A NEW EMBEDDING FOR THE AUGMENTED LAGRANGE METHOD

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ABSTRACT
Several algorithms such as penalti, barrier, Augmented Lagrangean and parametrical approaches are used in the solution of non-linear optimization problems. One of these approach construct of each optimization problem (P) or the results of some iterative algorithms. For this parametrical problem a necessary condition for a good behavior of the continuation or to define jumps is that the parametrical problem is JJT – regular. In this work we propose an embedding for the Augmented lagrangean Method, using the ideas of Bertsekas for this kind of algorithms and we proof that for almost every parameter, fixed the original optimization problem, the constructed parametric problem is JJT – regular. Some numerical examples to illustrate the solution are presented.

Key words: parametrical optimization problem, Augmented Lagrangean Method, JJT-regular, generalized critical points.

RESUMEN
Muchos algoritmos, como penalidad, barrera, lagrangeanos aumentados e embedding uniparamétricas, han aparecido para la solución de problemas de optimización no lineal (P). Este último consiste en la construcción para cada (P), de un problema de optimización uniparamétrica, cuya solución incluirá las soluciones de (P) o resultados de algún algoritmo iterativo. Para este tipo de problemas una condición necesaria para el buen comportamiento del método de continuación y saltos, es que el problema parámetrico sea JJT – regular. En este trabajo, proponemos una inmersión para el método de Lagrangeanos uaumentados, usando las ideas de Bertsekas para este tipo de algoritmos y probamos que, fijo el problema original, puede construirse una embedding tal que el problema paramétrico sea JJT – regular. Se presentan algunos ejemplos numéricos ilustrativos.

Palabras clave: problemas de optimización paramérica, método Lagrangeano Augmentado, JJT-regular, puntos criticos generalizados.

1. INTRODUCTION

There are different methods for solving the well known non linear optimization problem:

$$P = \min\{f(x) \mid x \in M\}$$

$$M = \left\{ x \in \mathbb{R}^n \,\middle|\, \begin{array}{l} h_i(x) = 0, i = 1\ldots m, \\ g_j(x) \geq 0, j = 1,\ldots, s \end{array} \right\} \qquad (1)$$

But as a global minimum is very difficult to obtain, many algorithms that pretend a convergence to it have appeared. Some examples are the barrier and the penalty algorithms. One important case of penalty method is the Augmented Lagrange Methods or Multipliers Method. In this algorithm a simpler optimization problem is constructed which objective function includes the Lagrange function and a quadratic penalty term. This function is an example of a smooth penalty function that is exact.

Since 1980 homotopy ideas for solving the non-linear programming problem has been used. This method construct an uniparametric problem P(t), and try to obtain a solution of the original problem, with a strategy of continuation and jumps on the set of solutions, generalized critical points, of P(t). In order to characterize the different situations Jongen, Jonker and Twilt have considered particular cases of solutions and classified them in five classes. They provides us of necessary conditionsunder which the continuation method may succeed. Some results about how strong is to assume that all the solutions of P(t) are of one of the fve defined classes are already known.

In this work we are going to propose a new immersion that describe the Augmented Lagrange methods. We also analyze its properties: the types of generalized critical points, defined by Jongen Jonker and Twilt, that may appear in P(t), under the assumption that the problem is JJT-regular, and the expected success of the continuation method applied to the parametric problem or perturbations of it. We also display some numeric examples and compare our results with the ones obtained for this method already known using other immersions. In the next section, we are going to describe the principal aspects of uniparametric optimization and the multipliers method. The third section is dedicated to the analysis of some simple properties of the immersions proposed by us. After this we proceed to proof the theorem that assure us that the immersion describe the Augmented Lagrange Method. In the fourth section, we proof theorems that told us how strong is the assumption that the parametric problem is JJT-regular. Finally, the last section is dedicated to the numerical experience, comparing the behavior of this inmersion with others.

