SAMPLING USING RANKED SETS: CONCEPTS, RESULTS AND PERSPECTIVES

Carlos N. Bouza[1], Facultad de Matemática y Computación, Universidad de La Habana

ABSTRACT
Traditionally we have used the simple random sampling to select samples. Most of the models of the statistic are supported by the use of samples selected by means of this design. During the last decade it has been begun to be used an alternative design that shows an improvement, in some cases, with respect to the sampling error and the accuracy. It is called Ranked Set Sampling. A random selection is made with the replacement of samples, which are ordered (ranked). Each order statistic is observed once. This process can be repeated or not. In this paper a review of the most significant results in this theme is made and some open problems related with this sampling design that should be studied are settled down.

Key words: estimation, order statistics, efficiency, sampling error, relative precision.

MSC: 62D05

RESUMEN
Tradicionalmente hemos utilizado el muestreo simple aleatorio para seleccionar muestras. La mayor parte de los modelos de la estadística se soportan por el uso de muestras seleccionadas mediante este diseño. En la última década se ha comenzado a utilizar un diseño alternativo que muestra una mejoría en algunos casos respecto al error de muestreo y a las precisiones. Este es llamado Muestreo de Conjuntos Ordenados (Ranked Set Sampling). En este se hace una selección aleatoria con reemplazo de muestras las que son ordenadas (rankeadas). Cada estadígrafo de rango es observado una vez. Este proceso puede ser repetido o no. En este trabajo se hace una revisión de los resultados más significativos en este tema y se establecen algunos problemas en los que se abre un campo de investigación utilizando este diseño muestral.

1. INTRODUCTION

Ranked set sampling (rss) was first proposed by McIntire (1952). He used this model for estimating the mean of pasture yields. This design appeared as a useful technique for improving the accuracy of the estimation of means. This fact was affirmed by McIntire but a mathematical prove of it was settled by Takahashi-Wakimoto (1968). In many situations the statistician deals with the need of combining some control and the implementation of some flexibility with the use of a random based sample. This is a common problem in the study of environmental and medical studies. In these cases the researcher generally has abundant and accurate information on the population units. It is related with the variable of interest Y and to rank the units using this information is cheap. The rss procedure is based on the selection of m independent samples, not necessarily of the same size, by using simple random sampling (srs). The sampled units are ranked and the selection of the units evaluated takes into account the order of them in the combined m samples. The proposal of McIntire (1952) was to use a prediction of Y. After some experiences with its application the lack of a coherent statistical theory appeared as an interesting theme of study to theoretical statisticians. An important role was played by Halls-Dell (1966) who established that rss was more efficient than srs for estimating the population mean from a large study of sampling forage yields. The interest for rss in applications is reflected not only in the initial papers but in the orientation of a series of papers to practice. See for example Chen (1999), Demir-Singh (2000), Kour **et al**. (1997) and Hall-Dell (1996) for examples. The interest in the development of a new statistical theory using rss can be illustrated by the contributions of Atalia (2000), Abu-Dayyeh and Muttlak (1996), Al-Saleh and Al-Khadari (1996), Barabasi and El-Shamawi (2001), Bouza (2002b), Chen (2001 a , 2001b), Kour **et al**. (2002), Kim-Barry (2000) Yu-Lam (1997).

The volume 12 of Handbook of Statistics dedicated a section to rss, see Patil et al. (1994). It reviewed the main results and provided an account of the contemporaneous main results. In that moment was established that to use an auxiliary variable for ranking allowed using rss. The efficiency of the rss estimator of the mean was not affected by the existence of errors in the ranking. A huge amount of papers is dedicated to the study of rss as an alternative to the use of srs. Different papers present a discussion of the State of the Art in rss.

E-mail: [1]bouza@matcom.uh.cu

Some of them are Bohn (1996) and Muttlak and Al-Saleh (2000). This paper has a similar aim and is based on the lecture given at the Applied Statistical Unit of the Indian Statistical Institute-Kolkata during a visit in 2002. We review some key results in estimation based on rss specially when the population is finite. Some areas open to research are pointed out using the trends of the recent contributions. The first section provides the basic ideas and procedures of the 'rss thinking' and the implementation of different selection algorithms is presented. The estimation theory is presented using a Horvitz-Thompson approach, see Hedayat-Sinha (1992), Cochran (1977), because the srs is a particular case of the corresponding estimator. Section 3 provides some variations to the most popular rss sampling designs and the following section plays a similar role by discussing the use of alternative estimators. Their efficiency with respect to the srs and the basic rss estimator of the mean is studied. The next section analyzes how rss is being used in classic statistical procedures. The study of parametric problems (behavior under distributions, Maximum Likelihood Estimation rss procedures, etc) and the robust and non-parametric approach (estimation of a distribution function, Sign Test, Mann-Whitney-Wilcoxon Tests, etc). is developed in the sequel. To follow the ideas and proofs involved in rss a knowledge of non parametric statistics and sampling is needed at a level which is covered by advanced text books as Arnold **et al**. (1992) and Hedayat-Sinha (1992).

