PARAMETRIC AND MULTIOBJECTIVE OPTIMISATION APPLIED IN AGRICULTURE: THE STUDY OF CROPPING PATTERN IN THE AMERIYA REGION IN WINTER CROPS

M. H. El Sayed Abdel Asís
Department of Engineering Mathematics and Physics, Faculty of Engineering
Cairo University

ABSTRACT:
This paper covers an important part of linear programming branch of Operations Research, which is Parametric Multiobjective Optimisation. It studies the following forms of linear parametric optimisation**:**

1. Parameter at the coefficients of the objective function.
2. Parameter at the right-hand side of the constrains..
3. Parameter at the left-hand side of the constrains.

This paper also introduces a brief study of the solution of non-linear problems using penalization methods, which transforms the problem into an unconstrained non-linear optimisation one. One of the main attentions of this research is oriented towards the study of Multiobjective Optimisation Problems. The study presents three basic techniques that seem to be used often in practice.
The paper presents the study of cropping pattern in the Ameriya region in winter crops. We were interested in investigating the problem of the planning production, of examining the process of allocating land on agriculture activities and determining the optimal cropping patterns to achieve maximum in the income, and minima of irrigation water, use of fertilizers and number of workers.

KEY WORDS: unconstrained non-linear optimisation, planning production, allocating land, maximum income, minimum irrigation water

MSC: 90C90

RESUMEN:
Este trabajo cubre una parte importante de la rama de Investigación Operacional conocida como Programación Lineal. Esta estudia las siguientes formas de optimización lineal paramétrica:

1. Parámetros en los coeficientes de la función objetivo.
2. Parámetros en el lado derecho de las restricciones
3. Parámetros en el lado izquierdo de las restricciones

Este trabajo también introduce un breve estudio de la solución de problemas no lineales usando métodos de penalización, los cuales transforman el problema en uno de optimización no lineal. La atención principal de esta investigación está orientada hacia el estudio de problemas de Optimización Multiobjetivo. Este estudio presenta tres técnicas básicas que son muy utilizadas en la práctica.
El trabajo presenta el estudio de patrones de cosecha en la región de Ameriya en cosechas de invierno.
Estamos interesados en investigar el problema del planeamiento de la producción, de examinar el proceso de localización de las tierras para actividades agrícolas y determinar el patrón óptimo de cosecha para obtener un ingreso máximo, el mínimo de agua de regadío el uso de fertilizantes y el número de trabajadores.

1 INTRODUCTION

Using the pattern of optimisation techniques as and administrative model is an optimal matter for the agriculture building, whether it is on the economic unit level or on the whole agricultural activity level of a country, a region or a continental economic complex for the following reasons:

- Agriculture activity is considered the most interdisciplinary occupation with the other elements and activities, as ´occupations´ , because the agricultural activity is related to the mixture of several elements such a as land, soil chemistry, diseases, insects, water and mechanical engineering, biology, economy, administration and accounting. The pattern of Linear Programming (LP) provides the best way of considering such elements in the search of the optimal choice concerning an agricultural activity. This optimal choice is the one based on natural , biological planning, engineering and economic conditions. See Aggrawal-Heady (1972), Ravindran et.al. (1987), Guddat et.al. (1985) and Gupta-Hirac (1999).

- The increased limitations and lack of agriculture elements sources such as land and water compared to the over-population.
- The nature of agriculture activity which presents too many choices for production elements and their rates with mixing together.

- Multiplying the choices of agriculture activity itself and the changing of it from season to season or form financial period to another, or from an agriculture cycle or program and animal together.

Especially the pattern of LP with its various forms and methods, can be used in both agriculture building and activity whether one is concerning with deterministic programming, stochastic programming, or other programs, see Fiacco-McCormick (1968),Ijiri (1965), Canoil (1965), Lee (1972), Nozicka et.al. (1985). Considering at the same time hypothesis and losses relative to the use of such models or patterns. We can summarize different uses of a deterministic model, especially LP, in agricultural activity as the following, see Mahmoud et.al. (2000), Nozicka et.al. (1985), Rao (1995), Taka (1992).

