolerance Limits Sampling Plans and Screening**. Marcel Dekker, New York.

OWEN, D.B. (1968): "A Survey of Properties and Applications of the Noncentral t-Distribution", **Technometrics**, 10, 445-478.

_____ (1985): "Noncentral t-Distribution", In: **Encyclopedia of Statistical Sciences**, 6, pp. 286-290. (Eds. S. Kotz and N.L. Johnson). John Wiley and Sons, New York.

RALSTON, A. and H. WILF (1967): **Mathematical Methods for Digital Computers**. John Wiley and Sons, New York.

RUBEN, H. (1966): "Some New Results on the Distribution of the Sample Correlation Coefficient", **Journal of the Royal Statistical Society**, Series B, 28, 513-525.

SAHAI, H. and M.M. OJEDA (1998): **Approximations to Percentiles of the Noncentral t, χ^2 and F. Distributions**. University of Veracruz Press, Xalapa, Veracruz, México.

SHIBATA, Y. (1981): **Normal Distributions**. Tokyo University Press, Tokyo. (In Japanese).

VAN EEDEN, C. (1961): "Some Approximations to the Percentage Points of the Noncentral t-Distribution", **International Statistical Review**, 29, 4-31.

VANGEL, M.G. (1996): "Confidence Intervals for a Normal Coefficient of Variation", **The American Statistician**, 50, 21-26.

WOLFOWITZ, J. (1946): "Confidence Limits for the Fraction of a Normal Population Which Lies Between Two Given Limits", **Annals of Mathematical Statistics**, 17, 483-488.