2. THE BASICS OF RSS

2.1. The basic ideas

We consider a population and a variable of interest Y. A sampling design d(s) is used for selecting a sample s. The inclusion probabilities π_i = Prob ($u_i \in$ s) and π_{ij} = Prob ($u_i \wedge u_j \in$ s) are perfectly calculable. Once s is selected we evaluate Y on the sampled units and $y_1,...,y_n$ are obtained. A well-known estimator of the population mean μ is

$$\mu_{HT} = \sum_{i \in S} Y_i / N\pi_i$$

if simple random sampling (srs) is used π_i = n/N and μ_{HT} is the sample mean μ_s. Note that if we rank the observation and define the order statistics (os) $Y_{(i)}$, i = 1,..,n we have that

$$\mu_s = \sum_{i=1}^{n} Y_{(i)} / n = \mu_{(s)}$$

Hence

$$E(\mu_{(s)}) = \sum_{i=1}^{n} E(Y_{(i)})/n = \sum_{i=1}^{n} \mu_{(i)})/n = \mu$$

If srs with replacement (srswr) is used the usual estimator of the population mean based on the observations is:

$$\mu_s = \sum_{i=1}^{n} Y_{(i)} / n$$

has variance

$$V\left[\sum_{i=1}^{n} Y_{(i)})/n\right] = \sigma^2 / n$$

If we base our inferences on the os's

$$V\left[\sum_{i=1}^{n} Y_{(i)})/n\right] = \sum_{i=1}^{n} Y_{(i)})/n^2 = \sum_{i=1}^{n} \sigma_{(i)}^2 / n^2$$

Note that the ranks do not intervene in the selection of the sample. We can define a map $g(u_i)$ such that it assigns to each sampled unit u_i a rank and only one. Each sampled unit may be ranked using g without measuring Y using some judgements. Say that the rank represents certain judgment on the value of Y. For example if we plan to study the stature of children we are able to rank them visually before selecting the sample. Similarly occurs when we use satellite information on the biomass for ecological studies. The first arising question is whether this ranking affects the behavior of a statistical procedure based in it. The first results in this theme considered that the rank was perfect, see McIntire (1952), Takahasi-Wakimoto (1968). Dell-Clutter (1972) studied this problem considering a cumulative distribution function (cdf) P(y) in each sample unit were measured Y_i and Rank $[Y_{(i)}]$. Taking $Y_{(i)}$ = i-th judgment rank of the order statistics and $f_{(i)}(y)$ as its probability density function (pdf) we have that, as g is a one-to-one map

$$P(y) = \sum_{i=1}^{n} f_{(i)}(y)/n$$

and

$$E[Y_{(i)}] = \sum_{t=1}^{n} Y_t f_{(y)}(y)/n = \mu_{(i)}$$

Hence when we deal with $\mu_{(s)}$ the unbiasedness property is maintained though judgements and not the values of Y makes the ranking. Therefore

$$\sum_{i=1}^{n}(\mu_{(i)} - \mu) = \sum_{i=1}^{n} \Delta_{(i)} = 0$$

The differences between the expected mean of the os's and the population mean play and important role in rss because $\sigma_{(i)}^2 = \sigma^2 - \Delta_{(i)}^2$. Then

$$V[\mu_{(s)}] = \sum_{t=1}^{n} \sigma_{(i)}^2 / n^2 = V[\mu_{(s)}] - \sum_{t=1}^{n} \Delta_{(i)}^2 / n^2$$

Note that as:

$$|\Delta_{(i)}|/\sigma \leq [\beta(2i - 1, 2n - 2i + 1) - (\beta(i, n - i + 1))^2]^{½} / (\beta(i, n - i + 1))$$

see Hartley-Davies (1954)

$$\sigma^2 \geq \Delta_{(i)}{}^2 \, (\beta(2i - 1, 2n - 2i + 1))^2 / [\beta(2i - 1, 2n - 2i + 1) - (\beta(i, n - i + 1))^2].$$

An extreme case is that in which none of the ranks assigned by judgement coincide with the true ones. The orders are considered as assigned by a random mechanism. Then $\Delta_{(i)} = 0$ for any i = 1,…,n and the rss design is equivalent to the srs design. Patil **et al**. (1997a) discussed the notion of coherent sampling. Taking into account that we are sampling a set of units and that any sample s is a subset of the population U; we can establish the following definition.

Definition 2.1. Define a protocol (a one-to-one map) g, which orders the units in a finite population $U(g(u_i) = rank(u_i))$ and induces an ordering on each $s \subset U$. It is called coherent if for any s and U the ranking induced on s is the same that the application of it directly in $s[g(u_i|U) < g(u_j|U) \Rightarrow g(u_i|s) < g(u_j|s), \forall s \subset U, \forall i \neq j]$.

We consider the use of coherent ordering protocols. It allows using a global ranking of the units for ordering the observed sample without inconsistencies. Hence census information permits to establish an ordering in the sampled units. As pointed out by Patil **et al**. (1997 a) if we have a coherent rss design we are implementing an imperfect stratification. The knowledge of the true ranks of all the population units allows using them for stratifying. Some kind of optimal stratification can be implemented and it will provide more accurate estimates than rss. Therefore g permits to stratify in 'small sets' where each member have very similar values of Y.

2.2. Some different ranking procedures and models

As quoted before we may rank using judgements. |It can be characterized by an auxiliary variable X related with Y. David-Levine (1972) quoted this problem. Dell-Clutter (1972) analyzed the case in which the ranking is made with errors and established that the usual estimates from the computed os's maintain the unbiasedness property. Stokes (1977) used this result by considering that X is known for any unit and is used for ranking. An apparent source of errors in rss is the use of X for ranking. A practical methodology is to consider that we select s and the sequence $X_{(1)},..,X_{(n)}$ is obtained.