2. PRELIMINARY ASPECTS AND NOTATIONS

At first we will define the following optimization problem

$$P = \min\{f(x) \mid x \in M\}$$

$$M = \left\{ x \in \mathbb{R}^n \,\middle|\, \begin{array}{l} h_i(x) = 0,\ i = 1\ldots m, \\ g_j(x) \geq 0,\ j = 1,\ldots,s \end{array} \right\} \tag{2}$$

Now we will present a theoretical background of the topics of parametric optimization that we are going to use in order to solve the problem (2) using the homotopy method. We will consider the problem:

$$P(t) = \min\{f(x,t) \mid x \in M\}$$

$$M = \left\{ x \in \mathbb{R}^n \,\middle|\, \begin{array}{l} h_i(x,t) = 0,\ i = 1\ldots m, \\ g_j(x,t) \geq 0,\ j = 1,\ldots,s \end{array} \right\} \tag{3}$$

It is shortly denoted as $P(t)_{f;H,G} = (f(x,t);\ h_1(x,t),\ldots,h_m(x,t),\ g_1(x,t)\ \ldots g_s(x,t))$. As we are interested in construct a parametric problem that should be related with the optimization problem P(2), the following properties should hold:

Properties 2.1

- P(1) and P are equivalents.
- There is a solution of the problem P(t) $\forall t \in [0,1]$.
- P(0) has an evident solution.

With this properties, we can begin with an easy point at t = 0. As there is a solution $\forall t \in [0,1]$, we can try to use a continuation, if it is possible, on the set of solutions and, if we reach t = 1, a solution of the original problem will be attained. The known definitions of Optimization are easily extended to this case, considering t as a parameter:

Definition 2.1.

Let $x \in M(t)$, (x, t) a feasible point P(t).

- $J_0(x, t) = \{ j : g_j(x, t) = 0\}$ is the active index set of (x, t).
- We will say that the LICQ holds at (x, t) if the following vectors $\{D_x h_i(x, t),\ i = 1\ldots m,\ \{D_x g_j(x, t),\ j \in J_0(x, t)$, are linear independent.
- $L(x,\lambda,\mu,t) = f(x, t) - \sum_{i=1}^{m} \lambda_i h_i(x,t) - \sum_{j \in J_0} \mu_j g_j(x,t)$ is the Lagrange function for P(t), and λ_i, i = 1…m, μ_j, $j \in J_0(x, t)$, are the Lagrange multipliers.

Now we are going to present the definitions of critical point and generalized critical point.

Definition 2.2

$$D_x f(x) = \sum_{i=1}^{m} \lambda_i D_z h_i(x) + \sum_{j\in J_0} \mu_j D_z g_j(x) \tag{4}$$

$$\mu_j \geq 0 \tag{5}$$

(x, t) $\in \Sigma_{crit}$ (i.e. is a critical point) if exist λ_i, i = 1…m, μ_j, j $\in J_0$(x,t) such that (x, λ,μ) satisfies the system (4). If (5) holds then we say it is a stationary point, (x, t) $\in \Sigma_{stat}$.(x, t) $\in \Sigma_{stat}$, (i.e. is a generalized critical point) if this system of vectors is linear dependent

$$\{D_z f(x, t), D_z h_i(x, t), i = 1\ldots m, D_x g_j(x, t), j \in J_0(x, t)\} \tag{6}$$

The solution of P(t) will be understood in this work as a generalized critical points. Actually we are going to deal with the structure of the set Σ_{gc}. In order to have an evident solution at t = 0 it is important to have an easy problem.

As we want to obtain a solution of the problem in t = 1, a continuation on Σ_{gc} is intended. Beginning at t = 0, we make a partition of the interval [0, 1], and using an active index strategy, we try to find from a solution at t = t_k a generalized critical point in t = t_{k+1}. This continuation depends on the structure of the set Σ_{gc}, as told in Jongen **et al**. [7]. That's why they define five class of generalized critical points:

Definition 2.3 $(\bar{x},\bar{y}) \in \sum_{gc}^{1}$, or is a generalized critical point of type 1

1a) LICQ holds.

1b) $J_0 J_0(\bar{x},\bar{t}) = J_+(\bar{\mu})$.

1c) $D_x^2(\bar{x},\bar{t})L(\bar{x},\bar{\lambda},\bar{\mu},\bar{t})\big|_{T_z M(t)}$ is non-singular.

