TRANSIENT AND STEADY STATE SOLUTION OF A SINGLE SERVER QUEUE WITH MODIFIED BERNOULLI SCHEDULE VACATIONS BASED ON EXHAUSTIVE SERVICE AND A SINGLE VACATION POLICY

Kailash C. Madan[1] and Adel Z. Abu Al-Rub, Department of Statistics, Faculty of Science Yarmouk University IRBID, JORDAN

ABSTRACT

We analyze a single server queue with modified Bernoulli server vacations based on exhaustive service. Unlike other vacation policies, we assume that only at the completion of service of the last customer in the system, the server has the option to take a vacation or to remain idle in the system waiting for the next customer to arrive. The service times of the customers have been assumed to be exponential and vacations are phase type exponential. We obtain explicit steady state results for the probability generating functions of the queue length, the expected number of customers in the queue and the expected waiting time of the customer. Some earlier known results of the single server have been derived as a particular case.

Key words: Bernoulli schedule server vacations, exhaustive service, phase type vacations, steady state, stability condition.

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RESUMEN

Analizamos una cola servidor simple con vacaciones Bernoulli modificada basada en un servicio exhaustivo. En forma diferente a otras políticas de vacaciones, asumimos que el servidor tiene la opción de tomar vacaciones o de continuar vacante en el sistema esperando el próximo cliente, solamente al completar el servicio del último cliente. Se asume que los tiempos de servicio de los clientes son exponenciales y las vacaciones son del tipo exponencial fásico. Obtenemos resultados explícitos para el estado estable para las funciones generatrices de probabilidad del largo de la cola, la esperanza del número de clientes en cola y la esperanza del tiempo de espera del cliente. Algunos resultados conocidos sobre servidores simples han sido derivados para este caso particular.

1. INTRODUCTION

In recent years, vacation queues has emerged as an important area of queueing theory. A number of researchers including Levy and Yechiali [1976], Fuhrman [1981], Doshi [1986, 1990], Keilson and Servi [1986], Baba [1986], Cramer [1989], Borthakur and Chaudhury [1997], Madan [1999], Madan and Al-Saleh 2001], Choi and Park [1990], Takagi [1991, 1992], Rosenberg and Yechiali [1993] and Choudhury [2000] and many others have studied vacation queues with different vacation policies with single or multiple server vacations. The different vacation policies include Bernoulli schedules, exhaustive service, generalized vacations, among others. In the present paper, we study a single server queue with modified server vacations based on exhaustive service, which means that the server must complete service of all customers present in the system before exercising his option to take a vacation. In other papers found in literature on this type of queues, the server must take a vacation after completing service of the last customer. However, in our model, the server may or may not take a vacation even after serving the last customer present in the system. Further, it may be noted that customers who arrive during the vacation period of the server have to wait in the queue for the server to be back. Under multiple vacation policy, it is often assumed that on returning back from vacation, if the server finds the system empty then he takes another vacation. But unlike this assumption of repeated vacations, we assume that whenever the server takes a vacation, it is always a single vacation.

E-mail: [1]kailashmadan@hotmail.com

The single server queueing system without server vacations is widely found in queueing literature, see Bunday [1986] and Kashyap and Chaudhry [1988], to mention a few. The mathematical model of our study is briefly described by the following assumptions:

2. ASSUMPTIONS UNDERLYING THE MODEL

The following assumptions describe the mathematical model:

Customers arrive at the system one by one in a Poisson process with a mean arrival rate λ (> 0). There is a single server who provides one by one service to customers on a first come, first-serverd basis. The service times are assumed to be exponential with mean service time $1/\mu$. The server does not have an option to take a vacation at every service completion like some other models studied by Madan and Al-Saleh [2001] and Madan [1999]. We assume that the server can take a vacation only when he has served the last customer present at the system. At such an instant when the server becomes idle he may take a vacation with probability p or he may continue to stay idle in the system with probability 1 –p and wait for the next customer to arrive. A queue with such a policy is called a vacation queue based on exhaustive service. The vacation time of the server is exponential with mean $1/\beta$, $\beta > 0$. It is further assumed that whenever the server takes a vacation, it is always a single vacation. Finally, we assume that inter-arrival times of customers, the service times of the customers and the vacation times of the server are all independent of each other.