Take the location model

$$Y_{(i)} = X_{(i)} + e_i , \quad i = 1,...,n.$$

and consider that the random errors have null expectation [$E(e_i) = 0$, $i = 1,...,n$]. A common assumption is that they are independently normal variables with variance σ_i^2. It is clear that the rss estimator is still unbiased. Another model is to consider that the regression

$$Y_i = a + bX_i + e_i , \qquad i = 1,...,n.$$

characterizes the relationship between two equally distributed variables X and Y. The correct os is $Y_{(i)}$ but the regression allows to fix that

$$E[Y_{(i)}|X_{(i)}] = \mu_Y + \rho\sigma_Y[X_{(I\,)} - \mu_X] / \sigma_X], \qquad i = 1,...,n.$$

and

$$[E[Y_{(i)}|X_{(i)}] - \mu_Y] / \sigma_Y = \rho[X_{(i)} - \mu_X] /\sigma_X], \qquad i = 1,...,n.$$

Then if X and Y are positively correlated the os determined by X and by Y will be the same.
If there is some clustering non-random selections should be made. Ribout-Cobby (1987) studied this problem. Assume that the effect of clustering is modeled by

$$Y_{(i)k} = U_k + V_{(i)k}.$$

Where the two variables describing $Y_{(i)k}$ are independent and identifies the cluster to which i belongs. A simple hypothesis is that the cluster has a null effect:

$$E[U_k] = 0$$

But

$$V[U_k] = \sigma_U^2$$

The unit i of cluster k is expected to be identified by

$$E[Y_{(i)k}] = E[V_{(i)k}] = \mu_Y, \quad i = 1,...,n.$$

and

$$V[Y_{(i)k}] = V[U_k] + V[V_{(i)k}] = \sigma_U^2 + \sigma_{(i)}^2, i = 1,...,n.$$

When the cluster has a constant effect:

$$\sum_{i=1}^{n} Y_{(i)k} / n = \mu_U + \mu_{V[k]}.$$

As a result to select purposively a member of the cluster do not affect the rss procedure because

$$E\left[\sum_{i=1}^{n} Y_{(i)k} / n\right] = \sum_{i=1}^{n} \mu_{(i)} = \mu_Y$$

and

$$V\left[\sum_{i=1}^{n} Y_{(i)k} / n\right] = \sum_{i=1}^{n} \sigma^2_{(i)} / m^2 + \sigma^2_U / m$$

Then there is an increase in the variance due to the existence of clustering. Taking $\sigma^2_{(.)} = \sigma^2 + \sigma^2_U$ we can evaluate the gain in accuracy with respect to srswr.

Muttlak-McDonald (1990) proposed to select a sample of size n* and to subsample. The assumptions used were that X and Y are non-negative with joint density f(x,y) and a weight function w(x,y) = x with

$$E[Y|X] = \mu_Y + \rho\sigma_Y (X - \mu_X)/\sigma_X$$

$$V[Y|X] = (1 - \rho^2)\, \sigma^2_Y$$

The sampling design assigns a probability $\pi_i = X_i/M$ to each unit. The Horvitz-Thompson estimator of the population mean using the first phase sample is

$$\sum_{i=1}^{n*} Y_i / \pi_i = \mu^*_{HT}$$

If the use of double sampling with srs in the second phase is implemented an unbiased estimator of the population mean is:

$$\mu^*_{(HT)srs} = Mn* \sum_{i=1}^{n} Y_i / X_i n$$

If rss sub-sampling is made they proposed the unbiased estimator

$$\mu^*_{(HT)rss} = Mn* \sum_{i=1}^{n} Y_i / X_i n$$

which is more efficient than its srs counterpart.

2.3. The basic rss strategies

The theoretical frame that permits to use the rss model is based on the hypothesis

1. We wish to enumerate the variable of interest Y.
2. The units can be ordered linearly without ties.
3. Any sample $s \subset U$ of size m can be enumerated.
4. To identify a unit, order the units in s and enumerate them is less costly than to evaluate $\{Y_i, i \in s\}$ or to order U.

The first hypothesis is common to the general sampling problem, the second fixes that the rank can be made without confusions and that any rank is assigned to only one of the sampled units. The third assumption is also common in the applications. The fourth has an economical and a statistical motivation: only if it is cheap to rank rss is a good alternative with respect to rank all the units of U and to stratify, which is more accurate. Some definitions are needed.

Definition 2.2. A statistical sampling unit (ssu) is a set s with m units of U.

Usually m ssu's are selected independently

The basic rss procedure is the following:

Procedure RSS1

While t < m do
Select a ssu independently from U using srswr.
Each unit in $s_{(t)}$ is ranked and the os's $Y_{(1:t)}$,...., $Y_{(r(t):t)}$ are determined.
END

Then the procedure generates the matrix

$Y_{\mathbf{(1:1)}}$	$Y_{(2:1)}$	• • •	$Y_{(t:1)}$	• • •	$Y_{(m:1)}$
$Y_{(1:2)}$	$Y_{\mathbf{(2:2)}}$	• • •	$Y_{(t:2)}$	• • •	$Y_{(m:2)}$
• • •	• • •	• • • • • • • • •	• • •	• • • • • • • • •	• • •
$Y_{(1:t)}$	$Y_{(2:t)}$	• • •	$Y_{\mathbf{(t:t)}}$	• • •	$Y_{(m:t)}$
• • •	• • •	• • • • • • • • •	• • •	• • • • • • • • •	• • •
$Y_{(1:m)}$	$Y_{(2:\ m)}$	• • •	$Y_{(t:\ m)}$	• • •	$Y_{\mathbf{(m:m)}}$

The ranked set sample is composed by the elements in the diagonal s(j) = {$Y_{(i:i)}$, i = 1,..,m}.

