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OPTIMAL ORDERING POLICY FOR DETERIORATING ITEMS UNDER THE DELAY IN PAYMENTS IN DEMAND DECLINING MARKET

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RESUMEN
En este trabajo se hace un enfoque para caracterizar un modelo de inventario para artículos que se deterioran en el mercado declinante cuando el suministrador ofrece una demora permisible en pagos. El algoritmo es mostrado, para un suministrador, para determinar la cantidad óptima a adquirir que minimiza el costo total del inventario por unidad de tiempo. Un ejemplo numérico es dado para determinar el flujo de la decisión óptima para el suministrador. La sensibilidad es analizada para analizar los cambios en la solución óptima con respecto a la tasa del empeoramiento de unidades en el inventario y la tasa del cambio de la demanda.

ABSTRACT:
In this paper, an attempt is made to characterize the inventory model for deteriorating items in declining market when the supplier offers a permissible delay in payments to the retailer to settle the account against the purchases. The algorithm is exhibited for a retailer to determine the optimal procurement quantity which minimizes the total inventory cost per time unit. A numerical example is given to demonstrate the flow of the optimal decision for the retailer. The sensitivity is carried out to analyse the changes in the optimal solution with respect to deterioration rate of units in inventory and the rate of change of demand.

KEY WORDS: Deterioration, lot-size, trade credit, declining demand.

MSC: 90B05

1. INTRODUCTION

The classical Wilson's economic ordering quantity (EOQ) model is derived under the assumption that the retailer settles the account immediately for the goods received in inventory. Brigham (1995) defined "net 30" which means a supplier allows 30-days time period to settle the total amount owed to him. The supplier does not charge any interest for the amount if it is paid within the allowable permissible delay period. However, if the payment is not settled within the allowable credit period, then interest is charged on the amount. However, a retailer can earn the interest on the revenue generated and delay the payment till the last allowable date of tread credit by the supplier. The permissible tread credit reduces the retailer's total cost, i.e. it is considered as a sales promotion tool for the supplier to attract new customers. However, the strategy of granting credit terms adds not only an additional cost to the supplier but also default risk to the supplier (Teng et al. (2005)).

Goyal (1985) developed an EOQ model under the condition of permissible delay in payments. He ignored the difference between the selling price and purchase cost, and concluded that the cycle time and order quantity increases marginally under the permissible delay in payments. Dave (1985) corrected the Goyal's model by assuming the fact that the selling price is higher than its purchase price. Shah (1993a) formulated a mathematical model when units in inventory are subject to constant deterioration and tread credit is offered to the retailer by the supplier. Shah (1993b, 1993c) derived probabilistic inventory model under the assumption of permissible delay in payment. Hwang and Shinn (1997) formulated the optimal pricing and ordering policies for the retailer under the scenario of allowable trade credit. Liao et al. (2000) developed an inventory model under same scenario when demand is stock-dependent. Most of the

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above stated articles ignored the difference between unit sale price and unit purchase cost, ending to the similar conclusions as those of Goyal (1985).

Jamal et al. (1997, 2000) and Sarker et al. (2000) considered the difference between the unit sale price and unit purchase cost and established that the retailer should settle the account somewhat sooner as the unit selling price increases relative to the unit cost. Teng (2002) provided an alternative conclusion that the well-established retailer should place order of smaller size to avail of the permissible delay more frequently. The most of the above stated study is done under the assumption of the constant and known deterministic demand. Chang et al. (2003) modelled the scenario when supplier offers trade credit to the buyer if the order quantity is greater than or equal to a pre-determined quantity. Ouyang et al. (2006) allowed partial shortages and credit period to settle the dues against the purchases. Chang et al. (2006) derived optimal ordering and pricing policies for deteriorating inventory problem when partial backlogging and trade credits are offered. Chung and Huang (2009) determined optimal ordering policy under conditions of allowable shortages and permissible delay in payments. Ouyang et al. (2009) considered partial trade credit linked to order quantity in deteriorating inventory model.

In this paper, the demand of a product is assumed to be decreasing with time. The decrease in demand is observed for fashionable garments, seasonal products etc. Shortages are not allowed and replenishment rate is infinite. It is assumed that the retailer generates revenue on unit selling price which is necessarily higher than the unit purchase cost. The total cost of an inventory system per time is minimized. The model is supported by a numerical example. The sensitivity analysis is carried out to observe the changes in the optimal solution.

2. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used to develop proposed mathematical model.

2.1 Notations:

$R(t) := a(1-bt)$; the annual demand as a decreasing function of time where a > 0 is fixed demand and $b(0 < b < 1)$ denotes the rate of change of demand.

C : the unit purchase cost.

P : the unit selling price with (P > C).

h : the inventory holding cost per unit per year excluding interest charges.

A : the ordering cost per order.

M : the permissible credit period offered by the supplier to the retailer for settling the account.

I_c : the interest charged per monetary unit in stock per annum by the supplier.

I_e : the interest earned per monetary unit per year.

Note : $I_c > I_e$

Q : the order quantity (a decision variable)

θ : the constant deterioration rate, where $0 < \theta < 1$.

$I(t)$: the inventory level at any instant of time t, $0 \le t \le T$.

T : the replenishment cycle time (a decision variable).

$K(T)$: the total inventory cost per time unit.

The total cost of inventory system consists of (a) ordering cost, (b) cost due to deterioration, (c) inventory holding cost (excluding interest charges), (d) interest charged on unsold item after the permissible trade credit when M < T, and (e) interest earned from sales revenue during the allowable permissible delay period.

2.2 Assumptions

1. The inventory system under consideration deals with the single item.
2. The planning horizon is infinite.
3. The demand of the product is declining function of the time.
4. Shortages are not allowed and lead-time is zero.
5. The deteriorated units can neither be repaired nor replaced during the cycle time.
6. The retailer can deposit generated sales revenue in an interest bearing account during the permissible credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day-to-day expenditure, and paying the interest charges on the unsold items in the stock.

3. MATHEMATICAL MODEL

The inventory level; $I(t)$ depletes to meet the demand and deterioration. The rate of change of inventory level is governed by the following differential equation:

$$\frac{dI(t)}{dt}+\theta I(t)=-R(t)\,, \qquad 0\le t\le T \tag{1}$$

with the initial condition $I(0)=Q$ and the boundary condition $I(T)=0$. Consequently, the solution of (1) is given by

$$I(t)=\left(\frac{a}{\theta}+\frac{b}{\theta^2}\right)\left(e^{\theta(T-t)}-1\right)-\frac{bTe^{\theta(T-t)}}{\theta}+\frac{bT}{\theta} \qquad 0\le t\le T \tag{2}$$

and the order quantity is

$$Q=I(0)=\left(\frac{a}{\theta}+\frac{b}{\theta^2}\right)\left(e^{\theta T}-1\right)-\frac{bTe^{\theta T}}{\theta} \tag{3}$$

The total cost of inventory system per time unit consists of the following:

a) Ordering cost; $OC=\dfrac{A}{T}$ (4)

b) Cost due to deterioration per time unit;

$$DC=\frac{C}{T}\left[Q-\int_0^T R(t)dt\right]=\frac{C}{T}\left[\left(\frac{a}{\theta}+\frac{b}{\theta^2}\right)\left(e^{\theta T}-1\right)-\frac{bTe^{\theta T}}{\theta}-aT+\frac{bT^2}{2}\right] \tag{5}$$

Inventory holding cost per unit per unit time;

c) $$IHC=\frac{h}{T}\int_0^T I(t)dt=\frac{-h}{2\theta^3 T}\begin{bmatrix}-2a\theta e^{\theta T}-2be^{\theta T}+2bT\theta e^{\theta T}\\ +2a\theta+2a\theta^2T+2b-b\theta^2T^2\end{bmatrix} \tag{6}$$

Regarding interest charges and earned (i.e., costs (d) and (e) in section 2.2), two cases may arise based on the length of T and M. These two cases are exhibited in Figure1 and Figure2.

