INFORMATION EXCHANGES
IN COURNOT OLIGOPOLIES

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ABSTRACT
In this work we analyse the profitability of information sharing among Cournot oligopolists receiving private information about a random demand. We model the random demand as a linear demand having an unknown intercept. In this scenario, firms observe private signals about the unknown parameter. We show that if the private signal observed by firms is accuracy enough, information exchange is profitable.

Key words: information exchange, Cournot equilibrium, accuracy effect, intercept demand uncertainty.

MSC: 91B42

RESUMEN
En este trabajo analizamos lo lucrativo de compartir información entre Oligopolistas Cournot que reciben información privada sobre la demanda aleatoria. Nosotros modelamos la demanda aleatoria como una demanda lineal que tiene un intercepto conocido. En este escenario, las firmas observan las señales privadas sobre el parámetro desconocido. Nosotros demostramos que si la señal privada observada por las firmas es suficientemente exacta, el intercambio de información es lucrativo.

Palabras clave: intercambio de información, equilibrio de Cournot, efecto de la exactitud, intercepto, incertidumbre en la demanda.

1. INTRODUCTION

Models of information exchange among oligopolists have assumed that market uncertainty is due to either unknown constant marginal cost for the firms or unknown market demand. There are vast economic literature that deal with both cases of uncertainty. In relation to the uncertainty about market demand, the most important contributions were made in the 80´s. Novshek y Sonnenschein (1982) have study the incentives of Cournot duopolists to share their private information about demand uncertainty. They found that firms would not benefit from sharing their information. Richard N. Clarke (1983); Xavier Vives (1984) confirmed Novshek and Sonnenschein´s results in Cournot oligopoly, but Vives (1984) found that allowing for price competition and differentiated products, exchange information about common demand intercept can increase firm´s profits. Lode Li (1985), showed that Cournot oligopolists producing homogeneous goods would not benefit from exchanging their information about demand uncertainty. Esther Gal-Or (1985), shows that firms will be strictly more profitable when they share their information. Alison J. Kirby (1988) found cases in which firms may have higher profit by sharing their information rather than keeping it private, she considered perfect substitutes but assumed marginal cost to be sufficiently steep.

With respect to the uncertainty about firm´s (constant) marginal cost of production, the most important contributions were made by Esther Gal-Or (1986) and Carl Shapiro (1986). They found that if firms are Cournot competitors producing substitutive products, and the only uncertainty is each firm´s (constant) marginal cost of production, then it will be an equilibrium for the firms to share their private information about their own costs. In the models developed by these authors the absence of information exchange profitability, it does not depended upon the accuracy of firm´s private information. We show that if each firm´s private information is enough accuracy, firms will be interested in share their information, because they increase their profits.

2. THE MODEL

We consider a symmetric duopoly model in which two firms, firm 1 and firm 2 producing identical products face an uncertain market demand. The inverse demand function is given by :

$$P(q^1 + q^2) = \alpha - \beta(q^1 + q^2)$$

where q^i denotes the amount of output produced by firm i, and α and β denote the demand intercept and the slope of market demand respectively. The inverse demand function is interpreted as net of costs. According to this demand function we study the following case:

Case: The uncertainty of market demand comes from the unknown demand intercept (α), then α is the random component and $\beta > 0$, is a positive parameter.

We assume firms have no fixed costs and their marginal cost are constant and equal to c, $c \geq 0$. Before making their output decisions and depending of the case of study, firms observe private signals about α. Firm i´s privately observed signal is denoted by s^i, i = 1, 2. We suppose that firms have common prior beliefs about the unknown random component of the market demand, α. Furthermore, we assume that the private signals received by the firms about α and is conditionally independents given α.

Finally, the above description of the environment is common knowledge among the firms.

3. UNKNOWN DEMAND INTERCEPT

3.1. Cournot Equilibrium and Information Exchange

We use the Bayesian Cournot equilibrium concept to solve the model: each firm chooses its output to maximize its expected profit conditional on its information, given the output strategic of its rival. Let I^i denote the information available to firm i when it chooses its output. Firm i´s expected profit, given I^i is equal to:

$$E\left[\left(P\left(q^i + q^j\right)\right)q^i \middle| I^i\right] = E\left[\left(\alpha - \beta\left(q^i + q^j\right)\right)q^i \middle| I^i\right], \tag{1}$$

where $i \neq j$, i, j = 1,2. The first order condition for profit maximization by firm i is, therefore:

$$E\left[\alpha \middle| I^i\right] = 2\beta q^i\left(I^i\right) + \beta E\left[q^j \middle| I^i\right]. \tag{2}$$