In the second type, $(\bar{x},\bar{t}) \in \sum_{gc}^{2}$, the 1b) condition doesn't hold. The condition 1c) fails at the points in the third class of generalized critical points. In the last two classes the LICQ is not satisfied, Gómez **et al**. [4].

The class $F = \left\{ \big(f(x,t), H(x,t), G(x,t)\big) : \sum_{gc}(f,H,G) \subset \bigcup_{i=1}^{5} \sum_{gc}^{i} \right\}$. It is also said that then the problem P(t) is JJT-regular.

A very important result says that this set is open and dense with the strong topology on $(C^3)^{1+m+s}$ and that for almost every quadratic perturbation of f(x,t), and linear on (H(x,t), G(x,t)), both in x, the parametric problem is in the class F. The results of this kind that we will obtain here are, as in the proved results, respect the Lebesgue's measure.

2.1. Lagrange Multipliers Method

This method constructs an optimization problem with non-negative restrictions and the objective function includes the Lagrange function and a quadratic penalty term. For example, in the problem P, (2), the Lagrange method solve $\forall$c >0, the problem:

$$\min x,\lambda,\mu \bar{F}(x,\lambda,\mu,c) \bar{F}(x,\lambda,\mu,c) = f(x) + \sum_{i=1}^{m} \lambda h_i^2(x) + c\sum_{i=1}^{m} h_i^2(x) + c\sum_{j=1}^{m} \frac{\mu_j^2}{4c^2} + c\sum_{j=1}^{m} \left(\min \left\{ 0, g_j(x) + \tfrac{\mu_j}{2c} \right\} \right)^2$$

and then as in a penalty algorithm, c $\to \infty$. This method obtains the multipliers too, having a dual interpretation. (x,λ,μ) is a saddle point of the Lagrange function.

3. EMBEDDING FOR AUGMENTED LAGRANGIANS

In this section we present a new embedding that construct a parametric problem such that construct a parametric problem such that, when we solve the obtained parametric problem, the obtained solutions should be "equivalent" to the application of the Lagrange Multipliers Embedding to P.

This immersion should have at t = 0 an evident saddle point, the start solution. It holds too that at least in a neighborhood V of t = 0, the continuation method, beginning at t = 0, will calculate, $\forall t \in V$, saddle points of the same characterof those obtained by the application of the original method. If it is possible to reach t = 1 with such a points, the procedure will be equivalent to the application of the Augmented Lagrange method. If not, the "equivalence" holds for some iterations of the classic method. In both cases we will made an study of the singularities in order to know which kind of jump are necessary to obtain a critical point at t = 1. The main diffrence between this parametric point of view and the original algorithm is that the parametric problem can have non-linear constrains in general. The immersion we are going to deal with construct the following parametric problem:

$$\text{PP1} = \min \left\{ \begin{array}{c} tL(y) + (1-t)[y^T A y - y^T y_0] + (v - v_0)^t D(v - v_0) + (w - w_0)^t E(w - w_0) \\ (y, v, w) \in M \end{array} \right\}$$

$$M = \left\{ \begin{array}{ll} (z,x,t) & \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \\ (\lambda,\mu) & \in \mathbb{R}^m \times \mathbb{R}^s \\ (v,w) & \in \mathbb{R}^m \times \mathbb{R}^s \end{array} \left| \begin{array}{cc} th_i(x) + (1-t)(v_i - v_{i0}) = 0 & i = 1...m. \\ t[g_j(x) - z_j^2] + (1-t)(w_j - w_{0j}) = 0 & i = 1...s. \\ \|y\|^2 \le q & \end{array} \right. \right\}$$

where

$Q = (A, D, E, v_0, w_0, q)$, $y = (x, z, \lambda, \mu)$ $A = \begin{pmatrix} A_{xx} & A_{x,\lambda} & A_{x,\mu} \\ A^T_{x,\lambda} & A_{\lambda,\lambda} & A_{\lambda,\mu} \\ A^T_{x,\mu} & A^T_{\lambda,\mu} & A^T_{\mu,\mu} \end{pmatrix}$, A_{zz} - $A_{\lambda,\lambda}$, $A_{\mu,\mu}$, D, E are positive definite,

q positive and A is non singular. In (7) the function L(y) is $L(y) = f(x) - \sum_{i=1}^m \lambda_i h_i(z) - \sum_{j=1}^s \mu[g_j(z) - z_j^2]$.