3. DEFINITIONS, NOTATIONS AND THE TIME-DEPENDENT EQUATIONS GOVERNING THE SYSTEM

We define

$W_n(t)$: probability that at time t, the server is providing service and there are $n \geq 0$ customers in the queue excluding one in service

$V_n(t)$: Probability that at time t, the server is on vacation and there are $n \geq 0$ customers in the queue.

$Q(t)$: Probability that at time t, there is no customer in the system and the server is idle but available in the system.

In addition, we define the following probability generating functions (PGFs):

$$W(z,t) = \sum_{n=0}^{\infty} W_n(t) z^n,\ V(z,t) = \sum_{n=0}^{\infty} V_n(t) z^n,\ |z| \leq 1. \tag{3.1}$$

Then, connecting states of the system at time t with those at time t+δt we have the following time–dependent forward system equations:

$$\frac{d}{dt} W_n(t) + (\lambda + \mu) W_n(t) = \lambda W_{n-1}(t) + \mu W_{n+1}(t) + \beta V_{n+1}(t),\ n \geq 1, \tag{3.2}$$

$$\frac{d}{dt} W_0(t) + (\lambda + \mu) W_0(t) = \lambda Q(t) + \mu W_1(t) + \beta V_1(t), \tag{3.3}$$

$$\frac{d}{dt} V_n(t) + (\lambda + \beta) V_n(t) = \lambda V_{n-1}(t),\ n \geq 1, \tag{3.4}$$

$$\frac{d}{dt} V_0(t) + (\lambda + \beta) V_0(t) = p\mu W_0(t) \tag{3.5}$$

$$\frac{d}{dt} Q(t) + \lambda Q(t) = \mu(1-p) W_0(t) + \beta V_0(t). \tag{3.6}$$

Next, we assume that initially the system starts when there is no customer in the system and the server is idle but available in the system so that the initial conditions are given by

$$Q(0) = 1,\ W_n(t) = 0,\ n \geq 0,\ V_n(t) = 0,\ n \geq 0. \tag{3.7}$$

Let $W_n^*(s)$, $V_n^*(s)$ and $Q^*(s)$ respectively denote the Laplace transform (LT) of $W_n(t)$, $V_n(t)$ and Q(t) and let W*(z,s) and V*(z,s) denote the LTs of the PGFs W(z,t) and V(z, t) respectively and are given by

$$W^*(z,s)=\sum_{n=0}^{\infty}W_n^*(s)z^n,\ V^*(z,s)=\sum_{n=0}^{\infty}V_n^*(s)z^n,\ |z|\leq 1. \tag{3.8}$$

Then taking LTs of equations (3.2) to (3.6) and utilizing the initial conditions (3.7), we obtain

$$(s+\lambda+\mu)W_n^*(s)=\lambda W_{n-1}^*(s)+\mu W_{n+1}^*(s)+\beta V_{n+1}^*(s),\ n\geq 1, \tag{3.9}$$

$$(s+\lambda+\mu)W_0^*(s)=\lambda Q^*(s)+\mu W_1^*(s)+\beta V_1^*(s), \tag{3.10}$$

$$(s+\lambda+\beta)V_n^*(s)=\lambda V_{n-1}^*(s),\quad n\geq 1, \tag{3.11}$$

$$(s+\lambda+\beta)V_0^*(s)=p\mu W_0^*(s), \tag{3.12}$$

$$(s+\lambda)Q^*(s)=\mu(1-p)W_0^*(s)+\beta V_0^*(s)+1. \tag{3.13}$$

Now, we multiply equations (3.9) and (3.10) by suitable powers of z, take summation over all possible values of n , add the two results, use (3.8) and simplify. We thus obtain

$$W^*(s,z)=\frac{\beta V^*(s,z)-\beta V_0^*(s)-\mu W_0^*(s)+\lambda zQ^*(s)}{[(s+\lambda+\mu-\lambda z)z-\mu]}. \tag{3.14}$$

A similar operation on equations (3.11) and (3.12) yields

$$V^*(s,z)=\frac{p\mu W_0^*(s)}{(s+\lambda-\lambda z+\beta)}\ . \tag{3.15}$$

Then we substitute for V*(s,z) from (3.15) into (3.14) and simplify, and have

$$W^*(s,z)=\frac{\left(\dfrac{\beta p\mu}{s+\lambda-\lambda z+p}-\mu\right)\ W_0^*(s)-\beta V_0^*(s)+\lambda zQ^*(s)}{[(s+\lambda+\mu-\lambda z)z-\mu]}. \tag{3.16}$$

Now, consider the denominator of the right side of (2.15). It has two zeros z_1 and z_2 such that $|z_1|\leq 1$ and $|z_2|>1$.

