# DETERIORATING INVENTORY MODEL IN DEMAND DECLINING MARKET UNDER INFLATION WHEN SUPPLIER CREDITS LINKED TO ORDER QUANTITY

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**RESUMEN**
En este artículo, un modelo de inventario es desarrollado cuando el proveedor ofrece al detallista un período de crédito para colocar la cuenta, si el detallista ordena una gran cantidad. En el estudio propuesto se considera la demanda que rehúsa el mercado. Las escaseces no son permitidas y el efecto de inflación es incorporado. Las unidades en el inventario están sujetas al precio constante del empeoramiento. El coste total es minimizado para deteriorar artículos en la demanda que rehúsa el mercado bajo la inflación cuando el proveedor ofrece un período de crédito al detallista si la cantidad de orden es mayor que o igual a una cantidad previamente especificada. Un algoritmo de sencillo uso es expuesto para encontrar la cantidad de orden óptima y el tiempo de relleno. La formulación matemática es explorada mediante un ejemplo numérico. El análisis de la sensibilidad de los parámetros en la solución óptima es realizado.

**ABSTRACT**
In this article, an inventory model is developed when supplier offers the retailer a credit period to settle the account, if the retailer orders a large quantity. The proposed study is meant for demand declining market. Shortages are not allowed and the effect of inflation is incorporated. The units in inventory are subject to constant rate of deterioration. The total cost is minimized for deteriorating items in demand declining market under inflation when the supplier offers a credit period to the retailer if the order quantity is greater than or equal to a pre – specified quantity. An easy – to – use algorithm is exhibited to find the optimal order quantity and the replenishment time. The mathematical formulation is explored by a numerical example. The sensitivity analysis of parameters on the optimal solution is carried out.

**KEY WORDS:** Inventory, inflation, trade credit, deterioration.

**MSC :** 90B05

## 1. INTRODUCTION

The classical economic order quantity model is derived under the assumption that the demand of the product is constant. However, the demand of seasonal goods, weather selected garments, blood during riots, accidents, etc. decreases after the particular phase is over. Another stringent assumption of the classical EOQ is that the retailer settles the accounts for the items as soon as items are received in his inventory system. In practice, the supplier offers a permissible credit period to the retailer if the outstanding amount is paid within the allowable fixed settlement period and the order quantity is large. The credit period is treated as a promotional tool to attract more customers. It can be expressed as a kind of price discount because paying later indirectly reduces

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the purchase cost which motivates retailers to increase their order quantity. Goyal (1985) developed an EOQ model under the conditions of permissible delay in payments. There are several interesting papers related to trade credits viz. Shah(1993a, 1993b), Aggarwal and Jaggi(1995), Arcelus and Srinivasan(1993), Davis and Gaither(1985), Hwang and Shinn(1997), Jamal et al. (1997), Sarkar et al. (1997), Khouja and Mehrez(1996), Liao et al.(2000), Teng (2002), Chang(2004) and their references.

The most of the above cited researchers have not considered influence of the inflation on inventory policy. However, from a financial point of view, an inventory represents a capital investment and must compete with other assets for a firm's limited capital funds. (Chang(2004)). Buzacott(1975), Bierman and Thomas(1977) and Mishra(1979) discussed the inventory decisions under an inflationary condition for the EOQ model. One can read Brahmbhatt(1982), Chandra and Bahner(1985), Datta and Pal(1991) and their references.

In this paper, an attempt is made to formulate inventory model in demand declining market under inflation when a supplier offers a permissible delay of payments for a large order that is greater than or equal to the pre – specified quantity $Q_d$ . It is assumed that if the order is less than $Q_d$ , then the retailer must settle the account for the items received immediately. An easy – to – use algorithm is given to decide optimal order quantity and replenishment time. The numerical example is provided to support the working rules for the optimal solution. The sensitivity analysis is carried out to study the effect of parameters on the optimal solution.

**2. ASSUMPTIONS AND NOTATIONS**

The following notations and assumptions are used in the development of the model.

**2.1 Notations**

| | |
|---|---|
| H | the length of finite planning horizon |
| $R(t)$ | $a(1-bt)$ ; The demand rate where a (>0) is fixed demand and, $b$ Denotes rate of change of demand. a > > b and 0 < b < 1 |
| I | the inventory carrying charge fraction per annum excluding interest charges |
| $r$ | Constant rate of inflation per unit time where 0 ≤ r < 1 |
| $P(t)$ | $Pe^{rt}$; the selling price per unit at time t, where P is the unit Selling price at t = 0. |
| $C(t)$ | $Ce^{rt}$; the purchase cost per unit at time t, where C is the unit Purchase cost at t = 0. |
| $A(t)$ | $Ae^{rt}$; the ordering cost per order at time t, where A is the ordering Cost at t = 0. |
| $M$ | The allowable credit to period in settling the account. |
| $I_c$ | The interest charged per \$ for un-sold stock per annum by the supplier |
| $I_e$ | The interest earned per \$ per annum. |
| $Q$ | The order quantity (a decision variable) |
| $Q_d$ | The minimum order quantity pre – specified by the supplier at which the delay in payment is permissible. |
| $T_d$ | The time length at which $Q_d$ – units are depleted to zero |

$\theta$ The constant deterioration rate, where $0 \le \theta \le 1$

$I(t)$ The inventory level at any instant of time t, $0 \le t \le T$.