Once the sample size n is fixed and if the i-th os is observed r(i) times, then:

$$\sum_{j=1}^{m} r(i) = n$$

A particular case is r = r(i), for any i = 1,...,m, and then mr = n.

Definition 2.3. The set of m samples s* = {s(1),...,s(m)} generated by the procedure:

Procedure RSS for a ranked set sample of size n generation

While j < m do
Procedure RSS1
End is a rss sample

Denote by $Y_{(i:i)j}$ the observation j of the i-th os of the rss s*. We can compute

$$\sum_{j=1}^{m} Y_{(i:i)j} z(i,j) = t(i)$$

where:

$$z(i,j) = \begin{cases} 1 \text{ if the } i-\text{th os of } s(j) \text{ is measured} \\ \\ 0 \text{ otherwise} \end{cases}$$

A rss estimator based in s* is:

$$\mu_{rss} = \sum_{j=1}^{m} t(i)/mr(i)$$

Note that $\mu_{rss} = \mu_{(s)}$ if m = 1 because we observe only a rss of size r = n.

Definition 2.4. When r(i) = r the rss design is denominated balanced and unbalanced otherwise.

For balanced rss designs we have that each s(j) is a srswr of size r and

$$\mu_{rss[r]} = \sum_{j=1}^{m}\sum_{i=1}^{r} Y_{(i:i)j} /n.$$

Noting that for any j $E[Y_{(i:i)j}] = \mu_{(i)}$

$$E[\mu_{rss[r]}] = \sum_{j=1}^{m}\left[\sum_{i=1}^{r}\mu_{(i)}/r\right]/m = \mu$$

The samples s(j) are independent , hence the variance of this unbiased estimator is:

$$V[\mu_{rss[r]}] = \sum_{i=1}^{m}\sigma_{(i)}^2/mr^2 = \sigma^2/n - \sum_{i=1}^{m}\Delta_{(i)}^2/nr$$

In the unbalanced design we have that

$$V[\mu_{rss}] = \sum_{i=1}^{m}\sigma_{(i)}^2/m^2 r(i)$$

Measures of the accuracy of the estimators are defined as follows:

Definition 2.5. The relative precision of μ_{rss} with respect to μ_s is:

$$RP[\mu_{s,}\ \mu_{rss}] = V[\mu_s]/V[\mu_{rss}]$$

And the relative saving due to rss is measured by

$$RS[\mu_{s,}\ \mu_{rss}] = 1 - 1/RP$$

In the balanced case RP$\in$ [1, (m + 1)/2] and in the unbalanced RP $\in$ [1,m]. The later depends on the allocation of the sample. Patil **et al**. (1997b) established that if we deal with a skew distribution or if an adequate stratification may be implemented the unbalanced design may not be so efficient. RS may be used with the purpose of evaluating the relative gain in accuracy due to the use of rss.

Kour **et al**. (1997) studied the allocation problem. When Neymann's allocation principle is used for determining r(i)'s and n is fixed the optimal sample sizes are given by:

$$r^*(i) = n\sigma_{(i)}^2 / \sum_{i=1}^{m}\sigma_{(i)}^2$$

Another approach is based on the knowledge of the existence of a large tail pdf. In the case of a heavy right tail, a skewed distribution we have that the os's variance are ordered and $\sigma_{(1)}^2 \leq \sigma_{(2)}^2 \leq \bullet\bullet \leq \sigma_{(m)}^2$. The statistician fixes a constant $\theta > 1$ and

$$r^* \equiv r(i) = r(m) / \theta, \qquad i = 1,...,m - 1.$$

Then

$$V[\mu_{rss}|\theta] = m^{-2}\left[\sum_{i=1}^{m-1}(\sigma^2_{(i)} / r^*) + (\sigma^2_{(m)} / \theta r^*\right]$$

Hence using a larger number of replicas seriously reduces the summand with larger variance of the os. Usually srswr is used for selecting the samples independently but srs without replacement may used (srswor). This is more important when we study a finite population because a correction should be introduced for computing the sampling error. The problem is certainly very complicated when compared with the usual one. Patil **et al**. (1995) derived the expression of the corresponding variance. A gain in precision due to rss now depends heavily on the replication factor. The theoretical problems associated with the use of os in finite population sampling using rss are the kernel of the behavior of the wor procedure. Lehman (1966) established some properties of the random variables generated by an univariate distribution and their os's. One of them is that any pair of os's has a joint pdf, which is positively likelihood ratio dependent. Then, if we sample a finite population of os's using srswor this property holds. Takasi-Fututsya (1998) used these results for deriving a method for computing the finite population correction factor

The existence of non responses was studied by Bouza (2002a). The existence of non responses (nr) establishes, see Cochran (1977) that the population is divided into two strata

$U_1 = \{u \in U|u$ responds at the first attempt$\}$

$U_2 = \{u \in U|u$ does not respond at the first attempt$\}$

Hence each sample s_i is divided into two subsamples $s_i = s_{i1} + s_{i2}$, $|s_i| = m$, where $s_{it} \subset U_t$ and for $t = 1,2$ $|s_{it}| = m(i,t)$.