For $|z_1|\leq 1$, the numerator of the right side of (3.16) must vanish, giving us

$$\left(\frac{\beta p\mu}{s+\lambda-\lambda z_1+\beta}-\mu\right)W_0^*(s)-\beta V_0^*(s)+\lambda z_1Q^*(s)=0\ . \tag{3.17}$$

The three unknowns Q*(s), V_0*(s) and W_0*(s), can be determined by solving the three equations (3.12), (3.13) and (3.17). Thus the LTs of the PGF's $V^*(z,s)$ in (3.15) and $W^*(z,s)$ in (3.16) can be completely determined.

4. STEADY STATE SOLUTION

If we assume that the steady state exists, then we let $\lim_{t\to\infty}W_n(t)=W_n$,$\forall_n$, $\lim_{t\to\infty}$ $V_n(t)$ = V_n, $\forall_n$ and $\lim_{t\to\infty}Q(t)=Q$ denote the steady state probabilities corresponding to $W_n(t)$, $V_n(t)$ and Q(t). Further, let W (z), V(z) denote the steady state PGFs corresponding to W(t,z) and V(t,z) respectively .

Now we shall use the well-known property of the LT

$$\lim_{s\to 0} sf^*(s) = \lim_{t\to\infty} f(t). \tag{4.1}$$

Then from (3.16), we have

$$W(z) = \lim_{s\to 0} sW^*(s,z)$$

$$= \frac{\left(\frac{\beta p\mu}{s+\lambda-\lambda z+\beta}-\mu\right)\lim_{s\to 0} sW_0^*(s)-\beta\lim_{s\to 0} sV_0^*(s)+\lambda z\lim_{s\to 0} sQ^*(s)}{\lim_{s\to 0}\left[(s+\lambda+\mu-\lambda z)z-\mu\right]}$$

$$= \frac{\left(\frac{\beta p\mu}{\lambda-\lambda z+\beta}-\mu\right)W_0-\beta V_0+\lambda zQ}{(\lambda+\mu-\lambda z)z-\mu}. \tag{4.2}$$

Similarly from (3.15), we have

$$V(z) = \lim_{s\to 0} V^*(s,z)$$

$$= \frac{p\mu\lim_{s\to 0} sW_0^*(s)}{\lim_{s\to 0}(s+\lambda-\lambda z+\beta)} = \frac{p\mu W_0}{\lambda-\lambda z+\beta}. \tag{4.3}$$

Now, for the steady state, equation (3.13) becomes

$$\lambda Q = \mu(1-p)W_0 + \beta V_0. \tag{4.4}$$

Then, using (4.4) in (4.2), it is easy to see that W(z) in (4.2) is indeterminate of the zero/zero form at z = 1, so that applying L' Hopital's rule, (4.2) yields

$$W(1) = \lim_{Z\to 1} W(z)$$

$$= \frac{\lambda p\mu W_0+\lambda\beta Q}{\beta(\mu-\lambda)}. \tag{4.5}$$

Similarly, (4.3) yields

$$V(1) = \frac{p\mu W_0}{\beta}. \tag{4.6}$$

Now we shall apply the normalizing condition

$$W(1) + V(1) + Q = 1. \tag{4.7}$$

Using (4.5) and (4.6) in (4.7) we have on simplifying

$$\lambda p\mu W_0 + \lambda\beta Q + (\mu-\lambda)p\mu\, W_0 + Q\beta(\mu-\lambda) = \beta(\mu-\lambda),$$

which again simplifies to

$$\mu^2 pW_0 + \beta\mu Q = \beta(\mu-\lambda). \tag{4.8}$$

Now, for the steady state, equation (3.12) becomes

$$(\lambda+\beta)V_0 = p\mu W_0\,,$$

which yields

$$V_0 = \frac{p\mu W_0}{\lambda+\beta}\,. \tag{4.9}$$

Using this value of V_0 from (4.9) into (4.4), we obtain on simplifying

$$Q = \frac{(\mu(1-p)(\lambda+\beta)+\beta p\mu)W_0}{\lambda(\lambda+\beta)}\,. \tag{4.10}$$

