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ORDERING POLICY FOR INVENTORY MANAGEMENT WHEN DEMAND IS STOCK-DEPENDENT AND A TEMPORARY PRICE DISCOUNT IS LINKED TO ORDER QUANTITY

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ABSTRACT
In this article, the effect of the sales promotional scheme viz. price discount offered by the supplier on the retailer's ordering policy is studied when demand is stock-dependent. It is assumed that the price discount rate depends on the order quantity of the retailer. This study will help the retailer to take the decision whether to adopt a regular or special order policy. The optimum special quantity is decided by maximizing the total cost saving between the special and regular orders for the cycle time. The algorithm is proposed to take the favorable decision. The numerical examples are given to validate the derived results. The sensitivity analysis is carried out to determine the critical inventory parameters.

KEYWORDS : Inventory model, stock-dependent demand, a temporary price discount, special order

MSC : 90B05

RESUMEN
En el presente artículo, el efecto del plan de promoción de ventas, a saber descuento sobre el precio ofrecido por el proveedor, con la política de pedidos del minorista, se estudia cuando la demanda es existencia-dependiente. Se supone que la tasa de descuento sobre el precio de la orden depende de la cantidad del pedido del minorista. Este estudio ayudará al minorista a aceptar la decisión de adoptar un período ordinario o extraordinario como política para comprar. La cantidad especial óptima es fijada maximizando el costo total del ahorro entre la orden especial y la regular para un ciclo de tiempo. El algoritmo propuesto determina la decisión favorable. Ejemplos numéricos son dados para validar los resultados. El análisis de sensibilidad se lleva a cabo para determinar los parámetros críticos óptimos del inventario.

1. **INTRODUCTION**

The offer of price discount by the supplier boost the demand, attracts more retailers, increase the cash-flow to reduce his inventory. But then the question "Is it always advantageous to avail of discount for a special order ?" is at the retailer end. Dixit and Shah (2005) gave a review article on inventory models when a temporary price discount is offered by the supplier to the retailer to study the relationship between price discounts and order policy.

The all-unit quantity discounts ordering policy is discussed by Arcelus et al. (2003), Shah et al. (2005), Bhaba and Mahmood (2006), Abad (2007), Dye et al. (2007), Shah et al. (2008), Mishra and Shah (2009). They assumed that the price discount rate is independent of the special order quantity. However, in market, it is observed that the supplier offers a quantity discount to encourage larger orders. For the larger order, the higher price discount rate is given by the supplier. As a result, the retailer has to settle the trade-off for purchase price savings against higher total holding cost.

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Ouyang et al. (2009) discussed the effects of a temporary price discount on a retailer's ordering policy by assuming that the price discount rate is linked to special order quantity. They assumed that the demand is constant and deterministic.

In this paper, we study the impact of a temporary price discount on a retailer's ordering policy when demand is stock-dependent and price discount rate is linked to special order quantity. This study will help the retailer to take decision about adopting or declining the sales promotion tool. The retailer's optimal special order quantity is obtained by maximizing the total cost savings between the special and regular orders during a special order cycle time. Two scenarios are discussed : (1) the special order time occurs at the retailer's replenishment time, and (2) the special order time occurs during the retailer's regular cycle time. An algorithm is derived to compute the optimal solutions. The numerical examples are given to validate the theoretical results. The sensitivity analysis of the optimal solutions is carried out with respect to the model parameters. The managerial issues are derived.

2. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used in this article :

Notations

$R(I(t))$	:	$(= \alpha + \beta I(t))$, stock-dependent demand rate where $\alpha > 0$ is scale demand and $0 < \beta < 1$ is stock-dependent parameter
C	:	Unit purchasing cost
A	:	Ordering cost per regular or special order
i	:	Holding cost rate per annum
Q	:	Order quantity under regular policy
Q^*	:	Optimal order quantity when regular order policy is adopted
T	:	Cycle time when regular order policy is adopted
T^*	:	Optimal cycle time for using a regular order policy
Q_s	:	Special order quantity at discounted price (a decision variable)
T_s	:	Cycle time for the special order quantity Q_s
q	:	Inventory level before the arrival of the special order quantity; $q \geq 0$
t_q	:	Cycle time when q – units deplete to zero
T_W	:	Cycle time for depletion of the inventory level $W = Q_s + q$
$I(t)$	:	Inventory level at time t when the regular order policy is adopted; $0 \leq t \leq T$
$I_s(t)$	:	Inventory level at time t when the special order policy is adopted; $0 \leq t \leq T_s$
$I_W(t)$	:	Inventory level at time t when the special order policy is adopted; $0 \leq t \leq T_W$ where $W = Q_s + q$

Assumptions

(a) The inventory system under consideration deals with a single item.