Two subsample strategies were considered:

1. Select a subsample s'_{i2} of size $m(i,2)$ from each s_{i2} , $i = 1,..,r$.

2. Select a subsample $s'_{2(j)}$ of size $r(i,2)$ from each s_{i2} , $j = 1,..,r.m$

The estimator proposed when the sample is selected using the strategy 1 is:

$$\mu_{rss(nr)} = \sum_{k=1}^{r} M(i) / r$$

where

$$M(i) = w(i,2)\left[\sum_{u=1}^{m(i,2)} Y'(i,u) / m(i,2)\right] + w(i,1)\left[\sum_{u=1}^{m} Y^*(i,u) / m(i,1)\right]$$

Defining $w(i,t) = m(i,t)/m$, $Y'(i,u)$ as the value of Y in the u-th unit of s'_{i2} and $Y^*(i,u) = y_{u(u)}$ if the unit with rank u in the u-ranked set responds and zero otherwise. The expected variance

$$EV[\mu_{rss(nr)}] = V + G$$

With

$$V = [\sigma^2 + W_2(K - 1)\, \sigma^2_2]/n$$

$$G = \Delta_1 - \Delta_2$$

$$\Delta_1 = \sum_{j=1}^{m} (\mu_{(j)} - \mu)^2 / m$$

$$\Delta_2 = \sum_{i=1}^{r} E\left[\sum_{j=1}^{m(i,2)} (\mu_{(j)} - \mu)^2\right] / n$$

K is the sub-sample parameter of Hansen-Hurwitz rule. A gain in accuracy is present when compared with the srs alternative.

The second alternative seemed to be inefficient due to that to subsample the non respondent order statistics creates a dependence that increases the variance.

A Monte Carlo experiment was performed and $RP[\mu_{s(nr)}, \mu_{rss(nr)}] \in [0{,}69\ 0{,}93]$ while for the second strategy. The RP was always larger than one.

3. SOME VARIATIONS IN THE SELECTION PROCEDURES

Some authors have changed the basic scheme seriously and other criteria are proposed for selecting the os. An improvement to the original rss design was developed by Li **et al**. (1999). They suggested selecting n independent samples of size n and ordering the observations within each sample. A sample of size n* < n is selected independently among the n ordered samples. For the j-th selected sample [s*(j)] the j-th os is measured. The new estimator of the population mean is:

$$\mu_{rss[n^*]} = \sum_{j=1}^{n^*} Y_{(j:j)j} / n^*$$

It is unbiased and its sampling error is:

$$V[\mu_{rss[n^*]}] = \sum_{i=1}^{r} \sigma^2_{(i)} / mn^* + [n - n^*] \sum_{i=1}^{m} \Delta^2_{(i)} / nn^*(n-1)$$

The new strategy is more efficient than the usual one if $n^* > n^2/(2n - 1)$. The estimation of the variance was also studied as an alternative for estimating σ^2. Their proposal

$$\sigma^2_{rss(n^*)} = [n - 1]\left[\sum_{j=1}^{n^*} (Y^2_{(j:j)} - \mu_{rss(n^*)})^2\right] / (n^* -1)\left(n - \sum_{i=1}^{n} v_{(i:n)} / n\right)$$

where v(i:n) is the variance of the i-th os of the standard normal distribution.

This estimator of the variance appears as a good alternative in the experiments developed under some friendly distributions.

The use of unequal probability in rss can be traced in the paper of Yanagawa-Shirahata (1976). The pdf of the variable F is sampled randomly and the mn observations are ranked. A matrix Kof mn elements is determined and a probability p_{ij} is assigned to each element of it. Then we have the matrix of selective probabilities

$$P = [p_{ij}]_{m \times n},\ p_{ij} \in [0,1]$$

For any i = 1,...,n

$$\sum_{j=1}^{m} p_{ij} = 1$$

The selection procedure works as follows.

Unequal selection procedure

Select an element $u(i,j_i)$ from the i-th row using $(p_{i1},....,p_{im})$

Measure Y in the selected unit

Compute

$$\mu_{Prss} = \sum_{i=1}^{n} Y[u(i, j_i)/n$$

The unbiasedness of this estimator holds if and only if

$$\sum_{i=1}^{n} p_{ij} = n/m, \text{ for any } j = 1,..,m.$$

Again in precision is due to the use of rss under certain distributions. Muttlak (1997) proposed to select the median of s(j) in each ssu. The pdf of Y has finite μ and σ and we observe $\{Y_{(1:1),...,}Y_{(1:n),}\ Y_{(2:1),...,}Y_{(n:n)}\}$. They considered only the case n = m, (r = 1). If n is odd the os's measured are

$$\{ Y^*_{(i:j)} = Y_{([n+1]/2:j)}, j = 1,...,n\}$$

If n is even is used

$$Y^*_{(i:j)} = \begin{cases} Y_{(0,5n:j)} \text{ if the } j \leq n/2 \text{ is measured} \\ \\ Y_{(0,5n+1:j)} \text{ otherwise} \end{cases}$$

The estimator is:

$$\mu_{rss[med]} = \sum_{j=1}^{n} Y^*_{(j:j)} / n$$

It is unbiased if the pdf is symmetric with respect to μ and $V[\mu_{rss[med]}] \leq V[\mu_{rss}] \leq V[\mu_s]$. The RP of this estimator is increased n. For not symmetric pdf's the estimator is still more precise then than μ_s but it is biased. The RP decreases if $n \geq 6$. The errors in the ranking do not affect seriously these results. Hence the use of median rss provides a gain in accuracy. This estimator can be used as a good alternative for estimating the population size.