First: Using the model in agriculture activity at the national or regional level (macro-level application). The LP, especially in the last two decades has a pioneering role in agriculture planning process, either on the national or regional level, for developing agriculture activity and increasing its production. In general there is imbalance or injustice concerning allocating the elements of agriculture production, nationally and regionally, which can be resulted in weakening the production or the agriculture sector. For example it has been shown, from a study conducted to recognize the needs of Arab countries from agriculture elements, to what extent these elements exist in other Arab countries by evaluating the agriculture production function in nine Arab countries between 1952 and 1968 (including land soil, number of working force, animal working force, fertilizers, machinery and technical experience) that there is a difference concerning the rates of mixing elements of agriculture production available in each country. Whereas there is an overuse of some elements beyond the current approaches, the quantities that used other elements are less than the quantities necessary for adequate production. Hence, we can see the significance of using LP to regulate the process of agriculture planning on the national and regional levels particularly in determining the optimal posits of agriculture sources and agriculture policy in general [Hazell-Norton (1988), Moncke-Pearson (1989)].

Second: Using the model at the level of agriculture building for planning production and marketing activities and examining the process of allocating land on agriculture activity and determining the optimal mixture for agriculture production, as well as allocating the available services on expected activities. It can be use as an aid tool when preparing the budget for operating the agriculture building [Arner et.al. a(1999), Karlin (1959), Murty (1996), Rao (1996)}].

Third: Using the model to regulate administration of the building concerning selection the optimal way for agricultural, technical processes such as the optimal way of feeding animals, fertilizers, scheduling times of milking and water resources during different seasons. A more advanced LP can be used such as (nearly optimal programming) and having them imply other goals and variables it is difficult to imply in mathematical equations in normal programming model. This model is known as the medal of modelling to generate alternatives. It is congruent with analysing agriculture problems in developing countries [Scott et.al. (1992), D´Alfonso-Rousskal (1990), Hiller-Davies (1995)], which includes searching for optimal solution for profits and production increases as well as its using in searching for the solution for the cost.

Fourth: Using the model to regulate administration of agriculture building concerning the financial and administrative decisions. For example the optimal relation between the daily request harvest production, the best ways for transporting products and financing a new agriculture activity. See Mansour et.al.. (2000a and 2000b), Rardin (1998).

Fifth: Investigating the ways of overcoming problems of weather and rains and determine optimal choices in that field.

A more developed LP can be used with respect to the optimal choices beyond uncertainty of atmosphere conditions. –This model is known as the model of stochastic programming, Tammer (1978), Guddat et.al. (1985). The study concerning regulations conduit of grains cultivators-farmers and their plans of production in Al Niger when they get different expectancies of rain falling and to what extent villagers can adopt production and administrative policies for challenging rains in a high risk environment by using a model of stochastic programming is an example of that model. . The study indicated that farmer-cultivators could adopt various patterns of agriculture with early, mediate or lately seasons chosen according to the conditions of rain falling, see Akinwumi et.al. (1991).

Sixth: Finally, an important usage of other forms of LP in agriculture activities is using parametric LP. It develops studying and determining the range of an agriculture project in an economy safety regional needs to maintain a closed basic solutions region. _In other words the resources and coefficients (costs or profits) of variables may be changed within certain limits without affecting the solution of the problem. This range of changes can be calculated using parametric programming. One of the main attentions of this research is oriented towards the study of this topic. See Shalaby (1989), Brosowsky (1983), Burkhard et.al. (1984), Jahin (1984) and Liska (1982).

2. PARAMETERS AT THE OBJECTIVE FUNCTION

Consider the following LP problem

Max{$\mathbf{Z=C^T X}$}
Subject to
$\mathbf{AX \leq b, X \geq 0,}$
Where
C is a (1×n) row vector of coefficients of the objective function
X is a (n×1) column vector of the variables
A is an (m×n) matrix of the constrains coefficients
B is an (m×1) column vector
X_{ir} is a basic variable $r \in \{1,\ldots,m\}$
x_{i_r} .is a no basic variable $i\alpha \in \{1,\ldots,m-n\}$

The optimal solution is

$$x_{i_{r_r}} = d_{r0} - \sum_{i=1}^{n-m} d_{ri} x_{i\alpha}$$

$$x_{i\alpha} = 0, \qquad r = 1,2,\ldots,m, \qquad i = 1,2\ldots,n-m$$

and the value of the objective function is:

$.\mathbf{z}=\mathbf{d}_{-1,0}-\sum^{n-m}_{i=1}\mathbf{d}_{-1,i}\mathbf{x}_{i\alpha}$

where

$\mathbf{d}_{-1,i}=\sum^{m}_{r=1}\mathbf{d}_{ri}\,c_{\alpha r}$
$-\,c_{\alpha r}$, i=0,1,…,m

and the condition of optimality of (2.1) is

$\mathbf{d}_{-1,i}\geq\mathbf{0}$, i=1,2…,m.
Now the parametric problem is of the form**:**