(b) The supplier offers the retailer a temporary price discount if the order quantity is larger than the regular order quantity Q^*. The discount rate depends on the quantity ordered and the discount schedule is as follows :

Class	Special order quantity	Discount rate
1	$Q_1 \leq Q_s < Q_2$	d_1
2	$Q_2 \leq Q_s < Q_3$	d_2
⋮	⋮	⋮
n	$Q_n \leq Q_s < Q_{n+1}$	d_n

where Q_k is the k-th discount rate breaking point, $k = 1, 2, \ldots, n+1$ and $Q^* < Q_1 < Q_2 < \ldots < Q_{n+1} = \infty$; d_k is the price discount rate offered by the supplier when the retailer's order quantity Q_s belongs to the interval $[Q_k, Q_{k+1})$ and $0 < d_1 < d_2 < \ldots < d_n$.

(c) The price discount is not passed on to the customers. Only one time price discount is offered.
(d) The replenishment rate is infinite.
(e) The lead-time is zero and shortages are not allowed.

Mathematical Model

The aim of the study is to decide the advantage of temporary price discount for larger order than the regular order, under the assumption of the stock-dependent demand. If the retailer adopts to follow regular order policy without a temporary price discount, then the inventory depletes in the retailer's inventory system due to the stock-dependent demand. The change in inventory level is governed by the differential equation

$$\frac{dI(t)}{dt} = -(\alpha + \beta I(t)), \quad 0 \le t \le T \tag{1}$$

With boundary condition $I(T) = 0$, the solution of (1) is

$$I(t) = \frac{\alpha}{\beta}\ \exp(\beta(T-t)) - 1\ ,\ 0 \le t \le T \tag{2}$$

Hence, the order quantity is

$$Q = I(0) = \frac{\alpha}{\beta}\ \exp(\beta T) - 1 \tag{3}$$

In this case, the retailer's total cost per order cycle is $A + CQ + Ci\int_0^T I(t)dt$. i.e.

$$A + \frac{C\alpha}{\beta}\ \exp(\beta T) - 1\ + \frac{Ci\alpha}{\beta^2}\ \exp(\beta T) - \beta T - 1 \tag{4}$$

Therefore, the total cost per unit time without temporary price discount is

$$TC(T) = \frac{1}{T}\left[A + \frac{C\alpha}{\beta}\ \exp(\beta T) - 1\ + \frac{Ci\alpha}{\beta^2}\ \exp(\beta T) - \beta T - 1\right] \tag{5}$$

The convexity of $TC(T)$ can be proved as given in Dye et al. (2007). It guarantees that there exists unique value of T (say) T^* that minimizes $TC(T)$. T^* can be obtained by setting

$$\frac{dTC(T)}{dT} = A - \frac{C(i+\beta)\alpha}{\beta^2}\ \ \beta T\exp(\beta T) - \exp(\beta T) + 1\ = 0 \tag{6}$$

Knowing the regular order cycle time T^*, the optimal order quantity without a temporary price discount, Q^* can be obtained as

$$Q^* = \frac{\alpha}{\beta}\left[\exp(\beta T^*) - 1\right] \tag{7}$$

The following two scenarios may arise when the supplier offers a temporary price discount and the retailer take advantage of this by ordering quantity greater than Q^* : (1) when the special order time occurs at the retailer's cycle time; and (2) when the special order time occurs during the retailer's cycle time. Next, we formulate the corresponding total cost savings for these two scenarios.

