JOINT OPTIMIZATION OF PRICE AND CYCLE LENGTH UNDER TWO-STAGE CREDIT POLICY WITH CREDIT-LINKED DEMAND

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ABSTRACT:
The objective of this paper is to jointly optimize the retailer's selling price and replenishment cycle under two-stage credit policy to reflect the real-life situations. Usually it is assumed that the supplier would offer the retailer a delay period but the retailer in turn would not offer the trade credit to his customers, which is unrealistic, because in reality retailer does offer the delay period to his customers in order to stimulate his own demand. Moreover the interaction between delay period and demand of an item ignored by the researchers, but it is observed that demand of an item does depend upon the credit period offered by the retailer to its customers. In order to incorporate this phenomenon, it is assumed that retailer's sales are divided in two categories (i) on cash, which is taken as a decreasing function of unit price and (ii) on credit, which is taken as a function of customer's credit period offered by the retailer. We then provide a solution procedure for determining the retailer's optimal price and cycle length simultaneously. Finally, a numerical example is given to illustrate the theoretical results followed by the sensitivity analysis of parameters on the optimal solution.

KEY WORDS: Inventory, credit-linked demand, delay in payments, pricing, two-stage credit policy

MSC: 90B05

RESUMEN:
El objetivo de este trabajo es el optimizar simultáneamente los precios de distribución y el ciclo de reabastecimiento con una política bietápica de crédito para reflejar situaciones de la vida real. Usualmente se asume que el suministrador ofrece al distribuidor un periodo de espera pero este deberá no ofrecer créditos comerciales a sus clientes, lo que es no realista, pues en la realidad el distribuidor ofrece en periodo de espera a sus clientes para estimular su propia demanda. Mas aun la interacción entre el periodo de espera y demanda de un ítem es ignorada por los investigadores, pero se observa que la demanda de un ítem depende del periodo de crédito ofertado por el suministrador a sus clientes. Para incorporar este fenómeno, se asume que las ventas del distribuidor se dividen en dos categorías (i) en efectivo, del que se considera como una función decreciente del precio por unidad y (ii) a crédito, el que se toma como una función del periodo de crédito ofertado por el distribuidor al cliente. Nosotros proveemos un procedimiento de solución para determinar el precio optimo del distribuidor y la amplitud del ciclo simultáneamente. Finalmente, un ejemplo numérico es dado para ilustrar los resultados teóricos seguido de un analiza de sensibilidad de los parámetros de la solución optima.

1. INTRODUCTION

In classical inventory analysis, it was tacitly assumed that the supplier is paid for the items as soon as the retailer receives the items. But in practice, the supplier allows a certain fixed credit period to settle the account for stimulating retailer's demand. During this credit period the retailer can start to accumulate revenues on the sales and earn interest on that revenue, but beyond this period the supplier charges interest. Hence, paying later indirectly reduces the cost of holding stock. On the other hand, trade credit offered by the supplier encourages the retailer to buy more and it is also a powerful promotional tool that attracts new customers, who consider it as an alternative incentive policy to quantity discounts. Hence, trade credit can play a major role in inventory control for both the supplier as well as retailer.

Owing to this fact, during the past few years, many articles dealing with various inventory models under trade credit have appeared in various research journals. *Haley and Higgins* (1973) introduced the first model to consider the economic order quantity under conditions of permissible delay in payments with deterministic demand, no shortages, and zero-lead time. *Goyal* (1985) considered a model similar to that of *Haley and Higgins* (1973) with the exclusion of the penalty cost due to a late payment. *Chung* (1989) presented the discounted cash flows (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. *Shah* (1993) and *Aggarwal and Jaggi* (1995) extended the *Goyal's* (1985) model to the case of deterioration. *Jamal et al*. (2000) further generalized the model to allow shortages. *Jaggi and Aggarwal* (1994) extended *Chung* (1989) to develop an inventory model for obtaining the optimal order quantity of deteriorating items in the presence of trade credit using the DCF approach. *Hwang and Shinn* (1997) considered the problem of determining the retailer's optimal price and lot size simultaneously when the supplier permits delay in payments. *Dye* (2002) in their paper considered the stock dependent demand for deteriorating items for partial backlogging and condition of permissible delay in payment. They assumed initial stock dependent demand function. *Teng* (2002) provided an alternative conclusion from *Goya*l (1985), and mathematically proved that it makes economic sense for a buyer to order less quantity and take benefits of the permissible delay more frequently. *Chang, Hung and Dye* (2004) considered an inventory model for deteriorating items with instantaneous stock-dependent demand and time-value of money when credit period is provided.