Max{$\mathbf{Z}=\sum^{n}_{i=1}(C_{\alpha i}+\eta\,C'_{\alpha i})X_i$
Subject to
$\mathbf{AX\leq B,\ X\geq 0,}\eta\in[\eta_{-},\eta_{+}]$

Where

η: is the parameter
η_{+} is the upper limit of η
η_{-} is the lower limit of η
C´: is a $(1\times n)$ variation row vector of coefficients of the objective function

We use

I. Available Resources

1. Total area of winter crops≤120000 fed.
2. The available irrigation water≤1.42E+08 m^3
3. The available number of workers≤4.2E+06 man/day
4. Available phosphate units≤1.8E+06 Units
5. Available azotic units≤8E+06 Units
6. Available potassium units≤0.6E+06 Units
7. Total finance≤80E+6 L.E.

II.. The capacity power of markets

1. $x_1\geq 34000$fed.
2. $x_3\geq 11000$fed.
3. Area of winter crops ≥15000fed
4. Area of winter vegetables ≥13000 fed.
5. $x_4\geq 14000$fed

where x_i: number of feddans (fed) of the i^{th} crop, i=1,…,13.

III **Requirements of water , workers , fertilizers and total cost of the** crops.

Table 1 shows the requirements of water , workers/day , fertilizers and total cost of the crops/fed. of the Ameriya region..

Each fed. of	Irrigation water m^3	Workers Man/day	Azotic units	Phosphate units	Potassium units	Total cost L.E.
$.x_1$	1609	18	99	18		720
x_2	1408	17		15	67	538
x_3	1281	38	49	21		903
x_4	878	19	50	22.5		385
x_5	1361	12		15		544
x_6	1361	50	33.5	30	31	1209
x_7	1361	31	33.5	15		1514
x_8	1361	18	324	15		655

x_9	1361	119	202.4	45		311
x_{10}	1361	28	33.5	66	60	2784
x_{11}	1361	38	33.5	15		871
x_{12}	1361	21	33			469
x_{13}	1361	32	15.5	15	24	745

Table 1 Requirements of water , workers/day , fertilizers and total cost of the crops/fed. of the Ameriya region

We formulate the following LP model with the previous objective function according to the previous data and constraints.

AMM:

Max $G_1=575\,x_1+207\,x_2+258.5\,x_3+403\,x_4+1949\,x_5+1041\,x_6+1040\,x_7+877\,x_8+4673\,x_9+1541\,x_{10}+320\,x_{11}+839\,x_{12}+1907\,x_{13}$

"Net Profit Objective Function"

Min $G_2=1609\,x_1+1408\,x_2+1281\,x_3+878\,x_4+1361\,x_5+1113\,x_6+1361\,x_7+1361\,x_8+1361\,x_9+1361\,x_{10}+1361\,x_{11}+1361\,x_{12}+1361\,x_{13}$

"Irrigation water Objective Function"

Min $G_3=18\,x_1+17\,x_2+38\,x_3+19\,x_4+12\,x_5+50\,x_6+31\,x_7+18\,x_8+119\,x_9+28\,x_{10}+38\,x_{11}+21\,x_{12}+32\,x_{13}$

"Workers Objective Function"

Min $G_4=99\,x_1+49\,x_3+50\,x_5+33.5\,x_7+33.5\,x_8+324\,x_9+202.4\,x_{10}+33.5\,x_{11}+33.5\,x_{12}+15.5\,x_{13}$

"Azotic units Objective Function"

Subject to:

$1609\,x_1+1408\,x_2+1281\,x_3+878\,x_4+1361\,x_5+1113\,x_6+1361\,x_7+1361\,x_8+1361\,x_9+1361\,x_{10}+1361\,x_{11}+1361\,x_{12}+1361\,x_{13}\le 1.42E+08$

"Irrigation water constraint"

$18\,x_1+17\,x_2+38\,x_3+19\,x_4+12\,x_5+50\,x_6+31\,x_7+18\,x_8+119\,x_9+28\,x_{10}+38\,x_{11}+21\,x_{12}+32\,x_{13}\le 4.2E+06$

"Workers constraint"

$99\,x_1+49\,x_3+50\,x_5+33.5\,x_7+33.5\,x_8+324\,x_9+202.4\,x_{10}+33.5\,x_{11}+33.5\,x_{12}+15.5\,x_{13}\le 8E+06$

"Azotic units constraint"

$18\,x_1+15\,x_2+21\,x_3+22.5\,x_4+15\,x_5+30\,x_6+15\,x_7+15\,x_8+45\,x_9+66\,x_{10}+15\,x_{11}+15x_{13}\le 1.8E+06$