Scenario 1 : When the special order time occurs at the retailer's cycle time **(Figure 1)**

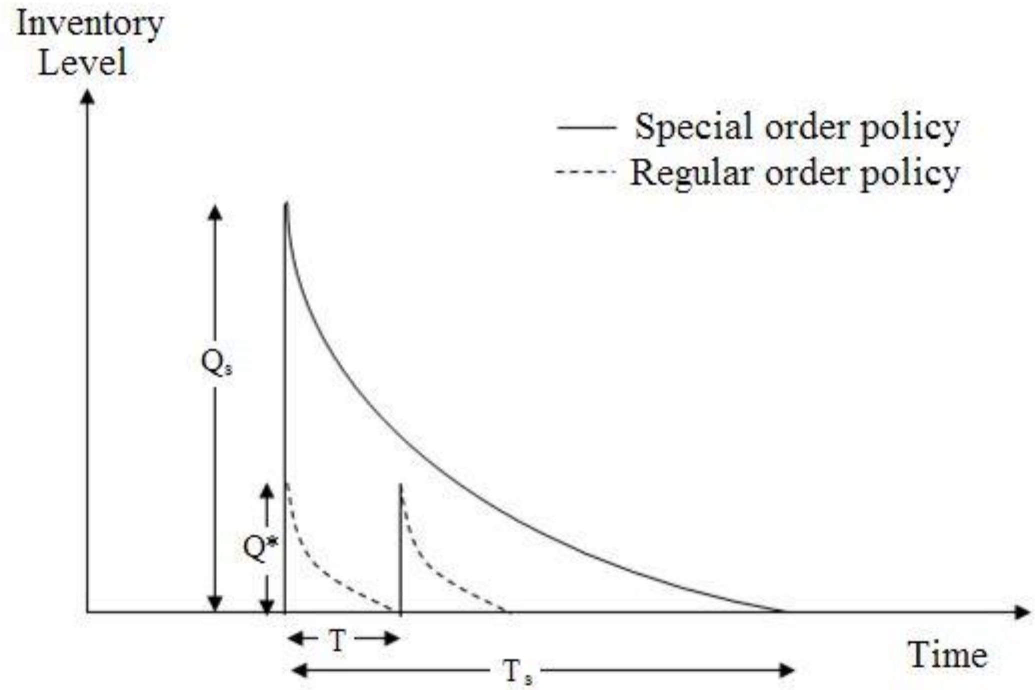

Figure 1 Special order time occurs at the retailer's cycle time

Here, if the retailer decides to give order of special quantity Q_s – units, arguing as above, the inventory level at time t is

$$I_s(t) = \frac{\alpha}{\beta} \ \exp(\beta(T_s - t)) - 1 \ , \ 0 \le t \le T_s \tag{8}$$

and the special order quantity is

$$Q_s = I_s(0) = \frac{\alpha}{\beta} \ \exp(\beta T_s) - 1 \tag{9}$$

For each price discount rate d_i, the total cost $TCS_{1i}(T_s)$ of the special order during the time interval [0, T_s] is

$$TCS_{1i}(T_s) = A + \frac{C(1-d_i)\alpha}{\beta} \ \exp(\beta T_s) - 1 \ + \frac{C(1-d_i)i\alpha}{\beta^2} \ \exp(\beta T_s) - \beta T_s - 1 \ , i=1, 2, \dots, n \tag{10}$$

On the other hand, if the retailer decides to follow regular order policy of Q^* - units instead of putting a large special order, the total cost during [0, T_s] is

$$TCN_1(T_s)=\frac{T_s}{T^*}\left[A+\frac{C\alpha}{\beta}\left[\exp(\beta T^*)-1\right]+\frac{Ci\alpha}{\beta^2}\left[\exp(\beta T^*)-\beta T^*-1\right]\right] \tag{11}$$

Obviously, $TCN_1(T_s) > TCS_{1i}(T_s)$ for given d_i. Hence, the total cost savings, $G_{1i}(T_s)$ because of the offer of a temporary price discount is

$$G_{1i}(T_s)=TCN_1(T_s)-TCS_{1i}(T_s) \tag{12}$$

Scenario 2 : When the special order time occurs during the retailer's cycle time **(Figure 2)**