Again, utilizing the value of Q from (4.10) into (4.8). we have

$$\mu^2 pW_0 + \beta\mu\left[\frac{(\mu(1-p)(\lambda+\beta+\beta p\mu)W_0}{\lambda(\lambda+\beta)}\right] = \beta(\mu-\lambda)\,, \tag{4.11}$$

which gives

$$W_0 = \frac{\lambda\beta(\mu-\lambda)(\lambda+\beta)}{\mu^2 p\lambda(\lambda+\beta)+\mu\beta\left[\mu(1-p)(\lambda+\beta)+\mu p\beta\right]},\quad \lambda<\mu\,. \tag{4.12}$$

Further, using (4.12) in to (4.9) and (4.10) we obtain

$$V_0 = \frac{p\beta\mu\lambda(\mu-\lambda)}{\mu^2 p\lambda(\lambda+\beta)+\mu\beta\left[\mu(1-p)(\lambda+\beta)+\mu p\beta\right]}\,. \tag{4.13}$$

$$Q = \frac{(\mu(1-p)(\lambda+\beta)+\beta p\mu)(\lambda\beta(\mu-\lambda))}{\lambda\,\left[\mu^2 p\lambda(\lambda+\beta)+\mu\beta\,\left[\mu(1-p)(\lambda+\beta)+\mu p\beta\right]\right]}\,. \tag{4.14}$$

We note that (4.13) gives the steady state probability that the server is on vacation and there are no customers in the queue and (4.14) gives the steady state probability that the system is empty and the server is idle but available in the system.

The utilization factor ρ being the proportion of time the server is busy is given by W(1) found in (4.5). Thus utilizing the value of W_0 from (4.12) and the value of Q from (4.14), equation (4.5), on simplifying, yields

$$\rho = \frac{\lambda(\lambda+\beta)\left[p\lambda+(1-p)\beta\right]+p\lambda^2\beta^2}{\mu p\lambda(\lambda+\beta)+\beta\,\left[\mu(1-p)(\lambda+\beta)+p\mu\beta\right]}\,. \tag{4.15}$$

Finally, utilizing the values of W_0, V_0, and Q from (4.12), (4.13) and (4.14) into (4.2) we obtain on simplifying

$$W(z) = \frac{(\mu-\lambda)\left\langle \left(\dfrac{p\mu\beta}{\lambda-\lambda z+\beta}-\mu\right)\lambda\beta(\lambda+\beta)-p\mu\lambda\beta^2 + \lambda\beta z\left[\mu(1-p)(\lambda+\beta)+p\mu\beta\right]\right\rangle}{\left[(\lambda-\lambda z+\mu)z-\mu\right]\;\left[p\lambda\mu^2(\lambda+\beta)+\mu\beta\{\mu(1-p)(\lambda+\beta)+p\mu\beta\}\right]},\ \lambda<\mu\,. \tag{4.16}$$

Next, substituting the value of W_0 from (4.12) into (4.3), we have

$$V(z) = \frac{p\mu[\lambda\beta(\mu-\lambda)(\lambda+\beta)]}{(\lambda-\lambda z+\beta)\ [\mu^2 p\lambda(\lambda+\beta)+\mu\beta[\mu(1-p)(\lambda+\beta)+\mu p\beta]]}, \ \lambda > \mu. \tag{4.17}$$