The Cournot equilibrium is given by a pair of outputs strategies, one for each firm, each of which satisfies Eq. (2) for each possible realization of a firm´s information. Given the conditions of this model the Cournot equilibrium is unique and symmetric. Eqs. (1) y (2) yield firm´s i (ex ante) equilibrium expected profit:

$$E\left[\left(P\left(q^i + q^j\right)\right)q^i\right] = E\left[E\left[\left(\alpha - \beta\left(q^i + q^j\right)\right)q^i \middle| I^i\right]\right] = \beta E\left[\left(q^i\right)^2\right]. \tag{3}$$

3.2. The influence of forecast accuracy and profitable information exchange

In this section we built an index to characterize the degree to which information sharing can improve a firm´s forecast of α. Let´s denote:

$e_{nc} = E\left[\alpha \middle| s^1\right] - \alpha$, denotes a firm´s (random) forecast error when firms do not exchange information and,

$e_c = E\left[\alpha \middle| s^1, s^2\right] - \alpha$, denotes the forecast error when do they do. Letting var(e) denote the variance of a random forecast error e, we define the index G by :

$$G = \left(var(e_{nc}) - var(e_c)\right)/var(e_{nc}) \text{ or alternatively } G = 1 - var(e_c)/var(e_{nc}).$$

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Index G measures the fraction of mean-squared forecasting error that can be eliminated by exchanging information; in other words, when index G is close to 1, mean-squared forecast error when firms share their information is much lesser than when they don´t do, then index G shows when firms would be interested in share their information. In these cases, the second signal essentially removes all residual uncertainty about demand. Informally, we view values of G close to 1 as akin to a sufficiently condition for profitability of information exchange. When G is close to 0, mean-squared forecast error when firms share their information is similar to the mean-squared forecast error when firms don´t share their private information, then firms don´t find profitable it, because there are not accuracy gains to information exchange.

We make some assumptions about market conditions and we investigate how variations in the quality of firm´s private information make their influence in Index G and therefore in the profitability of information exchange.

We assume firms know that market demand can be high or low, i.e., firms know α takes on one of two values α_b y α_a $(0 < \alpha_b < \alpha_a)$, indicating high demand and low demand respectively. In addition, we assume these parameter values are such that realized outputs and prices implied by Eq. (2) are nonnegative.

Distributions of the demand intercept and signals are specified below:

$\Pr(\alpha_b)$ = $\Pr(\alpha_a)$ = 1/2. High demand and low demand have the same likelihood.

Firm´s i private signal s^i, takes on one of three values:

s_b^i, s_n^i y s_a^i, i = 1, 2, indicating that firms can receive a low, medium or normal and high signal about market demand.

The conditional distributions of signal s^i, given α, are as follows:

$\Pr\left(s_b^i \mid \alpha_b\right) = \Pr\left(s_a^i \mid \alpha_a\right) = \sigma$ y $\Pr\left(s_n^i \mid \alpha_b\right) = \Pr\left(s_n^i \mid \alpha_a\right) = 1 - \sigma$ where $\sigma \in (0,1)$.

Thus, a firm´s realized signal either perfectly identifies the demand state (if s_b or s_a), or it provides no information if the signal is (s_n).

As the parameter σ increases from 0 to 1, the signal becomes increasingly informative.

Let $\overline{\alpha} = \dfrac{(\alpha_b + \alpha_a)}{2}$ denote the mean of the demand intercept α, and let $var(\alpha) = \dfrac{(\alpha_a - \alpha_b)^2}{4}$ denote the prior variance of α.

We solve Eq. (2) to derive the equilibrium strategies, getting the amount of output that each firm offer in the case of firms share their private information about market demand (case a) and in the case of firms do not share it (case b). The firm´s equilibrium expected profit is π_{nc}, when firms do not share their private information and π_c, when firms do share it. The expression for π_c and π_{nc} are obtained by substituting of the equilibrium outputs and appropriate probabilities into Eq. (3).

Case a (firms do not share their private information):

Let $I^i = \{s^i\}$, denote the information available to firm i when it chooses its output. At the equilibrium, the conditions of Eq. (2), from firm 1´s perspective, may be write as follows:

$$(1/\beta)\begin{pmatrix} E\left[\alpha\middle|s_b^1\right] \\ E\left[\alpha\middle|s_n^1\right] \\ E\left[\alpha\middle|s_a^1\right] \end{pmatrix} = \begin{pmatrix} 2 + \Pr\left(s_b^2\middle|s_b^1\right)\text{.......}\Pr\left(s_n^2\middle|s_b^1\right)\text{........}\Pr\left(s_a^2\middle|s_b^1\right) \\ \Pr\left(s_b^2\middle|s_n^1\right)\text{........}2 + \Pr\left(s_n^2\middle|s_n^1\right)\text{........}\Pr\left(s_a^2\middle|s_n^1\right) \\ \Pr\left(s_b^2\middle|s_a^1\right)\text{............}\Pr\left(s_n^2\middle|s_a^1\right)\text{.......}2 + \Pr\left(s_a^2\middle|s_n^1\right) \end{pmatrix} * \begin{pmatrix} q_b \\ q_n \\ q_a \end{pmatrix}$$