$T$ The cycle time (a decision variable)

$K(T)$ The total cost of an inventory system during the planning horizon

The cost of an inventory system is the sum of
(a) Ordering cost; $OC$
(b) Purchase cost; $PC$
(c) Inventory holding cost excluding interest charged; $IHC$
(d) Interest charges payable for unsold stock after the credit period $M$ ; $IC$ and minus
(e) interest earned from the sales revenue during the credit period; $IE$ .

**2.2. Assumptions**

1. The inventory system under considerations deals with a single item.
2. The demand is partially constant and partially decreases with time.
3. The inflation rate is constant.
4. Shortages are not allowed and the lead – time is zero.
5. The planning horizon is finite.
6. The dues for the items procured must be made immediately if the order quantity is less than $Q_d$ .

However, if the order quantity is greater than or equal to $Q_d$ , then the delay in payment up to M is allowed. During this credit period, the generated sales revenue is deposited in an interest bearing account. At the end of the delay period, the retailer can settle the account and after that supplier charges interest on the un-sold stock in the inventory system.

**3. MATHEMATICAL MODEL**

The retailer can make $n$ – replenishments after every $T$ – time units during the planning horizon $H$ . Thus, $H = nT$ where n is an integer. The inventory level depletes due to demand and the deterioration of units. This rate of change of inventory level is governed by the differential equation.

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t) \quad ; \quad 0 \le t \le T \tag{3.1}$$

With the boundary condition $I(0) = Q$ and $I(T) = 0$ . Hence the solution of (3.1) is given by

$$I(t) = \frac{a}{\theta^2}\left[\theta(1-bT)e^{\theta(T-t)} + be^{\theta(T-t)} - \theta(1-bt) - b\right] \quad ; \quad 0 \le t \le T \tag{3.2}$$

and the order quantity is

$$Q = I(0) = \frac{a}{\theta^2}\left[\theta e^{\theta T}(1-bT) + b(e^{\theta T} - 1) - b\right] \tag{3.3}$$

Using (3.3), the pre – specified $Q_d$ units are given by

$$Q_d = \frac{a}{\theta^2}\left[\theta e^{\theta T_d}(1-bT_d)+b(e^{\theta T_d}-1)-b\right] \tag{3.4}$$

Where $T_d$ is the time at which $Q_d$ units depletes to zero. The value of $T_d$ is given by

$$T_d = \frac{a\theta \pm \sqrt{a^2\theta^2 - 4ab(\theta^2 Q_d - \theta a + b)}}{2ab\theta} \tag{3.5}$$

Obviously, $Q < Q_d$ holds if and only if $T < T_d$ . Under the assumption that the lengths of time interval are equal, using (3.2), we have

$$I\left(kT+t\right) = a\left[T - t + \frac{b}{2}(t^2 - T^2)\right], \qquad 0 \le k \le n-1, \quad 0 \le t \le T \tag{3.6}$$

The cost components of the total cost of an inventory system during planning horizon of length are as follows:

1. Ordering Cost

$$OC = A(0) + A(T) + A(2T) + .... + A((n-1)T)$$
$$= A\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{3.7}$$

2. Purchase cost

$$PC = \left[C(0) + C(T) + C(2T) + .... + C((n-1)T)\right]Q$$
$$= CQ\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{3.8}$$

3. Inventory holding cost

$$IHC = h\sum_{k=0}^{n-1} C(kT)\int_0^T I(kT+t)dt$$

$$= \frac{Ca}{2\theta^3}\left(-2\theta - 2b - 2\theta^2 T + \theta^2 T^2 b + 2\theta e^{\theta T} - 2\theta b T e^{\theta T} + 2be^{\theta T}\right)\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{3.9}$$

Regarding interest charges and earned, the following four cases are possible depending on the lengths of $T, T_d$ and $M$ .