This immersion was motivated by the ideas presented in Luenberger [9] for the Lagrange Method. One immersion of this kind was already proposed in Dentcheva **et al**. [2], but is used the ideas of Bertsekas and do not convert the inequalities restrictions in equalities. Let's see some properties of immersion PP1 that allow us to say that it describe the method, at least in a neighborhood of 0.

Proposition 3.1 If $q >> 1$. Then:

1. We can obtain an easy saddle point at t = 0 and P(1) is equivalent to P.
2. The LICQ holds for all feasible point $(x, z, \lambda, \mu, v, w, t)$ $t \in [0; 1[$.
3. $\forall t \in [0,1[$, the feasible solution is bounded.
4. There is a global minimum of P(t), $\forall t \in [0,1]$.

Proof

1. It is evident if $||y_0||^2 < p$.
2. It' is enough to analyze the gradients of the active restrictions when the compactification restriction is active. That is: see that this matrix has full rank.

$$H = \begin{pmatrix} D_x h(x) & D_x g(x) & -2x \\ 0 & 0 & -2\lambda \\ 0 & 0 & -2\mu \\ 0 & -2tz & -2z \\ (1-t)I_m & 0 & 0 \\ 0 & (1-t)I_s & 0 \end{pmatrix}.$$

3. It'is sufficient to see that the variables (v, w) are bounded. It's obtained from the fact that $w_j = wo_j + \dfrac{t(gj(x) - x_j^2)}{1-t}$ (analogous for v), (x, z) are bounded and the functions are continuos.

4. If P has an optimal solution x_{opt}, then global minimum $\left(x_{opt}, \sqrt{g(x_{opt})}, 0, 0\ v_0\ w_0\right)$ solves P(1). ■

Remarks 3.2 From this proposition we obtain the three properties included in (2.1), and it is clear that the function involved in the restrictions and the objective function are of the class C^3 if (f; H; G) $\in [C^3]^{1+m+s}$.

Remarks 3.3 The main problem of this embedding is that there is not a compact set that includes M(t) $\forall t \in [0; 1]$.

Corollary 3.4 If the parametric problem P1(t), (7) is such that P(t) $\in$ F, then $\sum_{gc} \subset \bigcup_{i=1}^{3} \sum_{gc}^{i}$.

Proof It follows direct from assertion 2) of the last proposition. ■

For notations we are going to assume that $\zeta(y, v, w, \alpha, \beta, \gamma, t)$ is the hessian matrix of the Lagrange function corresponding to the parametric problem PP1 (7), respect to y. Here the multipliers are ($\alpha \in \mathbb{R}^m$, $\beta \in \mathbb{R}^s$, and $\gamma \in \mathbb{R}$. The last multiplier corresponds to the compactification.

There are two possibilities for the matrix M, if the compactificaction restriction is active or not, (8), (9) respectively.

$$M = \begin{pmatrix} \zeta + 2(1-t)A & 0 & 0 & tD_y^T h_i(x) & tD_y^T t[g_j(x) - z_j^2] & -2y \\ 0 & I_m & 0 & (1-t)I_m & 0 & 0 \\ 0 & 0 & I_s & 0 & (1-t)I_s & 0 \\ tD_y h_i(x) & (1-t)I_m & 0 & 0 & 0 & 0 \\ tD_y[g_j(x) - z_j^2] & 0 & (1-t)I_s & 0 & 0 & 0 \\ -2y^T & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{8}$$

$$M = \begin{pmatrix} \zeta + 2(1-t)A & 0 & 0 & D_y^T th_i(x) & D_y^T t[g_j(x) - z_j^2] \\ 0 & I_m & 0 & (1-t)I_s & 0 \\ 0 & 0 & I_s & 0 & (1-t)I_s \\ tD_y h_i(x) & (1-t)I_m & 0 & 0 & 0 \\ tD_y[g_j(x) - z_j^2] & 0 & (1-t)I_s & 0 & 0 \end{pmatrix} \tag{9}$$