Another procedure is to use the extreme os' of the samples. That is, in each ssu we measure $Y_{(1:j)}$ and $Y_{(n:j)}$. Take n even and

$$Y_{(j,e)} = [Y_{(1:j)} + Y_{(n:j)}]/2$$

The estimator proposed by Samawi **et al**. (1996) is:

$$\mu_{rss[e]} = \sum_{j=1}^{n} Y_{(j,e)} / n$$

as

$$E[\mu_{rss[e]}] = [\mu_{(1)} + \mu_{(n)}]/2$$

and

$$V[\mu_{rss[e]}] = [\sigma^2_{(1)} + \sigma^2_{(n)}]/2n$$

For n odd we introduce the variable

$$Y_{(i:j)e} = \begin{cases} Y_{(1:j)} \text{ if the } j < n \text{ and } n \text{ odd} \\ Y_{(n:j)} \text{ if the } j < n \text{ and } n \text{ even} \\ [Y_{(1n)} + Y_{(n:n)}]/2 \text{ if } j = n \end{cases}$$

They proposed the estimator

$$\mu^*_{rss[e]} = \sum_{j=1}^{n} Y_{(j:j)e}/n$$

Its expectation is equal to the expectation of $\mu_{rss[e]}$ but

$$V[\mu^*_{rss[e]}] = [2\sigma_{(1,n)} + (2n - 1)(\sigma^2_{(1)} + \sigma^2_{(n)})]/4n^2$$

Where $\sigma_{(1,n)} = Cov[Y_{(1:n)}, Y_{(n:n)}]$.

An alternative estimator analyzed for n odd was:

$$\mu^{**}_{rss[e]} = \left[\sum_{j=1}^{n-1} Y_{(j:j)e} + Y_{([n+1]/2:n)}\right]/n$$

which expectation and variance are:

$$E[\mu^{**}_{rss[e]}] = (n + 1)[\mu_{(1)} + \mu_{(n)}]/2n + \mu_{([n+1]/2)}/n$$

$$V[\mu^{**}_{rss[e]}] = [(n - 1)((\sigma^2_{(1)} + \sigma^2_{(n)})]/2n^2 + \sigma^2_{([n+1]/2)}/n^2$$

If the pdf is symmetric with respect to μ=0 the median is equal to zero. From the results of Arnold **et al**. (1992) we have that:

1. $\mu_{(1)} = -\mu_{(n)}$ for n even and $\mu_{([n+1=/2)} = 0$ if n is odd.
2. $\sigma^2{}_{(1)} = \sigma_{(n)}{}^2$

Therefore in this particular case:

$$E[\mu_{rss[e]}] = E[\mu^{**}_{rss[e]}] = E[\mu^{**}_{rss[e]}] = 0$$

And

$$V[\mu_{rss[e]}] = \sigma^2_{(1)}/n$$

$$V[\mu^*_{rss[e]}] = [(2n - 1)(\sigma^2_{(1)} + \sigma_{(1:n)})]/2n^2$$

$$V[\mu^{**}_{rss[e]}] = [(n - 1)(\sigma^2_{(1)} + \sigma^2_{([n+1]/2)}/n^2$$

When the distribution is a uniform these estimators have a smaller variance than $V[\mu_s]$. The preference of one or another estimator depends of the value of n including the usual μ_{rss}.

4. PARTICULAR ESTIMATORS

One of the most popular estimation problems is to estimate the ratio. It has received attention of different authors.

Bouza (2001b) used rss for selecting a sample using a third variable related with X and Y. Calculating

$$\mu_{rss[m]Z} = \sum_{i=1}^{m}\sum_{t=1}^{r} Z_{(j:j)t}/rm, \ Z = X,Y$$

the ratio estimator for rss is

$$\mu_{R(rss)} = \mu_X[\mu_{rss[m]Y}/\mu_{rss[m]X}]$$

Its comparison with the srs estimator

$$\mu_{R(srs)} = \mu_X[\mu_{srsY}/\mu_{srsX}]$$

leads that the variance

$$V[\mu_{R(srs)}] = \left[V_Y + V_X - [2R\rho V_Y^{1/2} V_X^{1/2}] + \sum_{i=1}^{m} (\mu_{(i)} - \mu_Y)^2 + \sum_{i=1}^{m} (\mu_{(i)X} - \mu_X)^2 \right]/mr$$

Is smaller than the srs estimator's variance whenever the correlation coefficient between Y and X satisfies that:

$$\rho \geq \left[\sum_{i=1}^{m} (\mu_{(i)} - \mu_Y)^2 + \sum_{i=1}^{m} (\mu_{(i)X} - \mu_X)^2 \right] / \ [2mRV_X^{1/2}] - RV_X^{1/2} / 2V_Y^{1/2}$$

Samawi-Muttlak (2001) assumed that the variables are ranked. The auxiliary variable X is ranked without error. The observation $(X_{(i:j)t}, Y_{(i:j)t})$ is the pair of values in the i-th judgement os in the rss sample s(j)at cycle t. Their proposal was to use not the pairs in the diagonal but the medians

$$Z^*_{(i:j)t} = \begin{cases} Z_{([m+1]/2:j)t} & \text{if m is odd} \\ [Z_{(m/2:j)t} + Z_{([m+2]/2:j)t\,]}/2 & \text{if m is even} \end{cases}$$

$$Z = X,Y.$$

The estimation of the mean is made by averaging the $Z^*_{(i:j)t}$´s.

$$\mu_{rss[m]Z^*} = \sum_{i=1}^{m}\sum_{t=1}^{r} Z^*_{(j:j)t}/rm . \qquad Z = X,Y$$

The estimator of the ratio based on these rss median based estimators of the mean is

$$R_m = \mu_{rss[m]Y^*}/\mu_{rss[m]X^*}.$$

Taking

$$V_{Z(h)} = \sigma^2_{Z(h)}/\mu_z^2, \qquad Z = X,Y$$

$$C_{(h)} = Cov[X_{(h)}, Y_{(Z)}]/\mu_X\mu_Y$$

The variance of this estimator is:

$$V[R_m]= R^2[V_{X([m+1]/2)} +V_{Y([m+1]/2)} - 2C_{([m+1]/2)}]/n$$

If m is odd and for m even

$$V[R_m] = R^2[V_{X(m/2)} + V_{Y(m/2)} – 2(C_{(m/2)} + C_{([m+2]/2)}]/n$$

The involved variances of the rss estimators are expressed as the difference between a function of the population variance of Z and a function of the sum of the Δ_i^{2}´s. $V[R_m] = R^2[V_{X([m+1]/2)} + V_{Y([m+1]/2)} - 2C_{([m+1]/2)}]/n$.