All the aforementioned inventory models implicitly assumed that the customer would pay for the items as soon as the items are received from the retailer. But, in most business transactions, this assumption is unrealistic and usually the supplier offers a credit period to the retailer and the retailer, in turn, passes on this credit period to his/her customers. Recently, *Huang* (2003) presented an inventory model assuming that the retailer also offers a credit period to his/her customer which is shorter than the credit period offered by the supplier, in order to stimulate the demand. Moreover, in all the above articles, although the presence of credit period has been incorporated in the mathematical models but the impact of credit period on demand is unfortunately ignored. In reality, it is observed that demand of an item does depend upon the length of the credit period offered by the retailer. In order to incorporate the above phenomena, a new form of credit-linked demand function has been coined using which an inventory model has been formulated to determine the retailer's optimal pricing and ordering policy when both the supplier as well as the retailer offers the credit period to stimulate customer demand.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are made to develop the mathematical model:

1. The supplier provides a fixed credit period *M* to settle the account to the retailer and retailer, in turn, passes on a maximum credit period *N* to its customers to settle the account.
2. At different points of time, the customer would get different lengths of credit period from the retailer. If the customer makes the purchases at time $t = 0$, then he gets the maximum credit period *N*; no credit period is offered to the customer if he makes the purchases at time $t = N$; and the customer gets credit period equal to ($N - t$) if he makes the purchases between time $t = 0$ and *N.* For simplicity, it is assumed that that the customer's credit period *N* is less than or equal to the retailer's credit period *M.* It is also assumed that the customers would settle their accounts only on the last day of the credit period *N*.
3. The annual demand rate consists of (i) regular cash-demand and (ii) credit-demand. Hence, demand function at any time *t* can be represented as

$$D(t) = \begin{cases} \lambda(P) + R(t) & 0 \le t \le N \\ \lambda(P) & N \le t \le T \end{cases}$$

   where

   $\lambda(P)$ is regular demand (i.e. cash sales) and is a decreasing function of unit price (*P*); we assume $\lambda(P) = kP^{-e}$ where k>0 and e>1

R(t) is the demand (i.e. credit sales) due to the maximum credit period *N* offered by the retailer ; we assume $R(t) = \alpha(N - t)$, where $0 \le t \le N$ and $\alpha > 0$.

4. Replenishment rate is instantaneous.
5. Shortages are not allowed.
6. Lead-time is negligible.

In addition following notation are also used in this paper:

Q	order quantity
T	inventory cycle length
$q(t)$	the inventory level at time t
A	ordering cost per order
C	unit purchase cost of the item
P	unit selling price of the item
I	out-of-pocket inventory carrying charge per \$ per year
I_e	interest that can be earned per \$ per year
I_p	interest payable per \$ per year
M	retailer's credit period offered by the supplier for settling the accounts
N	maximum credit period offered by the retailer to the customers, where $N \le M$ and also $N \le T$
$Z(T, P)$	retailer's annual profit which is a function of T and P. where the retailer's annual profit = (a) revenue from sales - (b) cost of purchasing units - (c) cost of placing orders - (d) cost of carrying inventory (excluding interest charges) + (e) interest earned from the sales during the permissible period - (f) cost of interest charges for the unsold items after the permissible delay.