Since the cycle time $T$ is less than $T_d$ , supplier will not facilitate the retailer for the trade credit to settle the account. The retailer will have to pay immediately for the units procured. This is the case of classical economic order quantity (EOQ). The interest charged for unsold items during finite planning horizon is

Case 1: $0 < T < T_d$ (Figure 1)

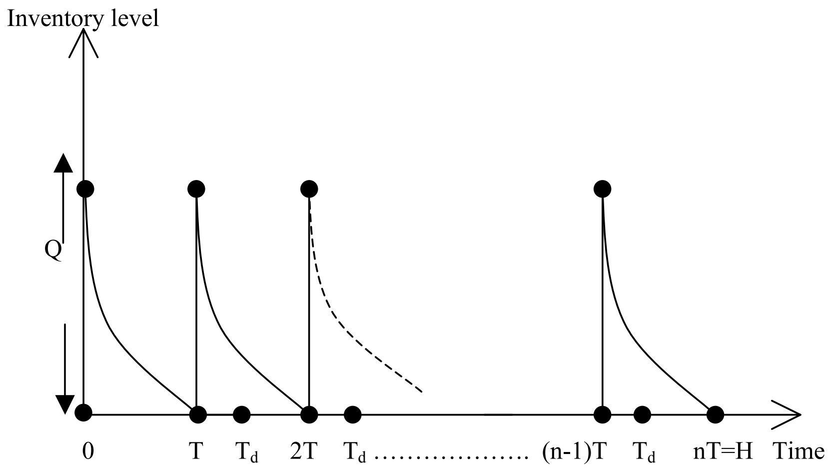


Figure: 1: $0 < T < T_d$

$$IC_1 = I_c \sum_{k=0}^{n-1} C(kT)\left[\int_0^T I(kT+t)dt\right]$$

$$= \frac{CI_c a}{2\theta^3}\begin{pmatrix} -2\theta - 2b - 2\theta^2 T + \theta^2 T^2 b \\ +2\theta e^{\theta T} - 2\theta bTe^{\theta T} + 2be^{\theta T}\end{pmatrix}\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{3.10}$$

Therefore, the total cost in [0, H] is

$$K_1(T) = OC + PC + IHC + IC_1 \tag{3.11}$$

The necessary condition for $K_1(T)$ to be minimum, is set derivative of $K_1(T)$ with respect to $T$ be zero;

$$\left(\begin{array}{l} -\dfrac{Are^{rT}(e^{rH}-1)}{(e^{rT}-1)^2} + \dfrac{Cae^{\theta T}(1-bT)(e^{rH}-1)}{(e^{rT}-1)} - \dfrac{(h+I_c)Ca(1-bT)(1-e^{\theta T})(e^{rH}-1)}{\theta(e^{rT}-1)} \\ +\dfrac{Care^{rT}(\theta + b + \theta bTe^{\theta T} - \theta e^{\theta T} - be^{\theta T})(e^{rH}-1)}{\theta^2(e^{rT}-1)^2} \\ -\dfrac{(h+I_c)Care^{rT}(-2\theta - 2b - 2\theta^2 T + \theta^2 T^2 b + 2\theta e^{\theta T} - 2\theta e^{\theta T} bT + 2be^{\theta T})(e^{rH}-1)}{2\theta^3(e^{rT}-1)^2} \end{array}\right) = 0$$

(3.12)

Solve equation (3.12) for $T = T_1$ by mathematical software. The obtained $T = T_1$ will minimize total cost provided

$$
\begin{aligned}
\frac{d^2K_1}{dT^2} &= \frac{Ar^2e^{rT}(e^{rH}-1)\left(e^{rT}+1\right)}{(e^{rT}-1)^3} + \frac{Car^2e^{rT}(-\theta e^{\theta T}+\theta e^{\theta T}bT-be^{\theta T}+\theta+b)(e^{rH}-1)}{\theta^2\left(e^{rT}-1\right)^2} \\
&-\frac{2Car^2e^{2rT}(-\theta e^{\theta T}+\theta e^{\theta T}bT-be^{\theta T}+\theta+b)(e^{rH}-1)}{\theta^2\left(e^{rT}-1\right)^3} + \frac{2Care^{rT}e^{\theta T}(bT-1)(e^{rH}-1)}{\left(e^{rT}-1\right)^2} \\
&-\frac{Cae^{\theta T}(\theta(bT-1)+b)(e^{rH}-1)}{\left(e^{rT}-1\right)} + \frac{hCa(b+\theta e^{\theta T}-\theta e^{\theta T}bT-be^{\theta T})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)} \\
&+\frac{2hCare^{rT}(1-bT)(1-e^{\theta T})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)^2} - \frac{I_cCa(b+\theta e^{\theta T}-\theta e^{\theta T}bT-e^{\theta T}b)(e^{rH}-1)}{\theta\left(e^{rT}-1\right)} \\
&+\frac{hCar^2e^{rT}(-2\theta-2b-2\theta^2T+\theta^2T^2b+2\theta e^{\theta T}-2\theta e^{\theta T}bT+2e^{\theta T}b)(e^{rH}-1)(e^{rT}+1)}{2\theta^3\left(e^{rT}-1\right)^3} \\
&-\frac{I_cCare^{rT}(-2\theta^2+2\theta^2Tb+2\theta^2e^{\theta T}-2\theta^2e^{\theta T}bT)(e^{rH}-1)}{\theta^3\left(e^{rT}-1\right)^2} \\
&+\frac{I_cCar^2e^{rT}(-2\theta-2b-2\theta^2T+\theta^2T^2b+2\theta e^{\theta T}-2\theta e^{\theta T}bT+2e^{\theta T}b)(e^{rH}-1)(e^{rT}+1)}{2\theta^3\left(e^{rT}-1\right)^3} > 0
\end{aligned}
\tag{3.13}
$$