For each case we define $\overline{M}$ as in (10), and (11), corresponding to the same two possibilities. In the first case k is such that $y_k \neq 0$.

$$M = \begin{pmatrix} \zeta_{k,k} + 2(1-t)A_{k,k} & 0 & 0 & D_{y_k}^T th_i(x) & tD_{y_k}^T [g_j(x) - z_j^2] & -2y \\ 0 & I_m & 0 & (1-t)I_m & 0 & 0 \\ 0 & 0 & I_s & 0 & (1-t)I_s & 0 \\ tD_{y_k}^T h_i(x) & (1-t)I_m & 0 & 0 & 0 & 0 \\ tD_{y_k}[g_j(x) - z_j^2] & 0 & (1-t)I_s & 0 & 0 & 0 \\ -2y_k & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

$$\overline{M} = \begin{pmatrix} I_m & 0 & (1-t)I_m & 0 \\ 0 & Is & 0 & (1-t)I_s \\ (1-t)I_m & 0 & 0 & 0 \\ 0 & (1-t)I_s & 0 & 0 \end{pmatrix} \tag{11}$$

Proposition 3.5 We can find a matrix $D \in R^{\frac{n(n+1)}{2}}$, a sub-matrix of $\overline{M}$ such than rank(D) = rank(M) = n – k and $\overline{M}$ is a sub-matrix of D.

Proof In the two cases the matrix $\overline{M}$ has full rank. It is enough to complete the basis. ■

4. MAIN RESULTS

Now we are going to proof how much strong is to assume that the parametric problem described as PP1(7) is in class F.

Theorem 4.1 For the immersion described in (7), for almost every Q = (A, y_0, v_0), its generalized critical points are of type 1, 2 or 3, (here the almost everywhere is taken in the sense of the Lebesgue's measure).

Proof Using the same ideas described in Gómez **et al**. [4], we can write every generalized critical point of the problem as the a solution of the system (12), where J_0 is its active index set, $J_+(\mu) = \{j: \mu_j \neq 0\}$ and D is a symmetric matrix, rank(D) = rank(M) = n – k. That means $\sum_{gc}$ is the finite union of sets described by the following system:

$$\begin{array}{ll}
H(x, \lambda, t, A, x_0) = 0 & \\
M(x, \lambda, t, A) = z, & \left(\text{taken as } M = \begin{pmatrix} B & C \\ C^T & D \end{pmatrix}\right) \quad (12) \\
B = CD^{-1}C^T & \text{(taken in z as a variable)} \\
\gamma = 0 & \text{iff } J_0 \neq \varnothing,\ J_0 = J_+(\mu) =\}
\end{array}$$

where $R \in R^{\frac{kxk}{2}}$, is a sub-matrix of ζ + 2(1 – t)A and depends of the parameter Q and the variables of the system, (Proposition 3.5). Then writing the jacobian of the system (12) respects to variables and parameters, we obtain:

$$\begin{pmatrix}
 & D_y & D_v & D_w & D_a & D_b & D_c & D_{y_0} & D_A & D_g \\
 & \otimes & 0 & 0 & \otimes & \otimes & -2y & -2\tau I_{n+m+2p} & \otimes & 0 \\
 & 0 & I_m & 0 & \tau I_m & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & I_p & 0 & \tau I_m & 0 & 0 & 0 & 0 \\
H = 0 & \otimes & \tau I_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & \otimes & 0 & \tau I_p & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 2y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
M = z & \otimes & 0 & 0 & 0 & 0 & 0 & 0 & \tau I. & I.. \\
B = CD^{-1}C^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I...|\otimes \\
c = 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix} \quad (13)$$

Here $\tau = (1 – t)$, $*$ = (n + m + s)(n + m + s+ 1)/2, $***$ = k(k + 1)/2 y $**$ = (n + 3m + 3s + δ)(n + 3m + 3s + δ +1)/2, $\delta = \begin{Bmatrix} 1 & \text{if } c = 0 \text{ and } J_0 \neq \varnothing \\ 0 & \text{if not} \end{Bmatrix}$.