3. MATHEMATICAL FORMULATION

As the demand function is

$$D(t) = \begin{cases} kP^{-e} + \alpha(N - t) & 0 \le t \le N \\ kP^{-e} & N \le t \le T \end{cases} \tag{1}$$

the order quantity can be calculated as

$$Q = \int_0^T D(t)dt = \lambda(P)T + \frac{\alpha}{2}N^2 \tag{2}$$

and the inventory level at any time t during the cycle is (*figure 1*)

$$q(t) = \begin{cases} q_1(t) = Q - \int_0^t \{\lambda(P) + \alpha(N - t)\}dt & 0 \le t \le N \\ q_2(t) = q_1(N) - \int_N^t \lambda(P)dt & N \le t \le T \end{cases}$$

$$= \begin{cases} q_1(t) = \lambda(P)(T - t) + \alpha(N - t)^2/2 & 0 \le t \le N \\ q_2(t) = \lambda(P)(T - t) & N \le t \le T \end{cases} \tag{3}$$

The retailer's annual profit consists of the following elements:

(a) Sales revenue $= PQ/T$

$$= \frac{P}{T}\left(\lambda(P)T + \frac{\alpha N^2}{2}\right) \tag{4}$$

(b) Cost of placing orders $= A/T$ (5)

(c) Cost of purchasing units $= CQ/T$

$$= \frac{C}{T}\left(\lambda(P)T + \frac{\alpha N^2}{2}\right) \tag{6}$$

(d) Cost of carrying inventory = $\frac{IC}{T}\left(\int_0^N q_1(t)dt + \int_N^T q_2(t)dt\right)$

$$= \frac{IC}{2T}\left(\lambda(P)T^2 + \alpha N^3/3\right) \tag{7}$$

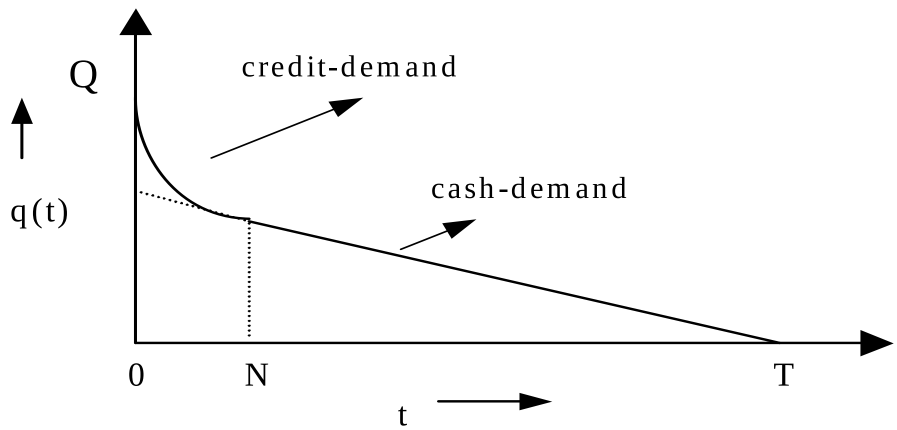

Figure 1

The computation for interest earned and payable [i.e. (e) and (f)] will depend on the following two possible cases based on the lengths of *T* and *M*:

Case1: $M \le T$

In this case, the retailer deposits the accumulated revenue from cash sales during the period (*0, M*) and also the accumulated revenue from the credit sales during the time period (*N, M*) into an account that earns an interest rate of I_e. At *M* the accounts have to be settled, it is assumed that accounts will be settled by proceeds of sales generated up to *M* and by taking a short term loan at an interest rate of I_p for the duration of (*T-M*) for financing the unsold stock.

Consequently the interest earned per year is

$$\frac{I_eP}{T}\left\{\int_0^M \lambda(P)tdt + \int_N^M \frac{\alpha N^2}{2}dt\right\} = \frac{I_eP}{2T}\left\{\lambda(P)M^2 + \alpha N^2(M-N)\right\} \tag{8}$$

and the interest payable per year is

$$\frac{I_pC}{T}\int_M^T q_2(t)dt = \frac{I_pC\lambda(P)}{2T}(T-M)^2 \tag{9}$$

Using the equations (4) to (9), the retailer's annual profit $Z_1(T,P)$ can be expressed as

$$Z_1(T,P) = \frac{1}{2T}\Big\{(P-C)\left(2\lambda(P)T + \alpha N^2\right) + I_eP\left(\lambda(P)M^2 + \alpha N^2(M-N)\right) - 2A - IC\left(\lambda(P)T^2 + \alpha N^3/3\right)$$
$$- I_pC\lambda(P)(T-M)^2\Big\} \tag{10}$$