"Phosphate constraint"

$67\,x_2+31\,x_6+60\,x_{10}+24\,x_{13}\le 0.6E+06$

"Potassium constraint"

$720\,x_1+538\,x_2+903\,x_3+385\,x_4+544\,x_5+1209\,x_6+1514\,x_7+655\,x_8+311\,x_9+2784\,x_{10}+871\,x_{11}+469\,x_{12}+745\,x_{13}\le 8E+07$

"Total changeable costs constraint"

$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}+x_{11}+x_{12}+x_{13}\le 120000$

"Crops area constraint"

$x_1\ge 34000$

"Wheat area constraint"

$x_3\ge 11000$

"Bean area constraint"

$x_2+x_5+x_6+x_7+x_8+x_9+x_{11}+x_{12}+x_{13}\ge 15000$

$x_7+x_8+x_9+x_{10}+x_{11}+x_{12}+x_{13}\ge 13000$

$x_4\ge 17000$

The feasible set is denoted by M. We refer to the previous objective function (OF) in the sequel by the following names

G_1: Net profit OF

G_2: Irrigation water OF

G_3: Workers OF

G_4: Azotic units OF

We will solve this problem using the previous study, Nozicka et.al. (1985), the WinQSB GP-IGIP software 1.0 for Windows and WinQSB LP-ILP software version 1.0 for windows. Table 3 shows the matrix inputs to the WinQSB program for the presented Ameriya LP mathematical model.,

We will apply the following procedure in solving the problem:

Step 1
Optimizing the individual OF
Step 2.

Applying the Goal Programming method with different orders Table 4 presents the parametric analysis for G1.
Parametric Analysis for Ameriya Model: Goal Programming—Goal GI

Range	μ (vector)	Goal Value G3	Goal Value G1	Goal Value G2	Goal GI Slope	Leaving Variable	Entering Varaible
1	-M				44,543,500	Basis	Persits
2	0	1,611,000	44,543,500	104,138,000	44,543,500	Original	Solution
3	M				44,543,500	Basis	Persits

Table 4. Parametric Analysis for Ameriya Model with Three Objective Functions Fourth Order-Parameter in Coefficient of the Profit Objective Function

Tables 5 and 6 presents the results for Irrigation water and number of workers constraints respectively.

Parametric Analysis for Ameriya Model: Three Objective Functions-Right Hand Side

Range	R.H.S. of CI	Goal Value G3	Goal Value G1	Goal Value G2	Goal G3 Slope	Goal G1 Slope	Goal G2 Slope	Leaving Variable	Entering Varaible
1	83,723,000	1,923,000	34,044,500	104,138,000	-0.02	1.20	0.00	Becomes	Feasible
2	86,445,000	1,871,000	37,302,500	104,138,000	-0.01	0.41	0.00	Surplus_C11	X8
3	104,138,000	1,611,000	44,543,500	104,138,000	0	0	0	BasisX11	Slack_C1
4	142,000,000	1,611,000	44,543,500	104,138,000	0	0	0	Original	Solution
5	M				0	0	0	Basis	persists

Table 5. Parametric Analysis for Ameriya Model with Three Objective Functions Fourth Order-Parameter in the irrigation Water Quantitiy Constraint

These results allowed to increase the efficiency of the management.

Parametric Analysis for Ameriya Model: Three Objective Funcitons-Right Hand Side

Range	R.H.S. of C2	Goal Value G3	Goal Value G1	Goal Value G2	Goal G3 Slope	Goal G1 Slope	Goal G2 Slope	Leaving Variable	Entering Varaible
1	1,611,000	1,611,000	44,543,000	104,138,000	0	0	0	Becomes	Feasible
2	4,200,000	1,611,000	44,543,000	104,138,000	0	0	0	Original	Solution
3	M				0	0	0	Basis	persits

Table 6. Parametric Analysis for Ameriya Model with Three Objective Functions Fourth Order-Parameter in the Number of Workers Constraint

REFERENCES

AGRAWAL, R.C. and E.O. HEADY (1972): **Operations Research Methods for Agricultural Decisions**. Iowa State University Press, Iowa**.**

AINER, N.G., W.I, MISENA, F.A. ABBS and H.M. EID (1999): A new concept of rationalization of irrigation water use in Egypt. **Third Conference of On Farm Irrigation** and **Agroclimatology**.