The matrix described in (13) has full rank. So we can apply the parametric Sard theorem and the rest of the proof follows the same steps of the Theorem 6.18, pp. 121 (Gómez **et al**. [4]). ■

Remarks 4.2 In this case we want to remark that as the set of the symmetric matrixes positive definite is open the Lebesgue´s measure restricted to this set is allowed, and the result holds with the restricted measure too.

Remarks 4.3 Another important problem is to know if the set of functions (f, H, G) $\in C^3_S$, are such that the parametric problem obtained by the application of the immersion PP1(7) are in class F is generic in C^3 with the strong topology. The proof of this result using quadratic perturbation and the parameterized Sard theorem as in theorem 6.25, pp. 137 (Gómez **et al**. [4]) is not valid.

5. NUMERICAL EXAMPLES

In this section we are going to present some examples of optimization problem, to whom the immersion PP1 (7) is applied and the parametric solved by the program PAFO. At first it was necessary to program in FORTRAN the proposed immersion. This program calculates, known the derivatives of the problem at t = 1, the third order derivatives of the parametric problem. The problem at t = 1 is not exactly the original optimization problem, that's why the program was made in such a way that the programmed optimization problems can be used. From a numerical point of view of the PAFO program with such a kind of immersion only small problem can be solved, with at most 5 inequalities restrictions. The same limitations are in the numerical solution of the immersion proposed for this method in Dentcheva **et al**. [2].

One of the chosen numeric examples was taken from Dentcheva **et al**. [2], to whom another immersion for the Augmented Lagrange method is applied. The original problem is:

$$\min(x_1 + 4)^2$$

S.T.

$$g(x) = -0.026573509\, x_1^8 + 0.21150527\, x_1^7 - 0.25753848\, x_1^6 - 1.34579642\, x_1^5 + 2.34222067\, x_1^4 - 0.45664738\, x_1^3 + 2.65029635\, x_1^2 - .091447716\, x_1 + x_3^2 + x_4^2 + x_2^2 + 5 \geq 0\,.$$

The immersion PP1 (7) applied to this problem, construct the following parametric problem:

$$\min\ t\big[(x_1 + 4)^2 + \mu(g(x) - z^2) + (1-t)[\|x - x_0\|^2 + (z - z_0)^2 - (\mu - \mu_0)^2] + (w - w_0)^2\big]$$

S.T.

$$t(g(x) - z^2) + (1-t)(w - w_0) = 0$$

$$\|x - x_0\|^2 + (z - z_0)2 + (\mu - \mu_0)^2 \leq 50$$

This problem was solved for two different initial points:

1. $(x_0, z_0, \mu_0, w_0) = (4, 0, 0, 0, 0, 0, 0, 0)$

2. $(x_0, z_0, \mu_0, w_0) = (0, 0, 0, 0, 0, 0, 0, 0)$

As can be seen at Figure 1, in the first case we can reach successfully the point t = 1. In the other case, Figure 2, is necessary to make some special considerations because there is a multiplier that tends to infinity. In the second example it is impossible to reach t =1 because of the presence of a turning point, point of type 3. We have considered the problem for two different initial points and the same situation was obtained.

$$\text{Min } -x_1$$

$$\text{S.T. } 1 + x_1(1 - x_1)^2 - x_2 \geq 0$$

is constructed the following parametric problem:

$$\min\ -x_1 + \mu(1 + x_1(1 - x_1)^2 - x_2) + (w - w_0)^2$$

$$\text{S.T.}\quad \begin{array}{l} 1 + x_1(1 - x_1)^2 - x_2 + x_3^2 = 0 \\ x_1^2 + x_2^2 + x_3^2 + \mu^2 \leq 50 \end{array}$$