Case2: M ≥ T

Here the credit period *M* is more or equal to the cycle *T*, so the retailer earns interest on cash-sales during the period (0, *M*) and also on credit-sales during the time period (*N, M*) but there is no interest payable. Therefore, the interest earned per year is

$$\frac{I_e P}{T}\left\{\int_0^T \lambda(P)t\,dt + \int_N^T \frac{\alpha N^2}{2}dt + \int_T^M Q\,dt\right\}$$

$$= \frac{I_e P}{2T}\left\{\lambda(P)T(2M - T) + \alpha N^2 (M - N)\right\} \tag{11}$$

As a result, using the equations (4), (5), (6), (7) and (11), the retailer's annual profit $Z_2(T,P)$ in this case is

$$Z_2(T,P) = \frac{1}{2T}\left\{(P-C)\left(2\lambda(P)T + \alpha N^2\right) + I_e P\left(\lambda(P)T(2M-T) + \alpha N^2 (M-N)\right) - 2A - IC\left(\lambda(P)T^2 + \alpha N^3/3\right)\right\} \tag{12}$$

Therefore, the retailer's annual profit *Z(T, P)* is

$$Z(T,P) = \begin{cases} Z_1(T,P) & \text{if} \quad M \le T \\ Z_2(T,P) & \text{if} \quad \mathrm{M} \ge \mathrm{T} \end{cases} \tag{13}$$

Since $Z_1(M,P) = Z_2(M,P)$, *Z(T, P)* is a continuous and well-defined on *T* > 0.

4. DETERMINATION OF THE OPTIMAL CYCLE LENGTH FOR A FIXED PRICE

Our problem is to determine the optimum value of *T* and *P* which maximizes *Z(T, P).* For a fixed value of *P*, by taking the first and second order derivatives of $Z_1(T, P)$ and $Z_2(T, P)$ with respect to *T*, we get

$$\frac{\partial Z_1(T,P)}{\partial T} = \frac{2A + \left(I_p C - I_e P\right)\lambda(P)M^2 - \alpha N^2 S - C\lambda(P)T^2\left(I + I_p\right)}{2T^2} \tag{14}$$

$$\frac{\partial Z_2(T,P)}{\partial T} = \frac{2A - \alpha N^2 S - \lambda(P)T^2\left(IC + I_e P\right)}{2T^2} \tag{15}$$

$$\frac{\partial^2 Z_1(T,P)}{\partial^2 T} = \frac{-2A - \left(I_p C - I_e P\right)\lambda(P)M^2 + \alpha N^2 S}{T^3} \tag{16}$$

and

$$\frac{\partial^2 Z_2(T,P)}{\partial^2 T} = \frac{-2A + \alpha N^2 S}{T^3} \quad \text{where } S = \left[(P - C) + I_e P(M - N) - ICN/3\right] \tag{17}$$

Consequently, $Z_1(T, P)$ is strictly concave on *T* > 0 for a fixed *P* if

$$2A > \left(I_e P - I_P C\right)\lambda(P)M^2 + \alpha N^2 S \tag{18}$$

and $Z_2(T, P)$ is strictly concave on *T* > 0 for a fixed *P* if

$$2A > \alpha N^2 S \tag{19}$$

Thus, there exists a unique value of T_1 which maximizes $Z_1(T, P)$ as

$$T_1 = \sqrt{\frac{2A + \left(I_p C - I_e P\right)\lambda(P)M^2 - \alpha N^2 S}{\left(I + I_p\right)C\lambda(P)}} \tag{20}$$