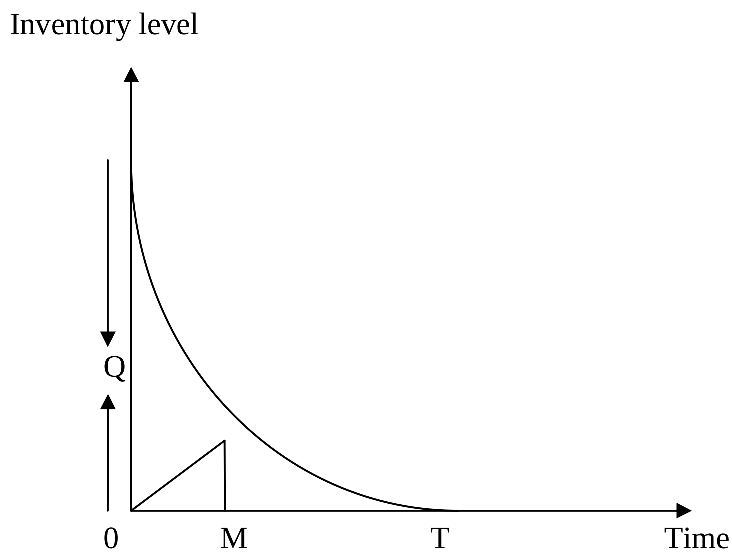

Figure.1 M< T

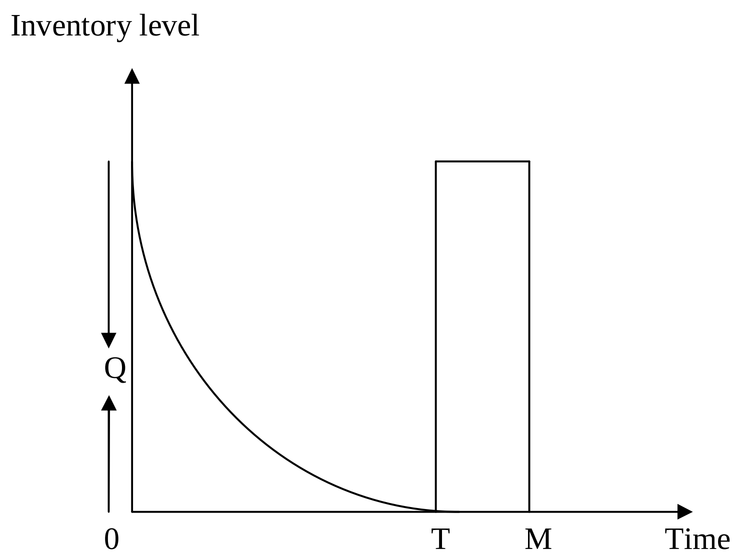

Figure. 2 M ≥ T

Case1: M < T

Under the assumption (b) above, the retailer sells R(M) M units by the end of the permissible tread credit M and has CR(M)M to pay the supplier. For the unsold items in the stock, the supplier charges an interest rate I_c from time M - onwards. Hence, the interest charged, IC_1 per time unit is

$$\text{d) } IC_1 = \frac{CI_c}{T}\int_m^T I(t)dt$$
$$= \frac{-CI_c}{2\theta^3 T}\begin{bmatrix} -2a\theta e^{\theta(T-M)} - 2a\theta^2 M - 2be^{\theta(T-M)} - 2bM\theta + 2bT\theta e^{\theta(T-M)} \\ +bM^2\theta^2 + 2a\theta + 2a\theta^2 T + 2b - b\theta^2 T^2 \end{bmatrix} \tag{7}$$

During [0, M], the retailer sells the product and deposits the revenue into an interest earning account at the rate I_e per monetary unit per year. Therefore, the interest earned, IE_1 per time unit is

$$\text{e) } IE_1 = \frac{PI_e}{T}\int_0^M R(t)tdt = \frac{PI_e}{T}\left[\frac{1}{2}aM^2 - \frac{1}{3}bM^3\right] \tag{8}$$

Hence, the total cost; $K_1(T)$ of an inventory system per time unit is

$$K_1(T) = OC + PC + IHC + IC_1 - IE_1 \tag{9}$$

Case2: T $\leq$ M

Here, the retailer sells R(T)T- units in all by the end of the cycle time and has CR(T)T to pay the supplier in full by the end of the credit period M. Hence, interest charges

$$\text{d) } IC_2 = 0 \tag{10}$$

and the interest earned per time unit is

$$\text{e) } IE_2 = \frac{PI_e}{T}\left[\int_0^T R(t)tdt + R(T)T(M-T)\right]$$
$$= \frac{PI_e}{T}\left[\frac{-abT^3}{3} + \frac{aT^2}{2} + aTM - aT^2 - abT^2M + abT^3\right] \tag{11}$$