The relative merit of this strategy is that the estimation is fitted in a non parametric sense and we need to rank only a part of the sample. Other intents in this line are presented in Patil **et al**. (1997a, 1998) Patil **et al**. (1993) studied the regression estimator when X is used for ranking. Different alternative strategies may be used. The rss alternatives appeared as more accurate than their srs counterparts. Another look to this problem is given by Yu-Lam (1997b) and Muttlak (2001).

Bouza-Prabhu Ajgaonkar (1993) studied the estimation of a difference within the frame proposed by Pi-Ehr (1971). The original sampling model considers the selection of 3 independent sub-samples from a srswr sample. The parameter to be estimated is $D = \mu_Y - \mu_X$. The sub-samples are

$$S(1) = \{j \in s|Y \text{ and } X \text{ are measured}\}, |S(1)| = m(1)$$

$$S(2) = \{j \in s| \text{ only } Y \text{ is measured}\}, |S(2)| = m(2)$$

$$S(3) = \{j \in s| \text{ only } X \text{ is measured}\}, |S(3)| = m(3)$$

An unbiased estimator of D is

$$D_{srs} = a\mu_{Ysrs(1)} + (1 - a)\mu_{Ysrs(2)} + b\mu_{Xsrs(1)} + (1 - b)\mu_{Xsrs(3)})$$

The use of

$$a = m(1)[[\sigma_Y(m(1) + m(3))/m(2)] - \rho\sigma_X][\sigma_Y((m(1) + m(2))(m(1) + m(3))/m(2)m(3)) + \rho^2]^{-1}$$

$$b = m(1)[[\sigma_X(m(1)+m(2))/m(3)] - \rho\sigma_Y][\sigma_X((m(1) + m(3))(m(1) + m(2))/m(2)m(3)) + \rho^2]^{-1}$$

grants that it is a Minimum Variance Unbiased Estimator. Taking

$$V(\mu_{Zsrs(t)}) = \sigma_Z^2 / m(t)$$

and

$$V(D_{srs}) = a^2V(\mu_{Ysrs(1)}) + b^2V(\mu_{Xsrs(1)}) + (1-a^2)V(\mu_{Ysrs(2)}) + (1-b)^2V(\mu_{Xsrs(1)}) - 2ab\rho[V(\mu_{Ysrs(1)})V(\mu_{Xsrs(1)})]^{-½}$$

Bouza (2001a) has studied the rss alternative using a third variable for ranking the units. The rss estimators were used and

$$D_{rss} = \mu_{Yrss(1)} - \mu_{Xrss(1)} + (\mu_{Yrss(2)} - \mu_{Xrss(3)})$$

Its variance is

$$V(D_{rss}) = A\sigma_Y^2 + B\sigma_X^2 - 2\rho[AB]^{-1/2}\sigma_Y\sigma_X - \Delta$$

Where

$$A = [m(1) + m(2)]/m(1)m(2)$$

$$B = [m(1) + m(3)]/m(1)m(3)$$

$$\Delta = A\Delta_Y + B\Delta_X$$

It represents a better alternative than the srs estimator of the difference does. Other alternative were propose using ratio and regression criteria but D_{rss} was the more precise estimator in a Monte Carlo experiments based analysis under Uniform, Exponential and Normal distributions of the variables.

The problem of non-responses has been studied in Bouza (2001c) using the usual frame. A srswr is selected and the non-response stratum is sub sampled using rss. Two strategies were proposed:

1. Select a sub-sample among the non-respondents units.

2. Select a sub-sample among the missing ranks.

The first strategy seemed to be the best in various applications. A theoretical analysis was developed by Bouza (2002a) and Monte Carlo experiments yielded similar results.
The problem of the estimation of the difference was

5. CLASSIC STATISTICAL APPROACHES

5.1. Parametric inferences

Rss is a method of sampling therefore it is not surprising that the performance of estimations based on rss samples has been studied using the more important pdf's.

The analysis of the Maximum Likelihood Estimation (MLE) when the sample is selected by rss is based on the likelihood function of the os's which is derived from the pdf of the original variable. The Fisher information for μ can be deduced and the asymptotic RP is given by:

$$ARP(\theta_{MLrss}, \theta_{MLsrs}) = \lim_{n\to\infty} RP(\theta_{MLrss}, \theta_{MLsrs}) = I(\theta_{MLrss})/I(\theta_{MLsrs})$$

A common hypothesis is that the pdf satisfies some mild regularity conditions. Stokes (1995) derived that $ARP(\mu_{MLrss}, \mu_{MLsrs}) \approx 1 + 0{,}4805(m - 1)$

When the pdf is a $N(\mu,1)$ and if it is a $N(0, \sigma^2)$

$$ARP(\sigma_{MLrss}, \sigma_{MLsrs}) \approx 1 + 0{,}13525(m - 1)$$

For an Exponential (σ) it is

$$ARP(\sigma_{MLrss}, \sigma_{MLsrs}) \approx 1 + 0{,}404(m - 1)$$

In the case of a two parameter symmetric pdf the MLE derived from a srs design is better than the corresponding MLE counterpart obtained for rss. If the pdf is a non-symmetric it is not clear which should be preferred.