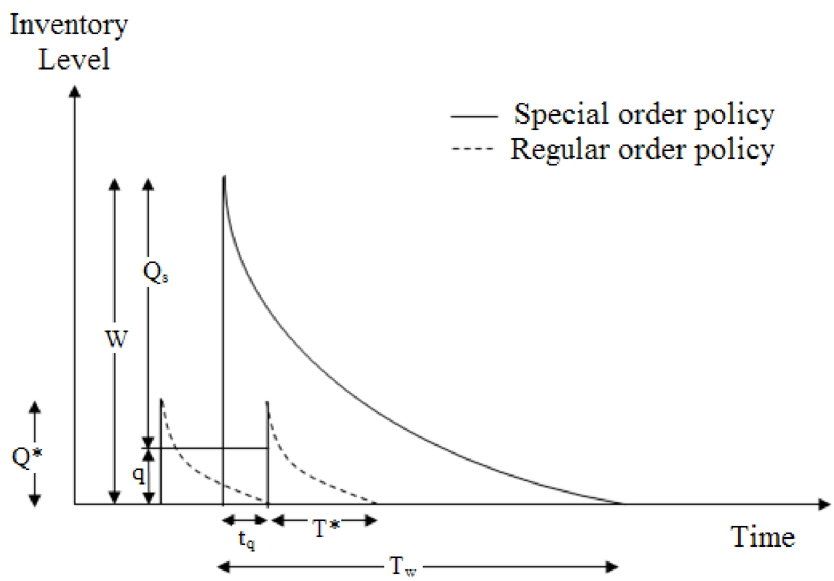

Figure 2 Special order time occurs during the retailer's cycle time

We want to analyze the situation when the special order time occurs during the retailer's cycle time. Here, the retailer has q – units and orders for Q_s – units which raises his inventory to $W = Q_s + q$. When the special order of Q_s - units is placed, the total cost during the interval [0, T_W] comprises of ordering cost; A, purchase cost as $\frac{C(1-d_i)\alpha}{\beta}\ \exp(\beta T_s)-1$ and the holding cost which is calculated as follows:

With the arrival of special order quantities, the stock on hand increase instantaneously from q to W, where

$$W=Q_s+q=\frac{\alpha}{\beta}\ \exp(\beta T_s)-1\ +\frac{\alpha}{\beta}\left[\exp(\beta t_q)-1\right]=\frac{\alpha}{\beta}\left[\exp(\beta T_s)+\exp(\beta t_q)-2\right] \tag{13}$$

The inventory level at any instant of time t is given by

$$I_W(t)=\frac{\alpha}{\beta}\ \exp(\beta(T_W-t))-1\ ,\ 0\le t\le T_W \tag{14}$$

and $W=I_W(0)=\frac{\alpha}{\beta}\ \exp(\beta T_W)-1$ (15)

From (13) and (15), we get

$$T_W = \frac{1}{\beta}\ln\left[\exp(\beta T_s) + \exp(\beta t_q) - 1\right] \tag{16}$$

The holding cost of q – units purchased at \$ C per unit is $\frac{Ci\alpha}{\beta^2}\left[\exp(\beta t_q) - \beta t_q - 1\right]$ and that of the special order quantity Q_s available at \$ $C(1\text{-}d_i)$ per unit is

$$C(1-d_i)i\left[\int_0^{T_W} I_W(t)dt - \frac{\alpha}{\beta^2}\left[\exp(\beta t_q) - \beta t_q - 1\right]\right]$$

$$= \frac{C(1-d_i)i\alpha}{\beta^2}\left[\exp(\beta T_s) + \exp(\beta t_q) - 2 - \ln(\exp(\beta T_s) + \exp(\beta t_q) - 1) - (\exp(\beta t_q) - \beta t_q - 1)\right] \tag{17}$$

Hence, the total holding cost of W – units during the time interval [0, T_W] is

$$\frac{C(1-d_i)i\alpha}{\beta^2}\left[\exp(\beta T_s) + \exp(\beta t_q) - 2 - \ln(\exp(\beta T_s) + \exp(\beta t_q) - 1)\right] + \frac{Cid_i\alpha}{\beta^2}(\exp(\beta t_q) - \beta t_q - 1) \tag{18}$$

Therefore, for the fixed price discount rate d_i, the total cost $TCS_{2i}(T_s)$ of the special order during the time interval [0, T_W] is

$$\begin{aligned} TCS_{2i}(T_s) = A + \frac{C(1-d_i)\alpha}{\beta}\ \exp(\beta T_s) - 1\ + \frac{Cid_i\alpha}{\beta^2}(\exp(\beta t_q) - \beta t_q - 1) \\ + \frac{C(1-d_i)i\alpha}{\beta^2}\left[\exp(\beta T_s) + \exp(\beta t_q) - 2 - \ln(\exp(\beta T_s) + \exp(\beta t_q) - 1)\right] \end{aligned} \tag{19}$$