Case 2: $T_d \le T < M$ (Figure 2)

Inventory Level

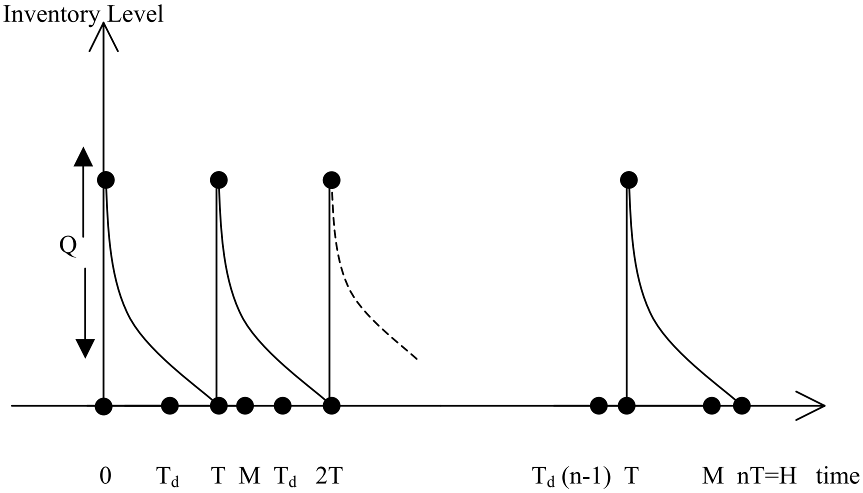


Figure: 2: $T_d \le T < M$

Since, $T_d \le T < M$ there is no interest charges. Interest earned during $[0, H]$ is

$$IE_2 = I_e\sum_{k=0}^{n-1}P(kT)\left[\int_0^T R(t)tdt + a(1-bT)T(M-T)\right]$$

$$= PI_e a\left[\frac{T^2}{2}-\frac{bT^3}{3}+(1-bT)T(M-T)\right]\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{3.14}$$

Therefore, the total cost during [0, H] is

$$K_2(T) = OC + PC + IHC - IE_2 \tag{3.15}$$

The necessary and sufficient conditions for $K_2(T)$ to be minimum at $T = T_2$ are

$$\frac{dK_2}{dT} = -\frac{Are^{rT}(e^{rH}-1)}{(e^{rT}-1)^2} + \frac{Care^{\theta T}(1-bT)(e^{rH}-1)}{(e^{rT}-1)} + \frac{I_e Pa(M-T)(1-2bT)(e^{rH}-1)}{(e^{rT}-1)}$$
$$-\frac{Care^{rT}(-\theta e^{\theta T}+\theta e^{\theta T}bT - be^{\theta T}+\theta+b)(e^{rH}-1)}{\theta^2(e^{rT}-1)^2} - \frac{hCa(1-bT)(1-e^{\theta T})(e^{rH}-1)}{\theta(e^{rT}-1)}$$
$$-\frac{hCare^{rT}(-2\theta-2b-2\theta^2T+\theta^2T^2b+2\theta e^{\theta T}-2\theta e^{\theta T}bT+2be^{\theta T})(e^{rH}-1)}{\theta^3(e^{rT}-1)^2}$$
$$-\frac{I_e Pare^{rT}(T^2(3-2bT)+6(1-bT)T(M-T))(e^{rH}-1)}{6(e^{rT}-1)^2} = 0 \tag{3.16}$$

and

$$\frac{d^2K_2}{dT^2} = \frac{Ar^2e^{rT}(e^{rH}-1)\left(e^{rT}+1\right)}{(e^{rT}-1)^3} + \frac{Car^2e^{rT}(-\theta e^{\theta T}+\theta e^{\theta T}bT-be^{\theta T}+\theta+b)(e^{rH}-1)}{\theta^2\left(e^{rT}-1\right)^2}$$
$$-\frac{Cae^{\theta T}(\theta(bT-1)+b)(e^{rH}-1)}{\left(e^{rT}-1\right)} - \frac{2Car^2e^{2rT}(-\theta e^{\theta T}+\theta e^{\theta T}bT-be^{\theta T}+\theta+b)(e^{rH}-1)}{\theta^2\left(e^{rT}-1\right)^3}$$
$$+\frac{2Care^{rT}e^{\theta T}(bT-1)(e^{rH}-1)}{\left(e^{rT}-1\right)^2} + \frac{hCa(b+\theta e^{\theta T}-\theta e^{\theta T}bT-be^{\theta T})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)}$$
$$+\frac{2hCare^{rT}(1-bT)(1-e^{\theta T})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)^2} - \frac{2I_e Pare^{rT}(2b(T-M)-T)(e^{rH}-1)}{\left(e^{rT}-1\right)^2}$$
$$+\frac{hCar^2e^{rT}(-2\theta-2b-2\theta^2T+\theta^2T^2b+2\theta e^{\theta T}-2\theta e^{\theta T}bT+2e^{\theta T}b)(e^{rH}-1)(e^{rT}+1)}{2\theta^3\left(e^{rT}-1\right)^3}$$
$$+\frac{I_e Par^2e^{rT}(3T^2-2bT^3+6(1-bT)T(M-T))(e^{rH}-1)(e^{rT}+1)}{6\left(e^{rT}-1\right)^3} > 0 \tag{3.17}$$