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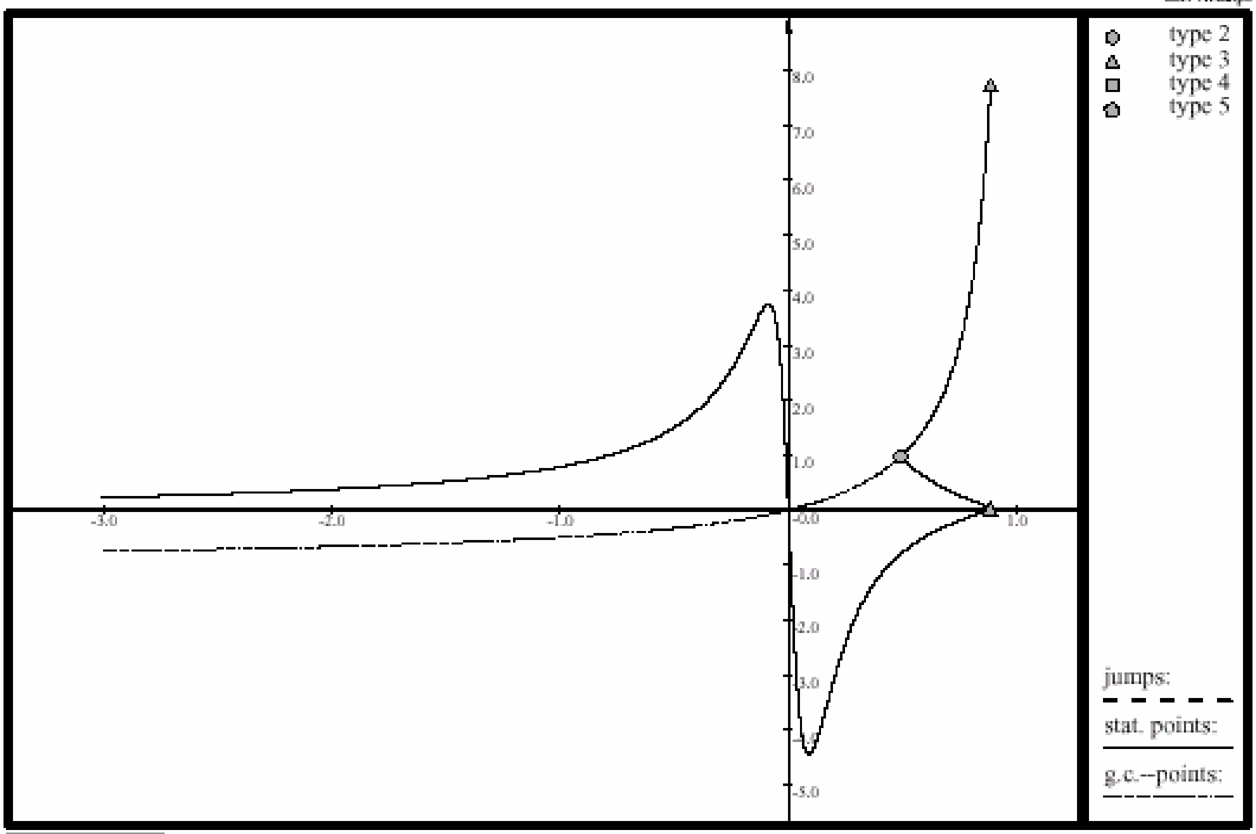
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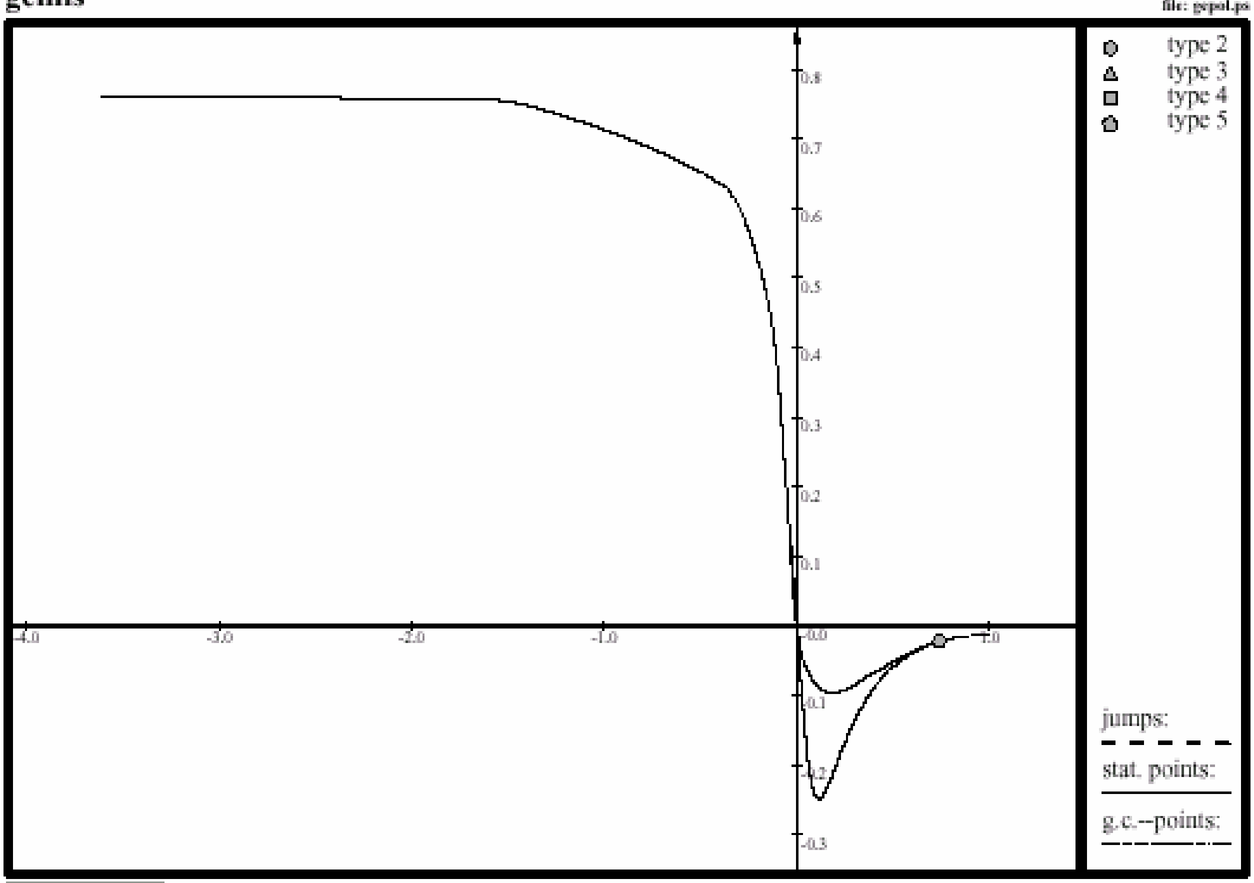
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6. CONCLUSIONS