Now, let $P_Q(z) = W(z) + V(z)$ denote the PGF at a random epoch of the queue size distribution irrespective of the state of the server. Then adding (4.16) and (4.17), we get

$$PQ(z) = \frac{(\mu-\lambda)\left\langle \begin{array}{c} \left(\dfrac{p\mu\beta}{\lambda-\lambda z+\beta}-\mu\right)\ \lambda\beta(\lambda+\beta)-p\mu\lambda\beta^2 \\ +\lambda\beta z\ [\mu(1-p)(\lambda+\beta)+p\mu\beta] \end{array} \right\rangle}{[(\lambda-\lambda z+\mu)z-\mu]\ [p\lambda\mu^2(\lambda+\beta)+\mu\beta\{\mu(1-p)(\lambda+\beta)+p\mu\beta\}]}$$

$$+ \frac{p\mu[\lambda\beta(\mu-\lambda)(\lambda+\beta)]}{(\lambda-\lambda z+\beta)\ [\mu^2 p\lambda(\lambda+\beta)+\mu\beta[\mu(1-p)(\lambda+\beta)+\mu p\beta]]}, \ \lambda < \mu. \tag{4.18}$$

We further note that when there are no server vacations, then for p = 0, (4.13) yields $V_0 = 0$, as it should be and from (4.12), we get

$$W_0 = \frac{\lambda}{\mu}.\frac{(\mu-\lambda)}{\mu} = \frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right), \tag{4.19}$$

$$Q = 1-\frac{\lambda}{\mu}, \qquad \frac{\lambda}{\mu} < 1. \tag{4.20}$$

The results in (4.19), (4.20) are the known results of the ordinary M/M/1 queueing system.

4.5.1. The Steady State Average Number of Customers in the Queue and the System

Let L_w, L_v respectively be the average number of customer in the queue when the server is working and when the server is on vacation. Then $L_q = L_v + L_w$ is the mean number customers in the queue irrespective of the state of the server. Then using (4.17) we have

$$L_v = \frac{d}{dZ} V(z)\Big|_{Z=1} = \frac{d}{dz}\left(\frac{p\mu W_0}{\lambda-\lambda z+\beta}\right)\Bigg|_{z=1} = \frac{\lambda p\mu W_0}{\beta^2}. \tag{4.21}$$

Using (4.12), (4.21) can be further simplified to

$$L_v = \frac{p\lambda^2(\mu-\lambda)(\lambda+\beta)}{\mu p\lambda\beta(\lambda+\beta)+\beta^2\ [\mu(1-p)(\lambda+\beta)+\mu\ p\beta]}, \ \lambda < \mu. \tag{4.22}$$

Now we shall use W(z) found in (4.2) to find L_w. However, we note that at z = 1, W(z) is indeterminate of the zero/zero form. Therefore we let $W(z) = \frac{N(z)}{D(z)}$, where N(z), D(z) are the numerator and the denominator of the right hand side of (4.2). Then using L' Hopital's rule twice we obtain

$$L_W = \lim_{z\to 1} \frac{D'(z)\ N''(z) - N'(z)\ D''(z)}{2(D'(z))^2} = \frac{D'(1)\ N''(1) - N'(1)\ D''(1)}{2(D'(1))^2}, \tag{4.23}$$

where primes stand for the derivatives with respect to z at z = 1. Carrying out the derivatives of N(z) and D(z) at z = 1, substituting in (4.23) and simplifying we obtain

$$L(w) = \frac{(\mu-\lambda)\left(\frac{2p\mu\lambda^3}{\beta}\right)\ (\lambda+\beta)+2\lambda^2\mu\ (p\lambda^2+\lambda\beta+\beta^2)}{2(\mu-\lambda)^2}\quad , \lambda<\mu . \tag{4.24}$$

where Q has been found in (4.14).

Then finally on adding (4.22) and (4.24), we get

$$L_Q = L_W + L_V$$

$$= \frac{(\mu-\lambda)\ \left(\frac{2p\mu\lambda^3}{\beta}\right)\ (\lambda+\beta)+2\lambda^2\mu(p\lambda^2+\lambda\beta+\beta^2)}{2(\mu-\lambda)^2}$$

$$+ \frac{p\lambda^2(\mu-\lambda)(\lambda+\beta)}{\mu\, p\lambda\, \beta(\lambda+\beta)+\beta^2\left[\mu(1-p)(\lambda+\beta)+\mu p\beta\right]},\ \lambda<\mu . \tag{4.25}$$

Further, let L denote the average number of customers in the system.