If the retailer does not opt for the price discount and continues to follow regular order policy, the total cost during the interval [0, T_W] will be computed for two periods. In the first period, he incurs the holding cost for q – units as $Ci\int_0^{t_q} I(t)dt = \frac{Ci\alpha}{\beta^2}\left[\exp(\beta t_q) - \beta t_q - 1\right]$ and in the next period total cost as

$$\frac{(T_W - t_q)}{T^*}\left[A + \frac{C\alpha}{\beta}\left[\exp(\beta T^*) - 1\right] + \frac{Ci\alpha}{\beta^2}\left[\exp(\beta T^*) - \beta T^* - 1\right]\right]$$

$$= \frac{(\ln(\exp(\beta T_s) + \exp(\beta t_q) - 1) - \beta t_q)}{\beta T^*}\left[A + \frac{C\alpha}{\beta}\left[\exp(\beta T^*) - 1\right] + \frac{Ci\alpha}{\beta^2}\left[\exp(\beta T^*) - \beta T^* - 1\right]\right]$$

Hence, the total cost of the inventory system is

$$TCN_2(T_s) = \frac{Ci\alpha}{\beta^2}\left[\exp(\beta t_q) - \beta t_q - 1\right]$$
$$+\frac{(\ln(\exp(\beta T_s) + \exp(\beta t_q) - 1) - \beta t_q)}{\beta T^*}\left[A + \frac{C\alpha}{\beta}\left[\exp(\beta T^*) - 1\right] + \frac{Ci\alpha}{\beta^2}\left[\exp(\beta T^*) - \beta T^* - 1\right]\right] \tag{20}$$

From (19) and (20), for a fixed discount rate d_i, the total cost savings; $G_{2i}(T_s)$ is

$$G_{2i}(T_s) = TCN_2(T_s) - TCS_{2i}(T_s) \tag{21}$$

WLOG, we assume that the total cost savings in (12) and (21) are both positive for special order policy.

3. ANALYTIC RESULTS

In this section, we will determine the optimal value of T_s that maximizes the total cost savings.

Scenario 1 : When the special order time occurs at the retailer's cycle time

For the fixed discount rate d_i, the derivative of $G_{1i}(T_s)$ in (12) with respect to T_s gives

$$\frac{dG_{1i}(T_s)}{dT_s} = \frac{1}{T^*}\left[A + \frac{C\alpha}{\beta}\left[\exp(\beta T^*) - 1\right] + \frac{Ci\alpha}{\beta^2}\left[\exp(\beta T^*) - \beta T^* - 1\right]\right]$$
$$- C(1-d_i)\alpha\exp(\beta T_s) - \frac{C(1-d_i)i\alpha}{\beta^2}\ \exp(\beta T_s) - 1 \tag{22}$$

and $$\frac{d^2G_{1i}(T_s)}{dT_s^{\,2}} = -C(1-d_i)(\beta + i)\alpha\exp(\beta T_s) < 0 \tag{23}$$

Eq. (23) proves that $G_{1i}(T_s)$ is a concave function of T_s. Hence, a unique value of $T_s = T_{s1i}$ (say) exists that maximizes $G_{1i}(T_s)$. Equating (22) to be zero gives value of T_{s1i} as

$$T_{s1i} = \frac{1}{\beta}\ln\left[\frac{C(1-d_i)i\alpha + \beta x}{C(1-d_i)\alpha(\beta + i)}\right] \tag{24}$$

where $x = \frac{1}{T^*}\left[A + \frac{C\alpha}{\beta}\left[\exp(\beta T^*) - 1\right] + \frac{Ci\alpha}{\beta^2}\left[\exp(\beta T^*) - \beta T^* - 1\right]\right] > 0$.

Clearly, $Q^* < Q_{s1i}$ if and only if $T^* < T_{s1i}$ .i.e. $\Delta_{1i} > 0$ (25)

where $\Delta_{1i} = x - C(1-d_i)\alpha\exp(\beta T^*) - \frac{C(1-d_i)i\alpha}{\beta^2}\left[\exp(\beta T^*) - 1\right]$

Using (24) in (12) gives the corresponding maximum total cost savings as

$$G_{1i}(T_{s1i}) = \frac{C(1-d_i)\alpha(\beta + i)}{\beta^2}\left[\beta T_{s1i}\exp(\beta T_{s1i}) - \exp\ \ \beta T_{s1i}\ \ + 1\right] - A \tag{26}$$