For the particular probabilities in this example, these conditions become:

$$(1/\beta)\begin{pmatrix} \alpha_b \\ \dfrac{\alpha_b + \alpha_a}{2} \\ \alpha_a \end{pmatrix} * \begin{pmatrix} 2+\sigma............1-\sigma............0 \\ \dfrac{\sigma}{2}...............3-\sigma.............\dfrac{\sigma}{2} \\ 0................1-\sigma.............2+\sigma \end{pmatrix} * \begin{pmatrix} q_b \\ q_n \\ q_a \end{pmatrix}$$

Solving this equation, we find the equilibrium outputs are:

$q_b^c = \dfrac{(5+\sigma)\alpha_b - (1-\sigma)\alpha_a}{6(2+\sigma)\beta}$. This expression represent the amount of Cournot produced by each firm when the signal received is low.

$q_n^c = \dfrac{\alpha_b + \alpha_a}{6\beta}$. This expression represent the amount of Cournot produced by each firm when the signal received is normal.

$q_a^c = \dfrac{(5+\sigma)\alpha_a - (1-\sigma)\alpha_b}{6(2+\sigma)\beta}$. This expression represent the amount of Cournot produced by each firm when the signal received is high.

Case b (firms share their information):

Let $I^i = \{s^1, s^2\}$, denote the information available to firm i when it chooses its output:

In this case, the equilibrium condition Eq. (2) is simply:

$$q\left(s^1, s^2\right) = E\left[\alpha \,\middle|\, s^1, s^2\right] \,/\, 3\beta$$

If either firm observes s_b or s_a, then the firms know for sure the value of α. If both firms observe s_n, then they will have gained no information about demand and will continue to assign probability ½ to each possible value of α. Let the Cournot equilibrium with information sharing (q_b^c, q_n^c, q_a^c), where q_b^c denotes each firm´s output when at least one firm has observed s_b, q_a^c denotes each firm´s output when at least one firm has observed s_a, and q_n^c denotes each firm´s output when at least one firm has observed s_n. Then it is immediate from Eq. (2) that:

$$q_b^c = \frac{\alpha_b}{3\beta}$$

$$q_n^c = \frac{\alpha_b + \alpha_a}{6\beta}$$

$$q_a^c = \frac{\alpha_a}{3\beta}$$

Once we derive the equilibrium strategies from each case (sharing and non sharing), then from Eq. (3) we find each firm´s equilibrium expected profit in both cases,

Expected profit when firms do not share their information:
$\pi_{nc} = \dfrac{1}{9\beta}\left\{\overline{\alpha}^2 + \dfrac{9\sigma}{(2+\sigma)^2} * var(\alpha)\right\}$

Expected profit when firms share their information:
$\pi_c = \dfrac{1}{9\beta}\left\{\overline{\alpha}^2 + \sigma(2-\sigma) * var(\alpha)\right\}$

Comparison of π_c and π_{nc} shows information exchange is profitable when private information is sufficiently accurate, i.e., there exists a $\sigma = \sigma^*$ such that $\pi_c > \pi_{nc}$ if and only if $\sigma > \sigma^*$.

The potential for profitable information sharing can be understood in terms of the index G and the effect of sharing on posterior beliefs. A firm´s forecast of demand is not perfectly accurate only if the signal(s) it observes is (are) equal to s_n. In this case, the firm´s posterior variance for α is equal to the prior variance of var(α). Without sharing, the chance that a firm does not know demand, given its signal, is $1 - \sigma$, so the expected posterior variance is $(1-\sigma)$var(α). With information sharing, the chance that there is any ex post forecast error is equal to $(1-\sigma)^2$, so the expected posterior variance of α is equal to $(1-\sigma)^2$var(α). Thus, as σ approaches 1, the expected forecast error goes to zero much faster when firms share their information than when they do not. Indeed, from the above calculations of expected posterior variance, it follows that G = σ:

$$G = \frac{\left(\mathrm{var}(e_{nc}) - \mathrm{var}(e_c)\right)}{\mathrm{var}(e_{nc})} = \frac{(1-\sigma)\mathrm{var}(\alpha) - (1-\sigma)^2 \mathrm{var}(\alpha)}{(1-\sigma)\mathrm{var}(\alpha)} = \sigma$$

Thus the improvement in forecasting accuracy resulting from information exchange run the gamut from essentially no improvement, when , $\sigma \approx 0$, to elimination of virtually all error, when $\sigma \approx 1$. Finally we can conclude that: *information sharing is profitable if and only if the accuracy gains as measured by G are sufficiently large*.

5. CONCLUSIONS

We have shown, in a simple linear Cournot model with uncertainty of demand, given by uncertain in demand intercept, that firms may have greater profit when sharing their information rather than keeping it private when their signals are sufficiently accurate.

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