Case 3: $T_d \le M \le T$ (Figure 3)

Here, cycle time; $T$ is greater than $T_d$ and $M$ both. Therefore, delay in payment is allowed. The interest earned in $[0, H]$ is

$$IE_3 = I_e \sum_{k=0}^{n-1} P(kT)\left[\int_0^M R(t)tdt\right]$$
$$= PI_e a\left(\frac{M^2}{2} - \frac{bM^3}{3}\right)\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{3.18}$$

And interest charged during $[0, H]$ is

$$IC_3 = I_c \sum_{k=0}^{n-1} C(kT)\left[\int_M^T I(kT+t)dt\right]$$
$$= \frac{CI_c a}{\theta^3}\left[\begin{array}{l} -2\theta - 2\theta^2 T + \theta^2 T^2 b + 2\theta e^{\theta(T-M)}(1-bT) + \\ 2b(e^{\theta(T-M)}-1) + 2\theta^2 M - \theta^2 M^2 b + 2bM\theta \end{array}\right]\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{3.19}$$

Consequently, the total cost in $[0, H]$ is

$$K_3(T) = OC + PC + IHC + IC_3 - IE_3 \tag{3.20}$$

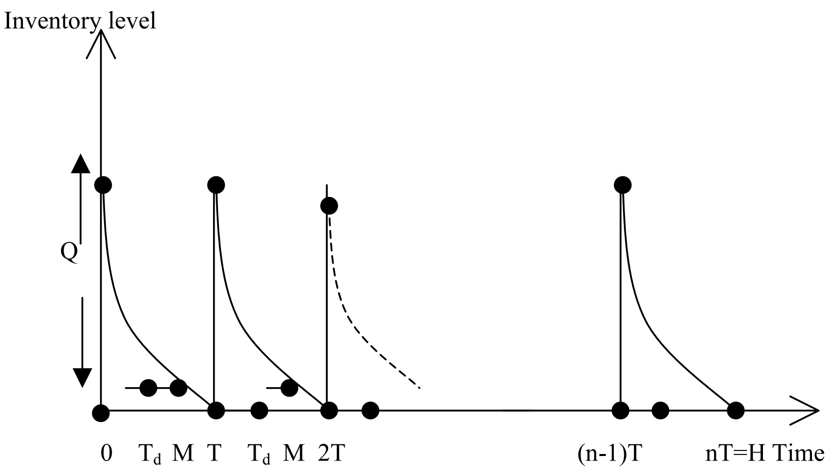


Figure: 3: $T_d \le M \le T$

The total cost can be minimized for $T = T_3$ (say) by setting

$$\frac{dK_3}{dT} = -\frac{Are^{rT}(e^{rH}-1)}{(e^{rT}-1)^2} + \frac{Care^{\theta T}(1-bT)(e^{rH}-1)}{(e^{rT}-1)} - \frac{hCa(1-bT)(1-e^{\theta T})(e^{rH}-1)}{\theta(e^{rT}-1)}$$
$$-\frac{Care^{rT}(-\theta e^{\theta T} + \theta e^{\theta T}bT - be^{\theta T} + \theta + b)(e^{rH}-1)}{\theta^2(e^{rT}-1)^2} + \frac{I_c Ca(1-bT)(1-e^{\theta(T-M)})(e^{rH}-1)}{\theta(e^{rT}-1)}$$
$$-\frac{hCare^{rT}(-2\theta - 2b - 2\theta^2 T + \theta^2 T^2 b + 2\theta e^{\theta T} - 2\theta e^{\theta T} bT + 2be^{\theta T})(e^{rH}-1)}{\theta^3(e^{rT}-1)^2}$$

$$-\frac{I_c Care^{rT}\begin{pmatrix}-2\theta-2b-2\theta^2T+\theta^2T^2b+2\theta e^{\theta(T-M)}\\ -2\theta e^{\theta(T-M)}bT+2be^{\theta(T-M)}+2\theta^2M-\theta^2M^2b+2bM\theta\end{pmatrix}(e^{rH}-1)}{\theta^3(e^{rT}-1)^2}$$

$$-\frac{I_e Pare^{rT}M^2(2bM-3)(e^{rH}-1)}{6(e^{rT}-1)^2} \tag{3.21}$$

Provided

$$\frac{d^2K_3}{dT^2}=\frac{Ar^2e^{rT}(e^{rH}-1)\left(e^{rT}+1\right)}{(e^{rT}-1)^3}+\frac{Car^2e^{rT}(-\theta e^{\theta T}+\theta e^{\theta T}bT-be^{\theta T}+\theta+b)(e^{rH}-1)}{\theta^2\left(e^{rT}-1\right)^2}$$