In this work we have presented an embedding that when the obtained parametric problem is solved, it describes the iterative process of solution of the original problem with the Multipliers Method. As the start solution is a saddle point of type 1, we can affirm that there is a neighborhood such that the obtained critical points are points of type 1 with the same characteristics. This neighborhood can be extended as far as we the critical points of the problem P(t) are of type 1 or non-turning point of type 2. In this neighborhood the equivalence between both algorithms holds. The functions involved in the constructed parametrical problem, are of class C^3 if $(f, h_1 \ldots g_1 \ldots g_m) \in [C^3]^{1+m+s}$, a property that doesn't holds for one of the embeddings proposed in Dentecheva **et al**. [2]. For the embedding P1(t)(7), proposed by us, we have proof that fixed (f; H; G) for almost every parameter, the parametric problem P1(t) (7) will be JJT-regular. That's why we can assume it for numerical needs. Something similar take place for the second embedding proposed in Dentcheva **et al**. [2] and as in that case we can not proof that the set (f; H; G) : Φ_3 (f; H; G) $\in$ F is generic using the Sard Lemma, where : Φ(f; H; G) s the parametrical problem defined at Dentcheva **et al**. [2]. The numerical examples, even in this limited cases, show us that we can not have a PC1 - path that connect a generalized critical point of t = 0, with other at t = 1. We want to remark that the same problem appears in one of the embeddings proposed by Dentcheva **et al**. [2]. We can not even assume that jumping to another component of Σ_{gc} we can obtain the desired solution. Nevertheless this embedding is very important because when the first singularity appears, there is a descending direction on $\sum_{gc}$, even if the singularity is of type 4.

In future work, we can think in the construction of an embedding for the Augmented Lagrangean Method $\Phi:[C^3]^{1+m+s} \rightarrow [C^3]^{1+m1+s1}$ in which $\Phi^{-1}(F)$ would be generic.

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