AN ORDER-LEVEL LOT-SIZE MODEL FOR DETERIORATING ITEMS FOR TWO STORAGE FACILITIES WHEN DEMAND IS EXPONENTIALLY DECLINING

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ABSTRACT
In this research study, an attempt is made to develop ordering policy for deteriorating items when demand is exponentially decreasing and retailer uses two warehouses to store items. It is assumed that the cycles form a regenerative process. The total cost per time unit is minimized. A numerical example is given to illustrate the model. Sensitivity analysis is carried out to study the changes in the decision variables and total cost of an inventory system.

KEY WORDS: Order-level lot-size model, deterioration, two ware-houses, exponentially declining demand.

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RESUMEN
En este estudio investigativo, se trata de desarrollar una política de ordenamiento pra ariticulos deterioroables en los que la demanda es exponencialmente decreciente y el minorista utiliza dos depósitos para almacenar articulos. Se supone que los ciclos con forman un proceso de regeneración. El costo total por unidad de tiempo es mínima. Un ejemplo numérico se da para ilustrar el modelo. El análisis de sensibilidad se lleva a cabo para estudiar los cambios en las variables de decisión y el coste total de un sistema de inventario.

1. INTRODUCTION

For extensive study of inventory models with deteriorating items refer to review articles by Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001) and the references there in. Most of the models, stated in aforesaid reviews assume that the retailer owns his warehouse.

Sarma (1983) developed an inventory model for a single item which is stored in two different warehouses viz. an owned warehouse (OW) and a rented warehouse (RW) for deteriorating items. The rented warehouse is used to store the excess units over the fixed capacity W of the OW which results in lowering deterioration rate. Also, the RW is assumed to charge higher inventory holding cost than

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the OW, and demand to be constant. Murdeshwar and Sathi (1985) extended Sarma's (1983) model for finite production rate. Goswami and Chaudhri (1992) allowed shortages and assumed demand to be linear function of time. Bhunia and Maiti (1994) modified and corrected assumptions of Goswami and Chaudhuri (1992). Sarma (1987) formulated a model for deteriorating items with constant demand and infinite replenishment rate allowing for shortages. Pakkala and Achary (1992) extended Sarma (1987)'s model under the assumption of finite replenishment for continuous release pattern in the rented warehouse. Benkherouf (1997) developed an inventory model for deteriorating items with two storage facilities by relaxing the assumption of fixed cycle length and known quantity to be stored in the OW.

In market, the demand of agriculture product decreases with time. In this paper, the assumption of fixed cycle length is relaxed. The demand of product is assumed to be exponentially decreasing with time. The optimal schedule is obtained to minimize total cost per time unit. The numerical example is given to support the model. The sensitivity analysis is carried out to observe the changes in the decision variables and total cost per time unit of an inventory system.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model of the aforesaid inventory system for deteriorating items and two warehouses is based on the following assumptions and notations:

- The inventory system deals with the single item.
- The replenishment rate is infinite.
- The demand rate is exponentially decreasing with time (say) $R(t) = ae^{-bt}$, $a > 0$ is fixed demand, $a >> b$, $0 < b < l$ is the rate at which demand is decreasing and t is time.
- Shortages are completely back-logged. *"π"* denotes the shortage cost per unit short.
- *"C"* denotes the purchase cost of an item.
- *"A"* denotes ordering cost per order.
- "h_o" is the inventory holding cost per unit per time unit in *OW* and "h_r" is the inventory holding cost per unit per time in *RW* with $h_r > h_o$
- The units in *RW* deteriorate at a constant rate *"β"* and in *OW* at a constant rate *"α"* with $0<\beta< \alpha < 1$.
- There is no repair or replacement of deteriorated units during the cycle under consideration.
- The capacity of the owned warehouse is denoted by *"W"*.

3. MATHEMATICAL MODEL:

The depletion of the inventory for one cycle is represented in Figure.1

The cycle starts with the quantity after clearing shortages of the previous cycle. The W_1 units of items are stored in the *OW* and the rest are sent to *RW* if $W_1 > W$. The inventory from *RW* is to be cleared first and then only from *OW*. The on-hand inventory $Q_r(t)$ in *RW* depletes due to demand and deterioration during $[0, t_w]$ until inventory level reaches to zero in the *RW*. During $[0, t_w]$ the stock in *OW* is subject to deterioration at a constant rate α. After the time point t_w, the on hand inventory depletes due to demand and deterioration up to time t_1 at which *OW* is empty. The system faces shortages during $[t_1, T]$. Note that if $W_1 < W$,
$t_w = 0$ then retailer should not go for *RW*.