$$-\frac{Cae^{\theta T}(\theta(bT-1)+b)(e^{rH}-1)}{\left(e^{rT}-1\right)}-\frac{2Car^2e^{2rT}(-\theta e^{\theta T}+\theta e^{\theta T}bT-be^{\theta T}+\theta+b)(e^{rH}-1)}{\theta^2\left(e^{rT}-1\right)^3}$$

$$+\frac{2Care^{rT}e^{\theta T}(bT-1)(e^{rH}-1)}{\left(e^{rT}-1\right)^2}+\frac{I_cCa(b+\theta e^{\theta(T-M)}-\theta e^{\theta(T-M)}Tb-be^{\theta(T-M)})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)}$$

$$+\frac{hCa(b+\theta e^{\theta T}-\theta e^{\theta T}bT-be^{\theta T})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)}+\frac{2hCare^{rT}(1-bT)(1-e^{\theta T})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)^2}$$

$$+\frac{hCar^2e^{rT}(-2\theta-2b-2\theta^2T+\theta^2T^2b+2\theta e^{\theta T}-2\theta e^{\theta T}bT+2e^{\theta T}b)(e^{rH}-1)(e^{rT}+1)}{2\theta^3\left(e^{rT}-1\right)^3}$$

$$+\frac{2I_cCare^{rT}(1-bT)(1-e^{\theta(T-M)})(e^{rH}-1)}{\theta\left(e^{rT}-1\right)^2}$$

$$+\frac{2I_ePaM^2(2bM-3)r^2e^{rT}(e^{rH}-1)(e^{rT}+1)}{6\left(e^{rT}-1\right)^3}$$

$$+\frac{I_cCar^2e^{rT}\begin{pmatrix}-2\theta-2b-2\theta^2T+\theta^2T^2b+2\theta e^{\theta(T-M)}-2\theta e^{\theta(T-M)}bT\\ +2e^{\theta(T-M)}b+2\theta^2M-\theta^2M^2b+2bM\theta\end{pmatrix}(e^{rH}-1)(e^{rT}+1)}{2\theta^3\left(e^{rT}-1\right)^3}>0$$

$$\tag{3.22}$$

Case 4: $M \le T_d \le T$ (Figure 4)

Here, also cycle time is greater than or equal to both $T_d$ and M. and hence case 4 is similar to case 3, therefore, the total cost during $[0, H]$ is

$$K_4(T)=OC+PC+IHC+IC_3-IE_3 \tag{3.23}$$

**4. COMPUTATIONAL ALGORITHM**

The retailer can decide optimal policy using following steps.
Step 1: Initialize all parameters.
Step 2: Compute $T = T_1$ using equation (3.12). If $T_1 < T_d$ then $K_1(T_1)$ is minimum; otherwise go to step 3.
Step 3: Compute $T = T_2$ using equation (3.16). If $T_d < T_2 < T$ then $K_2(T_2)$ is minimum; otherwise go to step 4.
Step 4: Compute $T = T_3$ using equation (3.21) and corresponding $K_3(T_3)$ is minimum; (Equivalently, $K_4(T_3)$ is minimum; otherwise go to step 4.)
Step 5: Stop.

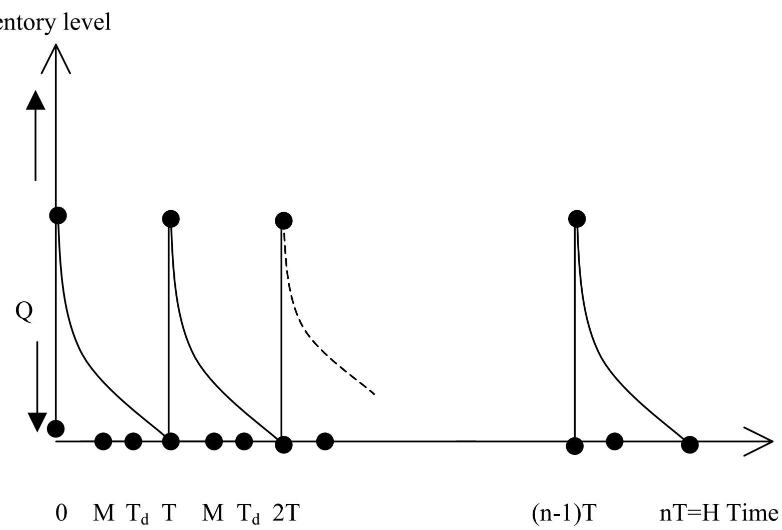

Figure: 4: $M \leq T_d \leq T$

**5. NUMERICAL EXAMPLES**

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**Example: 1**
Consider

$[H, a, b, h, I_c, I_e, r, C, P, M, Q_d, A, \theta]$ =
[1, 50, 0.10, 2, 0.10, 0.06, 0.05, 20, 30, 30/365, 20, 120, 0.05] in proper units.