Similarly, there exists a unique value of T_2 which maximizes $Z_2(T, P)$ as

$$T_2 = \sqrt{\frac{2A - \alpha N^2 S}{\lambda(P)(I_e P + IC)}} \tag{21}$$

5. DETERMINATION OF THE OPTIMAL PRICE

Since the cash demand rate $\lambda(P)$ is a function of P, therefore, each T_i can be represented by a real valued function of P *i.e.* $T_i = T_i(P)$. Substituting T with T_i in $Z_i(T, P)$, we have problem of maximizing $Z_i(T_i(P), P)$ which is single variable problem subject to optimality conditions. Hence from (10) and (12), the following single variable objective functions are obtained for the two possible cases.

$$Z_1(P) = (P - C + I_P CM)\lambda(P) - \sqrt{\{2A + \lambda(P)M^2(I_P C - I_e P) - \alpha N^2 S\}C\lambda(P)(I + I_P)} \tag{22}$$

and

$$Z_2(P) = (P - C + I_e PM)\lambda(P) - \sqrt{\{2A - \alpha N^2 S\}\lambda(P)(IC + I_e P)} \tag{23}$$

To determine the optimal price, consider the following mathematical programming problems for the two possible cases viz. $M \le T$ and $M \ge T$:

Problem1 (P1)

Max $Z_1(P)$

subject to

$2A > (I_e P - I_P C)\lambda(P)M^2 + \alpha N^2 S$ [using (18)]

$P \ge 0$

Problem1 (P2)

Max $Z_2(P)$

subject to

$2A > \alpha N^2 S$ [using (19)]

$P \ge 0$

The optimal values of unit price P (say P_1 for **P1** and P_2 for **P2)** can be calculated using any optimization software e.g. Lingo, Solver. To determine the optimal policy following procedure is adopted.

6. SOLUTION PROCEDURE

1. Determine P_1 by solving Problem **P1**. Now obtain the optimal value of T i.e. T_1 by substituting the P_1 in (20). If $T_1 \ge M$, then determine $Z_1(T_1, P_1)$ using (10). Otherwise set $Z_1(T_1, P_1) = 0$.
2. Determine P_2 by solving Problem **P2**. Now obtain the optimal value of T i.e. T_2 by substituting the P_2 in (21). If $M \ge T_2$, then determine $Z_2(T_2, P_2)$ using (12). Otherwise set $Z_2(T_2, P_2) = 0$.
3. If $Z_1(T_1, P_1) \ge Z_2(T_2, P_2)$ then the optimal annual profit is $Z^*(T, P) = Z_1(T_1, P_1)$ and stop. Otherwise, $Z^*(T, P) = Z_2(T_2, P_2)$ and stop.

7. NUMERICAL ANALYSIS

Let A = \$60/order, k = 400000, α = 10000, C = \$3/unit, I_p = 15%, I_e = 6% and I = 9%, e = 2.5. Using the above solution procedure, we obtain the results for various values of M and N, which is shown in *Table1*.

N (days)	M (days)	P^* (\$)	T^* (days)	$Q(T^*)$ (units)	Z^* (\$)
0	30	5.043	57.92	1111	13768
	45	5.026	59.68	1155	13871
	60	5.015	62.22	1211	13964
10	30	5.050	54.35	1043	13818
	45	5.034	56.26	1088	13919
	60	5.022	58.69	1142	14011
20	30	5.085	41.55	796	13995
	45	5.069	43.89	846	14088
	60	5.056	43.79	850	14176

Table1

Table1 shows that as the M increases for any fixed N there is there is increase in cycle length, order quantity and annual net profit but there is marginal decrease in unit price while as the N increases for any fixed M, cycle length and order quantity decreases but unit price and annual net profit increases.

8. CONCLUSION

This paper develops an inventory model for the joint optimization of the retailer's optimal price and cycle length under two-stage credit policy with credit-linked demand. A solution procedure is proposed which gives the decision rule for obtaining the retailer's optimal price and cycle length. Finally numerical examples are presented to illustrate the theoretical results. Results suggest that retailer should order more and charge lower unit price as the retailer's credit period (M) increases. Further, results indicate that when the customer's credit period (N) increases then he should order more frequently and charge higher unit price. In further research, this paper can be extended for the deteriorating item.

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