As a result, the total cost; $K_2(T)$ of an inventory system per time unit is

$$K_2(T) = OC + PC + IHC + IC_2 - IE_2 \tag{12}$$

Hence, the total cost; $K(T)$ of an inventory system per time unit is

$$K(T) = \begin{cases} K_1(T), M \leq T \\ K_2(T), M \geq T \end{cases} \tag{13}$$

For T = M, we have

$$K_1(M) = K_2(M) = \frac{1}{M}\begin{bmatrix} C\left(\frac{a}{\theta}+\frac{ab}{\theta^2}\right)\left(e^{\theta M}-1\right)-\frac{abCMe^{\theta M}}{\theta}-aMC+\frac{abCM^2}{2}+A \\ -PI_e\left(\frac{-abM^3}{3}+\frac{aM^2}{2}\right) \\ +\frac{ha}{2\theta^3}\left[\left(2\theta+2b-2bM\theta\right)e^{\theta M}-2\theta-2M\theta^2-2b+bM^2\theta^2\right] \end{bmatrix}. \quad (14)$$

The optimum value of $T = T_1$ is the solution of

$$\frac{\partial K_1(T)}{\partial T} = \begin{bmatrix} \frac{C}{T}\left[\left(a-abT\right)e^{\theta T}+abT-a\right]-\frac{C}{T^2}\left[\left(\frac{a}{\theta}+\frac{ab}{\theta^2}\right)\left(e^{\theta T}-1\right)-\frac{abTe^{\theta T}}{\theta}+\frac{abT^2}{2}-aT\right] \\ +\frac{A}{T^2}+\frac{ha}{2T\theta^3}\left[\left(2\theta^2-2bT\theta^2\right)e^{\theta T}-2\theta^2+2b\theta^2T\right] \\ -\frac{ha}{2T^2\theta^3}\left[\left(2\theta+2b-2bT\right)e^{\theta T}-2\theta-2\theta^2T-2b+b\theta^2T^2\right] \\ +\frac{CI_c}{2T\theta^3}\left[\left(2\theta^2-2bT\theta^2\right)e^{\theta(T-M)}-2\theta^2+2b\theta^2T\right]+\frac{PI_e}{T^2}\left(\frac{-abM^3}{3}+\frac{aM^2}{2}\right) \end{bmatrix}$$

(15)

which minimizes $K_1(T)$

$$K_1(T_1) = \begin{bmatrix} \frac{C}{T_1}\left[\left(\frac{a}{\theta}+\frac{ab}{\theta^2}\right)\left(e^{\theta T_1}-1\right)-\frac{abT_1e^{\theta T_1}}{\theta}+\frac{abT_1^2}{2}-aT_1\right]+\frac{A}{T_1^2} \\ \frac{ha}{2T_1\theta^3}\left[\left(2\theta+2b-2bT\right)e^{\theta T_1}-2\theta-2\theta^2T_1-2b+b\theta^2T_1^2\right] \\ +\frac{CI_ca}{2T_1\theta^3}\left[\left(2\theta-2bT_1\theta+2b\right)e^{\theta(T_1-M)}+2M\theta^2+2bM\theta+b\theta^2T_1^2\right] \\ -\frac{CI_ca}{2T_1\theta^3}[2b+2\theta+2\theta^2T_1+bM^2\theta^2]-\frac{PI_e}{T_1^2}\left(\frac{-abM^3}{3}+\frac{aM^2}{2}\right) \end{bmatrix}$$

. (16)

The optimum value of $T = T_2$ is the solution of

$$\frac{\partial K_2(T)}{\partial T}=\left[\begin{array}{l}\frac{C}{T}\left[(a-abT)e^{\theta T}+abT-a\right]-\frac{C}{T^2}\left[\left(\frac{a}{\theta}+\frac{ab}{\theta^2}\right)\left(e^{\theta T}-1\right)-\frac{abTe^{\theta T}}{\theta}+\frac{abT^2}{2}-aT\right]+\\ \frac{A}{T^2}+\frac{ha}{2T\theta^3}\left[(2\theta^2-2bT\theta^2)e^{\theta T}-2\theta^2+2b\theta^2T\right]-\\ -\frac{ha}{2T^2\theta^3}\left[(2\theta+2b-2bT)e^{\theta T}-2\theta-2\theta^2T-2b+b\theta^2T^2\right]\\ +\frac{PI_{c}}{T}\left[-abT^2+aT-abT(M-T)+a(1-bT)(M-T)-a(1-bT)T\right]+\\ \frac{PI_e}{T^2}\left(\frac{-abT^3}{3}+\frac{aT^2}{2}+a(1-bT)T(M-T)\right)\end{array}\right]$$