Following, algorithm defined in section 4, $T_1 = 0.3566 < T_d = 0.4042$ years. Hence case 1 is optimal decision. The minimum cost is $ 1722.78 and optimum purchase quantity is 17.67 units.

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**Example: 2**
Consider

$[H, a, b, h, I_c, I_e, r, C, P, M, Q_d, A, \theta]$ =
[1, 50, 0.10, 2, 0.10, 0.06, 0.05, 20, 30, 120/365, 10, 120, 0.05] in proper units.

Then $T_d = 0.2010 < M = 0.3288$ years. Thus, case 2 is optimal decision policy. The optimal cycle time $T_2 = 0.3571$, minimum cost is $K_2(T_2) = \$\ 1691.53$ and purchase quantity is 17.69 units.

**Example: 3**
Consider

$[H, a, b, h, I_c, I_e, r, C, P, M, Q_d, A, \theta] =$
[1, 50, 0.10, 2, 0.10, 0.06, 0.05, 20, 30, 30/365, 15, 120, 0.05] in proper units.
Then $T_d = 0.3023$ *years* $> M = 0.0822$ *years*. From case 4, optimal cycle time $T_4$ is 0.3566 years, minimum cost $K_4(T_4)$ is \$ 1714.62. See Figure 5

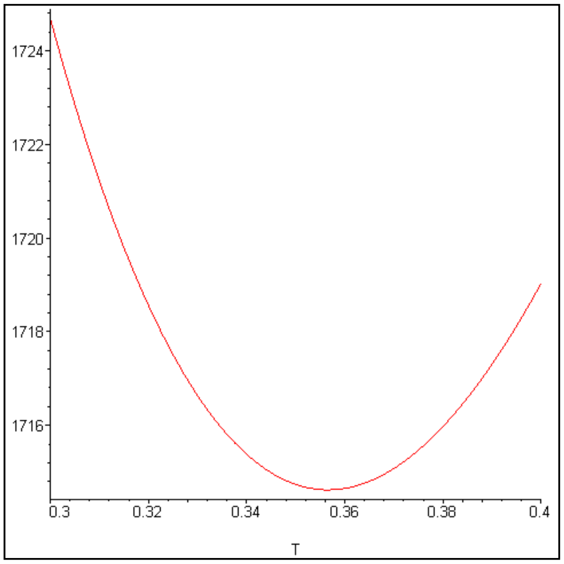


Figure 5 Convexity of total cost

Next, we carry out variations in critical parameter to study effects on decision variable and total cost during $[0, H]$.

It is observed that increase in deterioration rate decreases cycle time and increases total cost of an inventory system during finite planning horizon. (Fig 6). The model is very sensitive to changes in the fixed demand 'a'. (Fig 7) Increase in fixed demand increases total cost significantly and decreases cycle time. Increase in demand rate 'b' increases cycle time and decreases total cost of an inventory system. (Fig 8). The model is very sensitive to changes in ordering cost (Fig 9) and inflation rate (Fig 10). The total cost of inventory system decreases if supplier allows longer credit period. The decrease in total cost is because the retailer can earn more interest on the generated sales revenue.

| Parameter | | Cycle time T (in years) | Total Cost $K(T)$ (in \$) |
|---|---|---|---|
| $\theta$ | 0.05 | 0.3566 | 1714.62 |
| | 0.10 | 0.3498 | 1725.67 |
| | 0.15 | 0.3433 | 1736.59 |
| $a$ | 50 | 0.3566 | 1714.62 |
| | 60 | 0.3246 | 1985.90 |
| | 70 | 0.2999 | 2251.73 |
| $b$ | 0.10 | 0.3566 | 1714.62 |
| | 0.15 | 0.3668 | 1700.72 |
| | 0.20 | 0.3785 | 1686.25 |
| $A$ | 120 | 0.3566 | 1714.62 |
| | 150 | 0.4000 | 1795.17 |
| | 180 | 0.4397 | 1867.68 |
| $r$ | 0.05 | 0.3566 | 1714.62 |
| | 0.10 | 0.3651 | 1742.72 |
| | 0.15 | 0.3743 | 1771.20 |
| $M$ | 30/365 | 0.35658 | 1714.62 |
| | 45/365 | 0.35658 | 1710.63 |
| | 60/365 | 0.35663 | 1706.69 |
| | | | |
| $Q_d$ | 15 | $T_4$ = 0.3566 | $K_4(T_4)$ = 1714.62 |
| | 18 | $T_1$ = 0.3566 | $K_1(T_1)$ = 1722.78 |
| | 20 | $T_1$ = 0.3566 | $K_1(T_1)$ = 1722.78 |