Barnett-Mendez Barreto (2001) developed a MLE estimator for the parameter of a Poisson pdf for rss. It appears as more efficient than the usual MLE and it is expected that the same property is valid for the usual rss estimator. A ML-estimator was also derived using rss values and optimization methods.

Stokes (1995) analyzed the behavior of the Maximum Likelihood Estimator (MLE) of μ and σ^2 of the rss estimators versus the classic obtained in the literature for the same family of distributions that depends of these parameters: $F = \{F(x|\mu,\sigma),\ \mu \in \Re$ and $\sigma^2 \in \Re^+\}$. The rss estimator of μ is more efficient but the estimator of the variance

$$\sigma^2_{s(95)} = \sum_{t=1}^{r}\sum_{i=1}^{m}(Y_{(i:i)t} - \mu_{rss})^2 / rm$$

is more efficient than the usual estimators only if rm = n is sufficiently large. A study of this estimator when the distribution is a $N(\mu,1)$ and $N(0,\sigma)$ was developed for establishing what e "large m" means. The hypothesis $m \to \infty$ is impractical. Previously Stokes (1980) has analyzed a variance estimator in a broader sense.

Chen-Bai (1998) developed another strategy for deriving MLE and Best Linear Unbiased Estimator (BLUE) determining optimal unbalanced procedures. Assuming some regularity conditions the MLE estimator tends to have a normal distribution with null expectation and asymptotic variance covariance matrix $I(\theta,p)^{-1}$. An optimal unbiased rss scheme was developed. It grants strategies which minimizes $|I(\theta,p)^{-1}|$ the asymptotic variance covariance matrix. The hypothesis used is that $n \to \infty$. They were evaluated using two location-scale families: the normal and the extreme value pdf´s. They used also an unbalanced rss design for obtaining a BLUE with a larger RP for estimating parameters of the pdf´s belonging to location-scale families.

Bhoj (1997) developed an interesting rss protocol. The surveyor selects m samples of size and n = 2m and evaluates two os´s, say $Y_{(i:j)}$ and $Y_{(k:j)}$ in the samples. Then the mean and variance are estimated unbiasedly by

$$\mu_{rss(B)} = \sum_{j=1}^{m}[Y_{(i:j)} + Y_{(k:j)}]/n$$

$$\sigma^2_{rss(B)} = \sum_{j=1}^{m}[Y_{(i:j)} - Y_{(k:j)}]^2 (n + 1)/n[n(n - 1)]^{½}$$

Particular expressions are derived for the Rectangular and the Logistic pdf's. A sequel is the paper of Bhoj (1999).

5.2. Non Parametric and Robust Inference

The importance of rss has generated the study of the design effect in non parametric inference. Though it is not the objective of this paper to review of the contributions in this area. It is interesting to quote some lines that are being defined. The evident connections of rss and non parametric methods has stimulated the evaluation of new models.

Goodness of fit for testing normality has been studied by Shen(1994). The derived tests appeared as better alternatives to the traditional ones.

One of them is the use of well known tests . The hypothesis are not much stronger than the usual in non parametric theory based on srs: the distribution must be continuous, symmetric and unimodal.

Oztürk-Wolfe (2000) proposed a rss protocol which maximizes Pitman's Asymptotic Relative Efficiency (ARE). The search for a non parametric MLE of the distribution was developed by Kvan-Samaniego (1994). The statistic can not be expressed in a closed form and algorithms should be used for obtaining approximate values are needed. The existence and unicity of the estimator was derived under the balanced and unbalanced rss.

Stokes-Sager (1988) estimated the empirical pdf using rss and a Kolmogorov-Smirnov type test was developed for establishing the parent pdf of the data. The estimation based on rss is more precise than the use of the srs estimator. Another result is the convergence in distribution of it to a standard normal random variable when n is sufficiently large. The rss estimator of the distribution function, Stokes –Sager (1988) is more efficient than the empirical distribution.

Bohn-Wolfe (1992) developed a 2-sample Mann-Whitney-Wilcoxon test for rss sampling and its ARE was determined. The distribution of the test statistic is independent of the unknown pdf and if it is symmetric, with respect to its expectation and asymptotically normal. Unfortunately the asymptotic variance has a rather complicated expression, see Bohn-Wolfe (1994). Under some mild conditions the ARE is (m+1)/2. Similar results follow for the One-sample test statistic: the ARE is equal to the 2-sample statistic if the pdf is continuous and symmetric. Another look to this problem is given in Yu-Lam (1997a).

Hettmansperger (1995) and Kati-Babu (1996) made a serious study of Bohn-Wolfe sign test. The experiments sustained that rss was a good alternative. The statistic was developed by Hettmansperger (1992) the expectation and variance under the null hypothesis was derived. It has a smaller variance than the usual Sign Test statistic. Its distribution is a normal and the values of the ARE for different sample size values are tabulated.

Barnett-Moore (1997) used the results of Sinha **et al**. (1996) and developed an L-estimator when the nuisance parameter is unknown. The optimal L-estimator is more efficient than the usual one without needing that the density function be symmetric.

Chen (2001) proposed an unbiased rss design. It is specially suited for estimating quantiles. It is in the saga of another paper of Chen (2000). He derived strong consistency, Bahadur representation asymptotic normality. When the ranking is perfect optimal procedures can be implemented and the ARE were computed. The optimal procedure is sensitive to errors in the ranks.

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