Denote $\Delta_{2i} = G_{1i}(T_{s1i})$. The retailer opts for special order only if $\Delta_{2i} > 0$. Otherwise, he will continue with the regular order policy of Q^* - units. Hence, the optimal value of T_{s1i} (denoted by $T_{s1i}{}^*$ for scenario 1 is

$$T_{s1i}{}^* = \begin{cases} T_{s1i}, & \text{if } \Delta_{1i} > 0 \text{ and } \Delta_{2i} > 0 \\ T^*, & \text{otherwise} \end{cases} \tag{27}$$

Scenario 2 : When the special order time occurs during the retailer's cycle time

For the fixed price discount rate d_i, setting the first order derivative of $G_{2i}(T_s)$ in (21) with respect to T_s to be zero gives

$$\frac{dG_{2i}(T_s)}{dT_s} = \left[x + \frac{C(1-d_i)i\alpha}{\beta} \right] \frac{\exp(\beta T_s)}{\exp(\beta T_s) + \exp(\beta t_q) - 1} - \frac{C(1-d_i)(\beta+i)\alpha}{\beta} \exp(\beta T_s) = 0 \tag{28}$$

$T_s = T_{s2i}$ (say)

$$T_{s2i} = \frac{1}{\beta} \ln\left[\frac{\beta x + C(1-d_i)i\alpha - C(1-d_i)(\beta+i)\alpha \exp(\beta T_s)}{C(1-d_i)(\beta+i)\alpha} \right] \tag{29}$$

The second order derivative

$$\left. \frac{d^2 G_{2i}(T_s)}{dT_s^2} \right|_{T_s = T_{s2i}} = -\frac{C(1-d_i)(\beta+i)\alpha}{\exp(\beta T_s) + \exp(\beta t_q) - 1} \exp(2\beta T_{s2i}) < 0$$

guarantees that $G_{2i}(T_{s2i})$ is maximum. Next to ensure that $Q^* < Q_{s2i}$ i.e. $T^* < T_{s2i}$, substitute (29) into this inequality which results in

$T^* < T_{s2i}$ if and only if $\Delta_{3i} > 0$ (30)

where

$$\Delta_{3i} = x - \frac{C(1-d_i)(\beta+i)\alpha}{\beta} \ \exp(\beta T^*) + \exp(\beta t_q) - 1 \ + \frac{C(1-d_i)i\alpha}{\beta}$$

Using (29) into (21) gives the corresponding maximum total cost savings as

$$G_{2i}(T_{s2i}) = \frac{1}{\beta^2} C(1-d_i)(\beta+i)\alpha \left[\ln(\exp(\beta T_{s2i}) + \exp(\beta t_q) - 1) - \beta t_q \right] (\exp(\beta T_{s2i}) + \exp(\beta t_q) - 1)$$
$$- \frac{1}{\beta^2} C(1-d_i)(\beta+i)\alpha(\exp(\beta T_{s2i}) - 1) - A \tag{31}$$

Clearly, $\Delta_{4i} = G_{2i}(T_{s2i}) > 0$ to qualify for special order otherwise retailer should follow the regular order policy. Hence, the optimal value of T_{s2i} (denoted by $T_{s2i}{}^*$) for scenario 2 is

$$T_{s2i}{}^* = \begin{cases} T_{s2i}, & \text{if } \Delta_{3i} > 0 \text{ and } \Delta_{4i} > 0 \\ T^*, & \text{otherwise} \end{cases} \tag{32}$$

Next we outline computational procedure to obtain the optimal cycle time T_s^* and the optimal special order quantity Q_s^* for the two scenarios.

Computational Procedure

Step 1. If $q = 0$, then compute T^* and go to step 2. Otherwise calculate t_q from $t_q = \frac{1}{\beta}\ln\left(1+\frac{\beta q}{\alpha}\right)$, and go to step 4.

Step 2. For each d_i, $i = 1, 2, \ldots, n$ obtain T_{s1i} from (24), Δ_{1i} from (25) and Δ_{2i} from (26). If $\Delta_{1i} > 0$ and $\Delta_{2i} > 0$ then substitute T_{s1i} into (9) and obtain Q_{s1i}. Check Q_{s1i} under di. If

(i) $Q_i \le Q_{s1i} < Q_{i+1}$, then Q_{s1i} is a feasible solution. Set $Q_{s1i}^* = Q_{s1i}$ and compute $G_{1i}(T_{s1i}^*)$.

(ii) $Q_{s1i} \ge Q_{i+1}$, then larger price discount rate is possible and thus Q_{s1i} is not a feasible solution. Set $G_{1i}(T_{s1i}^*) = -\infty$.