Then adding (4.25) and (4.15), we obtain

$$L = L_q + \rho$$

$$= \frac{(\mu-\lambda)\left(\frac{2p\mu\lambda^3}{\beta}\right)\ (\lambda+\beta)+2\lambda^2\mu\left(p\lambda^2+\lambda\beta+\beta^2\right)}{2(\mu-\lambda)^2}$$

$$+ \frac{p\lambda^2(\mu-\lambda)(\lambda+\beta)}{\mu p\lambda\beta(\lambda+\beta)+\beta^2\left[\mu(1-p)(\lambda+\beta)+\mu p\beta\right]}$$

$$+ \frac{\lambda(\lambda+\beta)\ \left[p\lambda+(1-p)\beta\right]+p\lambda^2\beta^2}{\mu p\lambda(\lambda+\beta)+\beta\left[\mu(1-p)(\lambda+\beta)+p\mu\beta\right]},\ \lambda<\mu . \tag{4.26}$$

Further note that when p = 0, we get

$$L_Q = \frac{\lambda^2}{\mu(\mu-\lambda)};\quad L = \frac{\lambda}{\mu-\lambda} \tag{4.27}$$

Note that the results in (4.27) are know results of the ordinary M/M/1 queueing system.

4.5.2. Average Waiting Time in the Queue and the system

Let W_q and W denote the average waiting time in the queue and the system respectively. Then using the values of L and L_q obtained above we get

$$Wq = \frac{Lq}{\lambda},\qquad W = \frac{L}{\lambda} \tag{4.28}$$

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REFERENCES

BABA, Y. (1986): "On the $M^X/G/1$ queue with vacation time", **Operations Research Letters**, 5, 93-98.

BORTHAKUR, A. and G. CHAUDHURY (1997): "On a batch arrival Poisson queue with generalized vacation", **Sankhya Ser. B**, 59, 369-383.

BUNDAY, B.D. (1986): **Basic Queueing Theory**, Edward Arnold, Australia.

CHAUDHURY, G. (2000): "An $M^X/G/1$ queueing system with a set up period and a vacation period", **Queueing Systems**, 36, 23-38.

CHOI, B.D. and K.K. PARK (1990): "The M/G/1 retrial queue with Bernoulli schedule", **Queueing Systems**, 7, 219-228.

CRAMER, M. (1989): "Stationary distributions in a queueing system with vacation times and limited service", **Queueing Systems** , 4(1), 57-68.

DOSHI, B.T. (1986): "Queuing systems with vacations- a survey", **Queueing Systems**, 1, 29-66.

DOSHI, B.T. (1990): "Conditional and unconditional distributions for M/G/1 type queues with server vacation", **Queueing Systems**, 7, 229-252.

FUHRMAN, S. (1981): "A note on the M/G/1 queue with server vacations", **Operations Research**, 31, 1368.

KASHYAP, B.R.K. and M.L. CHAUDHRY (1988): "An Introduction to Queueing Theory", **A & A Publications**, Kingston, Ontario, Canada.

KEILSON, J. and L.D. SERVI (1986): "Oscillating random walk models for GI/G/1 vacation systems with Bernoulli schedules", **Journal of Applied Probability**, 23, 790-802.

LEVI, Y. and U. YECHILAI (1976): "An M/M/s queue with servers' vacations", **Infor**, 14(2), 153-163.

MADAN, K.C. (1999): "An M/G/1 queue with optional deterministic server vacations", **Metron LVII** (3-4), 83-95.

___________ (2002): "Balking phenomenon in the $M^X/G/1$ queue", **Korean J. Statistical Society**, 31(4), 1-17.

___________ and M.F. SALEH (2001): "On single server vacation queues with deterministic service or deterministic vacations", **Calcutta Statistical Association Bulletin**, 51, (203-204), 225-241.

___________ and M.F. SALEH (2001): "On M/D/1 queue with deterministic server vacations, **Systems Science**, 27 (2), 107-118.

MEDHI, J. (1982): **Stochastic Processes**, Wiley, Eastern.

ROSENBERG, E. and U. YECHIALI (1993): "The $M^X/G/1$ queue with single and multiple vacations under LIFO service regime", **Operational Research Letters**, 14, 171-179.

TAKAGI, H. (1991): **Queueing Analysis: A foundation of performance evaluation**, 1 North Holand, Amsterdam.

__________ (1992): "Time dependent process of M/G/1 vacation models with exhaustive service", **Journal of Applied Probability**, 29, 418-429.