(17)

which minimizes $K_2(T)$

$$K_2(T_2)=\frac{1}{T_2}\left[\begin{array}{l}C\left(\frac{a}{\theta}+\frac{ab}{\theta^2}\right)\left(e^{\theta T_2}-1\right)-\frac{abCT_2e^{\theta T_2}}{\theta}-aT_2C+\frac{abCT_2^{\,2}}{2}+A-\\ PI_e\left(\frac{-abT_2^{\,3}}{3}+\frac{aT_2^{\,2}}{2}+a(1-bT_2)T_2(M-T_2)\right)\\ +\frac{ha}{2\theta^3}\left[\left(2\theta+2b-2bT_2\theta\right)e^{\theta T_2}-2\theta-2T_2\theta^2-2b+bT_2^{\,2}\theta^2\right]\end{array}\right] \tag{18}$$

4. COMPUTATIONAL ALGORITHM

To obtain optimal solution, decision maker is advised to observe the following steps.

Step1: Initialize all parametric values.
Step2: Compute T_1 from equation. (15). If $M < T_1$ then $K_1(T_1)$ (equation. 16) gives minimum cost else go to step3.
Step3: Compute T_2 from equation. (17). If $M > T_2$ then $K_2(T_2)$ (eq.18) gives minimum cost for decision maker else.
Step4: $K_1(M) = K_2(M)$ (equation. 14) is the minimum cost.
Step5: Stop.

5. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

To validate the proposed model, let us consider following examples.

Example1: Consider the parametric values:
$[a, b, A, C, P, h, I_c, I_e, M, \theta] = [1000, 0.2, 250, 20, 40, 1, 0.12, 0.09, 30/365, 0.10]$. Using the algorithm exhibited in section 4, $T_1 = 0.3185$ years which is greater than $M = 0.082$ years. Hence

corresponding minimum cost is $K_1(T_1) = \$1396.44$ (see Figure3) and optimum procurement units are 313.

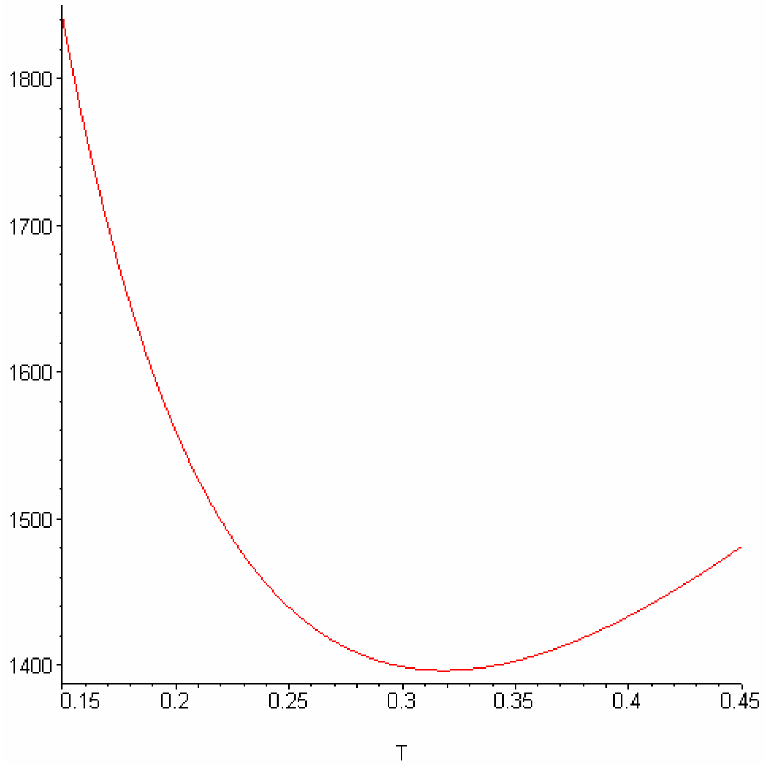

Figure 3 Convexity of total cost when M < T

Example2: Consider a= 600 units/year, b = 0.10, A = \$50 per order, C = \$30/ unit, P = \$35/ unit, h = \$1.00 units/annum, I_c =\$0.15/year, M = 60/365 years and θ =0.20/annum. Then $T_2 = 0.122$ years which is less than $M = 0.1644$ years. Hence using algorithm stated in section4, the minimum cost is $K_2(T_2) = 405.36$ \$ (see Figure 4) and optimum purchase quantity is 74 units. The remaining parametric values are same as in example1.