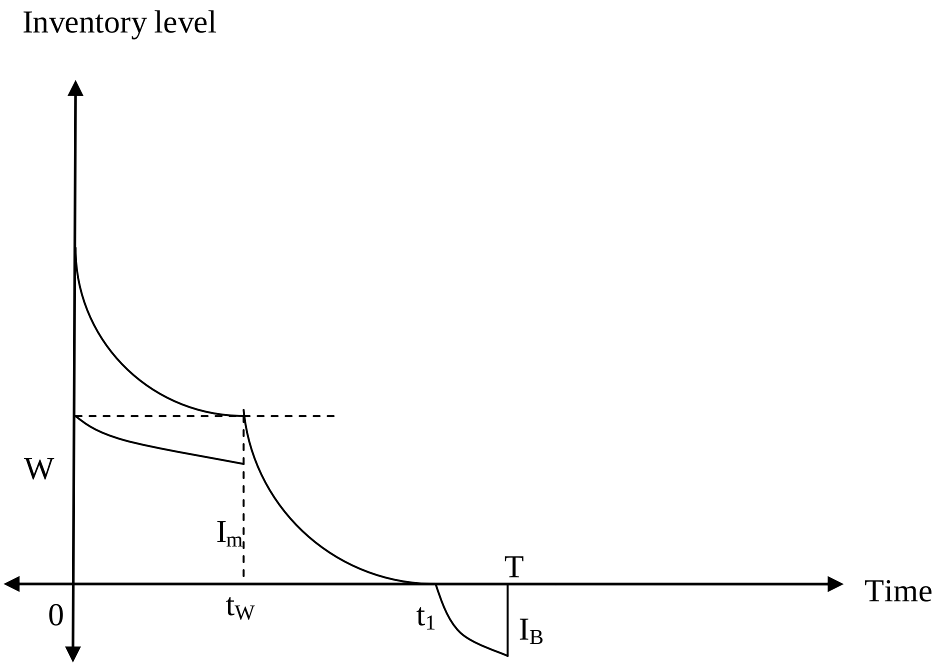

Figure.1: Time-inventory status.

3.1 Inventory status at RW

The on-hand inventory $Q_r(t)$ in RW depletes due to demand and deterioration during $[0, t_w]$. Therefore, instantaneous state of $Q_r(t)$ is governed by the differential equation:

$$\frac{dQ_r(t)}{dt} = -R(t) - \beta Q_r(t), \quad 0 \le t < t_w \tag{1}$$

with boundary condition $Q_r(t_w) = 0$.

The solution of (1) is given by

$$Q_r(t) = \frac{a}{\beta - b}[e^{(\beta-b)t_w-\beta t} - e^{-bt}], \quad 0 \le t < t_w \tag{2}$$

Hence, total inventory during $[0, t_w]$ is

$$I_r = \int_0^{t_w} Q_r(t)dt = \frac{a}{\beta}[\frac{1}{\beta-b}(e^{(\beta-b)tw}-1)+\frac{1}{\beta}(e^{-btw}-1)] \tag{3}$$

3.2 Inventory status at OW

During [o, t_w], the inventory in OW depletes only due to deterioration. The instantaneous state of inventory at any time t is governed by the differential equation:

$$\frac{dQ_o(t)}{dt} = -\alpha Q_o(t), \quad 0 \le t < t_w \tag{4}$$

with initial condition $Q_o(0) = W$. The solution of (4) is

$$Q_o(t) = W_1 e^{-\alpha t}, \ 0 \le t < t_w \tag{5}$$

Note that for $t_w > t$, $W_1 = W$.

Total inventory in *OW* during $[0, t_w]$ is

$$I_{o1} = \frac{W_1}{\alpha}[1 - e^{-\alpha t_w}] \tag{6}$$

The on-hand inventory in *OW* during $[t_w, t_1]$ depletes due to demand and deterioration. Hence, the corresponding differential equation, representing inventory level at any instant of time *t* during $[t_w, t_1]$ is

$$\frac{dQ_o(t)}{dt} = -R(t) - \alpha Q_o(t), \ t_w \le t < t_1 \tag{7}$$

with boundary condition $Q_o(t_1) = 0$. The solution of (7) is

$$Q_o(t) = \frac{a}{\alpha - b}[e^{(\alpha-b)t_1 - \alpha t} - e^{-bt}], \ t_w \le t < t_1 \tag{8}$$

and hence the inventory in the system during $[t_w, t_1]$ is

$$I_{o2} = \frac{1}{\alpha}\int_{t_w}^{t_1} (e^{\alpha(u-t_w)} - 1)R(u)du$$

$$= \frac{a}{\alpha}[\frac{1}{\alpha-b}(e^{(\alpha-b)t_1-\alpha t_w} - e^{-bt_w}) + \frac{1}{b}(e^{-bt_1} - e^{-bt_w})] \tag{9}$$

Since $Q(t)$ is continuous function, $Q_o(t_w)$ from equation (5) and (8) gives

$$W_1 = \frac{a}{\alpha - b}[e^{(\alpha-b)t_1} - e^{(\alpha-b)t_W}] \tag{10}$$

During $[t_1, T]$, the system suffers with shortages. The instantaneous state of the inventory system in $[t_1, T]$ is governed by the differential equation

$$\frac{dQ_o(t)}{dt} = -R(t) \ , t_1 \le t \le T \tag{11}$$

with $Q_o(t_1) = 0$ Hence, total shortages during $[t_1, T]$ are

$$SU = \int_{t_1}^{T}(T-u)R(u)du = \frac{a}{b^2}[e^{-bT} - e^{-bt_1} + b(T - t_1)e^{-bt_1}] \tag{12}$$