AKINWUMI, A, J, ADESINA and S. SANDERS (1991): Peasant farmer behaviour and cereal technologies: stochastic programming analysis in Niger. **Agricultural Economics**, **5**, 21-38.

BROSOWSKI , B. (1983): On vector optimisation and parametric programming. **Proc. Of the Conference of Parametric Optimisation and Approximation. Oberwolfach.**

BURKHARD, R. E., J KARUP and P.M PRUZAN (1984): Some relationships between Multicriteria and Parametric discrete optimisation problems with bottleneck criteria. **Math. Oper. Sta. Series Optimisation 3**, 383-395

CAN-OIL, C.W. (1981): The created response surface technique for optimising non-linear restraint systems. **Oper. Res. 9**, 169-184.

D´ALFONSO, T.H. and W.B. ROUSH (1990): A comparison of stochastic programming, linear programming with a margin of safety for least const poultry rations. **Poultry Sc. Journal. 1**, 35-42

FIACCO, A.V. and G.P MCCORMICK (1968): **Non-linear programming sequential unconstrained minimization techniques.** Wiley, N. York

GEFFERY, S.R. , R.R. GIBSON and M.D. FAMMINOW(1992): Nearly optimal programming as a guide to agricultural planning. **Agricultural Economics. 8**, 1-19

GUDDAT, J., F. GUERRA-, K.TAMMER and K. WENDLER (1985**): Multiobjective and stochastic optimisation based on parametric optimisation.** Akedemie Verlag, Berlin

GUPTA P.K and D.S. HIRA (1999): **Operations Research.** S. Chand and Company Ltd, Bombay

HAZELL, P.B.R. and R. D. NORTON (1988**): Mathematical Programming for Economic Analysis in Agriculture.** McMillan , N. York.

HILLIER, F. S.and G. J. LIEBERMAN (1995): **Introduction to Operations Research.** McGraw Hill, N. York.

IJIRI, Y. (1965): **Management Goals an Accounting for Control.** Rand McNally, Chicago

JAHAN, J, (1984): Secularization vector optimisation. **Math Programming, 29**, 203-218

KARLIN, s. (1959): **Mathematical Methods and Theory in Games, Programming and Economics. Vol. 1.** Addison Wesley, Cambridge.

LEE, S. (19672): **Goal Programming for Decision Analysis.** Auerbanc Pub., Philadelphia

LIN, J. G.. (1976): Maximal vector multiobjective optimization. **J. of Optimization Theory and App. 18,** 41-64

LYSKA, W. (1982): Ein Neuer Parametrischer Zugang zur Vektoroptimierung. **Wiss. Zeitschriftt Th. 28,** 77-91

MANSOUR M.E. , A. O. EL-BALACY and S. ADIY (2000): Impact of economic liberalization policies on forming Egypt's sold land cropping patterns**. Agricultural Economics Research Institute.**

MANSOUR M.E. , E. M. ELGAMASSY AND A. M. ELSHATER (2000): The effect of economic liberalization policies on cropping pattern in the new lands, **. Agricultural Economics Research Institute**

MONKE, E.A, and S. R. PEARSON (1989): The policy Analysis Matrix for Agriculture Development. **Cornel Univ. Press**, Ithaca.

MURTY, K. G. (1996): **Operations Research Deterministic Optimization Model**. McGraw Hill, N. York

NASH S. G. and A. SOFER (1996**): Linear and Non-linear Programming.** McGraw Hill, N. York

Nozicka, F. , J. Guddat, H. Hollatz and B. Bank (1985): **Theorie Linearen Parametrischen Optimierung.** Akademie Verlag. Berlin.

RAO, S.S. (1995): **Optimisation Theory and Applications**. Formerly Eastern Limited Publications ., Bombay

RARDI, R.L. (1998): Optimization in Operations Research. **Prentice Hall**, N. York
RAVINDRAN, D., T. PHILLIPS and J.J. SOLBERG (1987): Operations Research Principles and Practice. **Wiley and Sons**, N. York

SHALABY, A.E. (1989): A study and Modification in Solving some Parametric linear Programming Problems. (**PhD Thesis), Cairo University , Faculty of Engineering**.

TAHA, H.A. (1992): Operations Research: An Introduction. **McMillan Pub. House**, N. York.

TAMMER K. (1978): Relations between stochastic and parametric programming for decision problems with random objective function. **Math. Optimization and Statistics. . Ser Opt.. 9**, 523-535

WIERZBICKI, A.P. (1977): Basic properties of secularising functions for multiobjective optimisation. **Math. Optimization and Statistics. . Ser Opt.. 8,** 55-60

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