Next, we carry out sensitivity analysis by varying parameters b, θ and M as - 40%, - 20%, 20%, 40%. The corresponding changes in the cycle time, purchase quantities and total cost are exhibited in table1.

It is observed that as rate of change of demand increases, cycle time increases while total cost of an inventory system decreases. Increases in deterioration rate forces retailer to buy more number of units frequently and hence increases total cost of an inventory system. Increases in delay period decreases retailer's cycle time and total cost of inventory system. The reduction in total cost is obvious because retailer can earn more interest during this permissible delay period for settlement of accounts against his dues.

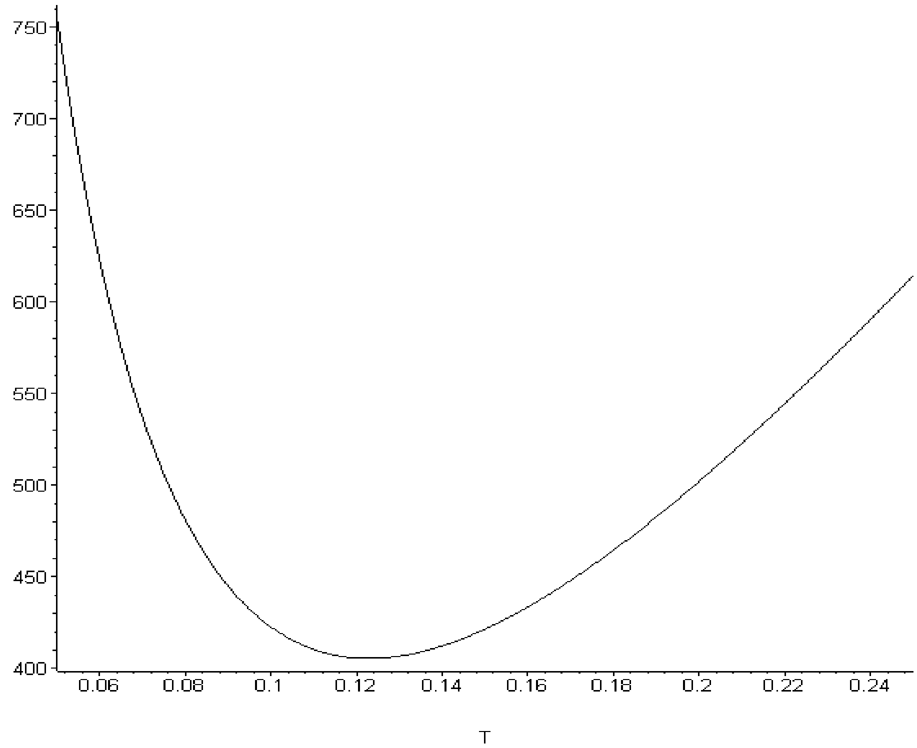

Figure 4 Convexity of total cost when $M \geq T$

Table1: sensitivity analysis

Parameter	% changes	% change in		
		T	Q	$K_1(T)$
b	-40	-2.3	-1.2	1.1
	-20	-1.2	-0.6	0.5
	20	1.2	0.6	-0.5
	40	2.7	1.2	-1.2
θ	-40	8.7	8.9	-8.9
	-20	4.1	4.1	-4.3
	20	-3.6	-3.5	4.2
	40	-	-	-
M	-40	0.9	0.6	5.9
	-20	0.4	0.3	3.0
	20	-0.5	-0.6	-3.0
	40	-1.1	-1.2	-6.2

Note: ‘ – ‘ stands for infeasible solution

6. CONCLUSIONS

The effect of delay period offered by the supplier to retailer is analyzed when the demand of the product is decreasing in the market. The units in inventory are assumed to deteriorate at a constant rate. It is observed that incentive of credit period is advantageous to the retailer for lowering the total cost of an inventory system.

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