The total cost K_r in *RW* is

$$\begin{aligned} K_r &= C\int_0^{t_W} e^{\beta u}R(u)du + \frac{h_r}{\beta}\int_0^{t_W}(e^{\beta u} - 1)R(u)du \\ &= \frac{aC}{\beta - b}[e^{(\beta-b)t_W} - 1] + \frac{ah_r}{\beta}[\frac{1}{\beta - b}(^{(\beta-b)t_W} - 1) + \frac{1}{b}(e^{-bt_W} - 1)] \end{aligned} \tag{13}$$

and total cost K_o in *OW* is

$$K_o \quad = \quad A + CW_1 + h_o W_1 + \int_0^{t_W} e^{-\alpha u}du + \frac{h_o}{\alpha}\int_{t_W}^{t_1}(e^{\alpha(u-t_W)} - 1)R(u)du$$

$$\begin{aligned} = \quad & A + \frac{Ca}{\alpha - b}[e^{(\alpha-b)t_1} - e^{(\alpha-b)t_W}] + \frac{h_o a}{\alpha(\alpha - b)}[e^{(\alpha-b)t_1} - e^{(\alpha-b)t_W}](1 - e^{-\alpha t_W}) \\ & + \frac{h_o a}{\alpha}[\frac{1}{\alpha - b}e^{(\alpha-b)t_1 - \alpha t_W} - e^{-bt_W}) + \frac{1}{b}(e^{-bt_1} - e^{-bt_W})] \end{aligned} \tag{14}$$

Hence, total cost $K(t_w, t_1, T)$ per time unit of an inventory system is

$$K = K(t_w, t_1, T) = \frac{1}{T}(K_r + K_o + \pi(SU) + C\int_{t_1}^{T}R(u)du)$$

To find the optional solution for the total cost to be minimum, the necessary condition is

$$\frac{\partial K}{\partial t_W} = 0, \frac{\partial K}{\partial t_1} = 0, \frac{\partial K}{\partial T} = 0, \tag{16}$$

Provided

$$\begin{vmatrix} \dfrac{\partial^2 K}{\partial t_w^2} & \dfrac{\partial^2 K}{\partial t_w \partial t_1} & \dfrac{\partial^2 K}{\partial t_w \partial T} \\ \dfrac{\partial^2 K}{\partial t_1 \partial t_w} & \dfrac{\partial^2 K}{\partial t_1^2} & \dfrac{\partial^2 K}{\partial t_1 \partial T} \\ \dfrac{\partial^2 K}{\partial T \partial t_w} & \dfrac{\partial^2 K}{\partial t_1 \partial T} & \dfrac{\partial^2 K}{\partial T^2} \end{vmatrix} > 0 \tag{17}$$

The non-linear equations in (16) can be solved by software. In next section, the changes in parameters are considered to study the effects in decision variables and objective function.

4. NUMERICAL EXAMPLE

Consider the following parametric values in proper units: *[a, b, A, π, C, h_o, h_r, α, β]= [2000, 0.51, 10, 2.5, 0.085, 0.11, 0.01, 0.006]*

Then $t_w = 0.13669yrs$, $t_1 = 0.28410yrs$, $T = 0.66213yrs$, $W = 265.44$ units and total cost $K = \$10589.71$.

Table 1: Changes in parameters

Changes in		t_w	t_1	T	W	K
a	1000	0.13759	0.28599	0.66424	133.53	5295.12
	1500	0.13698	0.28473	0.66355	199.48	7942.41
	2000	0.13669	0.28410	0.66213	265.44	10589.71
b	0.49	0.14546	0.30255	0.72698	282.21	11872.01
	0.50	0.14103	0.29321	0.69592	273.72	11749.36
	0.51	0.13669	0.28410	0.66213	265.44	10589.71
α	0.010	0.13669	0.28410	0.66213	265.44	10589.71
	0.015	0.15126	0.28294	0.66280	236.57	10598.80
	0.020	0.16957	0.28139	0.66352	201.14	10607.03
β	0.0060	0.13669	0.28410	0.66213	265.44	10589.71
	0.0065	0.13532	0.28406	0.66217	267.93	10590.01
	0.0070	0.13398	0.28402	0.66219	270.38	10590.31
Π	2.0	0.13669	0.28410	0.66213	265.44	10589.71
	2.5	0.10723	0.22194	0.73232	211.31	11724.89
	3.0	0.08126	0.16757	0.78913	162.21	13036.81

It is observed that decision variable W and the total cost K are very sensitive to changes in constant demand a. With increase in the demand, optimum time t_w for inventory in RW, t_1, inventory in OW and total cycle time T for replenishment decreases. The rate of change of demand 'b' is inversely proportional to changes in decision variables and total cost of an inventory system. The deterioration rate increases total cost and cycle time T. Increase in rate of deterioration does not produce significant changes in the optimal solution. Increase in shortage cost decreases t_w, t_1, W and increases cycle time T and total cost of an inventory system significantly.

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