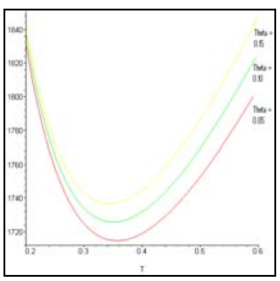

**Figure 6**

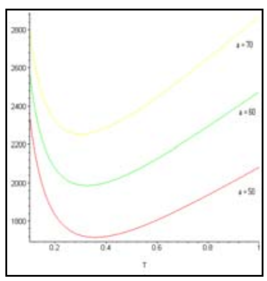

**Figure 7**

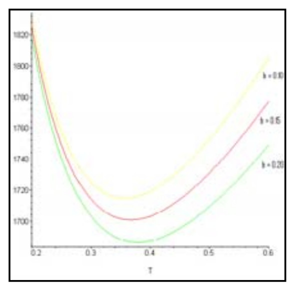

**Figure 8**

## 6. CONCLUSIONS

In this study, optimal policy is derived for deteriorating items when the supplier provides a permissible delay in payments if ordered units are more than pre specified number by the supplier. The effect inflation is incorporated. The proposed model can be extended to a two – parameter Weibull distribution. It can be generalized to allow for shortages. The comparison of quantity discounts and trade credit is also an interesting future scope of research.

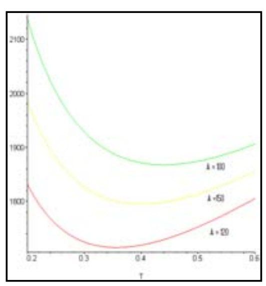

**Figure 9**

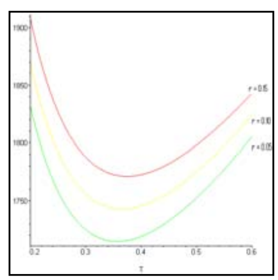

**Figure 10**

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**REVISED AUGUST 2009**

## REFERENCES


[1] AGGARWAL, S.P. and JAGGI, C.K. (1995): Ordering policies of deteriorating items under permissible delay in payments. **Journal of the Operational Research Society**, 46, 658 – 662.

[2] ARCELUS, F. J. and SRINIVASAN, G. (1993): Delay of payments for extra ordinary purchases. **Journal of the Operational Research Society**, 44, 785 – 795.

[3] BIERMAN, H. and THOMAS, J.(1977): Inventory decisions under inflationary condition. **Decision Sciences**, 8, 151 – 155.

[4] BRAHMBHATT, A. C.(1982): Economic order quantity under variable rate of inflation and mark – up prices. **Productivity**, 23, 127 – 130.

[5] BUZACOTT, J.A. (1975):. Economic order quantities with inflation. **Operational Research Quarterly**, 26, 553 – 558.

[6] CHANDRA, M.J. and BAHNER, M.J. (1985): The effects of inflation and time value money on some inventory systems. **International Journal of Production Research**, 23, 723 – 729.

[7] Chung, C. T.,(2004): An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. **International Journal of Production Economics**, 88, 307 – 316.

[8] DATTA, T.K. and PAL, A.K. (1991): Effects of inflation and time value of money on an inventory model with linear time dependent demand rate and shortages. **European Journal of Operational research**, 52, 326 – 333.

[9] DAVIS, R.A. and GAITHER, N. (1985). Optimal ordering policies under conditions of extended payment privileges. **Management Science**, 31, 499 – 509.

[10] GOYAL, S.K. (1985): Economic Order Quantity under conditions of permissible delay in payments. **Journal of Operational Research Society**, 36, 335 – 338.

[11] HWANG, H. and SHINN, S.W. (1997): Retailer's pricing and lot – sizing policy for exponentially deteriorating products under the conditions of permissible delay in payments. **Computers and Operations Research**, 24, 539 – 547.

[12] JAMAL, A.M., SARKAR, B.R. and WANG, S. (1997): An ordering policy for deteriorating item with allowable shortage and permissible delay in payment. **Journal of the Operational Research Society**, 48, 826 – 833.

[13] KHOUJA, M. and MEHREZ, A. (1996): Optimal inventory policy under different supplier credit policies, **Journal of Manufacturing systems**,15, 334 – 339.

[14] LIAO, H.-C., TSAI, C.-H. and SU, C. -T. (2000): An inventory model with deteriorating items under inflation when a delay in payments is permissible. **International Journal of Production Economics**, 63, 207 – 214.

[15] MISHRA, R.B. (1979): A study of inflation effects on inventory system. **Logistic Spectrum**, 9, 260 – 268.

[16] SHAH, NITA H. (1993): A. A lot – size model for exponentially decaying inventory when delay in payments is permissible. **CERO**, 35, 1 – 9.

[17] SHAH, NITA H., 1993 b. Probabilistic time scheduling model for an exponentially decaying inventory when delay in payments are permissible. **International Journal of Production Economics,** 32, 77 – 82.

[18] TENG, I. T., 2002. On Economic Order Quantity under conditions of permissible delay in payments. **Journal of the Operational Research Society**, 53, 915 – 918.