(iii) $Q_{s1i} < Q_i$ then set $Q_{s1i}^* = Q_i$. Substitute Q_{s1i}^* into (9) and find T_{s1i}^* and hence compute $G_{1i}(T_{s1i}^*)$. If $G_{1i}(T_{s1i}^*) > 0$, go to step 3; otherwise set $T_{s1i}^* = T^*$, $Q_{s1i}^* = Q^*$ and $G_{1i}(T_{s1i}^*) = 0$.

Step 3. Find $\max\limits_{i=1,2,\ldots,n} G_{1i}(T_{s1i}^*)$. Go to step 6.

Step 4. For each d_i, $i = 1, 2, \ldots, n$ obtain T_{s2i} from (29), Δ_{3i} and Δ_{4i}. If $\Delta_{3i} > 0$ and $\Delta_{4i} > 0$ then substitute T_{s2i} into (13) and obtain Q_{s2i}. Check Q_{s2i} under di. If

(i) $Q_i \le Q_{s2i} < Q_{i+1}$, then Q_{s2i} is a feasible solution. Set $Q_{s2i}^* = Q_{s2i}$ and compute $G_{2i}(T_{s2i}^*)$.

(ii) $Q_{s2i} \ge Q_{i+1}$, then larger price discount rate is possible and thus Q_{s2i} is not a feasible solution. Set $G_{2i}(T_{s2i}^*) = -\infty$.

(iii) $Q_{s2i} < Q_i$ then set $Q_{s2i}^* = Q_i$. Substitute Q_{s2i}^* into (9) and find T_{s2i}^* and hence compute $G_{2i}(T_{s2i}^*)$. If $G_{2i}(T_{s2i}^*) > 0$, go to step 5; otherwise set $T_{s2i}^* = 0$, $Q_{s2i}^* = 0$ and $G_{2i}(T_{s2i}^*) = 0$.

Step 5. Find $\max\limits_{i=1,2,\ldots,n} G_{2i}(T_{s2i}^*)$. Go to step 6.

Step 6. Stop.

In the next section, numerical examples are presented to validate the proposed problem.

4. NUMERICAL EXAMPLES

Example 1 Consider the following parametric values for the retailer inventory system when the special order is due at the regular order cycle time : C = \$ 10 / unit, α = 1000 units / year, A = \$ 150 / order, i = 30 % per annum, β = 10 %. Using step 1 of computational procedure, the optimum cycle time T^* = 0.271 years and regular order quantity is Q^* is 275 units per order. The price discount rate offered by the supplier is given in Table 1.

Table 1. Price discount rate schedule

Class I	Special order quantity Q_s	Discount rate d_i
1	$500 \le Q_s < 1000$	10 %
2	$1000 \le Q_s < 2400$	20 %
3	$Q_s \ge 2400$	28 %

Using steps 2 and 3, the solution is obtained as given in Table 2.

Table 2. Optimal solutions for Example 1

d_i	Q_{s1i}	T_{s1i}^{*}	Q_{s1i}^{*}	G_{1i}^{*}
10 %	583	0.567	583	451.17
15 %	765	0.953	1000	733.94
28 %	969	2.151	2400	1072.40

Shaded solution is the optimal solution.

From Table 2, it is observed that the retailer saves \$ 1072.40 by ordering 2400 – units available at the discount rate 28 %.

Example 2. Consider the data as given in example 1 except for *q*. Here, we want to validate scenario 2 when the special order time is during the retailer's cycle time. The optimal ordering policies for q = 50, 100 and 200 are given in Table 3.

Table 3. Optimal solutions for Example 2 for different values of q

q	T_{s2}^{*}	Q_{s2}^{*}	G_{2}^{*}
50	2.151	583	757.37
100	2.151	1000	445.16
200	0.953	2400	80.95

From Table 3, it is can be seen that the total cost savings is negatively very sensitive to the remnant inventory. It directs the logistic manager to keep remnant inventory as low as possible when the special order time occurs during the cycle time.

Example 3. In Table 4, we study the effect of changes in the inventory parameters C, α, A, i and β on the optimal price discount rate, special order quantity and total cost savings. The data is taken as that of Example 2 and q = 50.

Table 4 Sensitivity analysis

Parameter	Value	d_i^{*}	Q_{s2}^{*}	G_{2}^{*}
C	5.0	0.28	2400	1073.76
	7.5	0.28	2400	945.51
	12.5	0.28	2400	714.65
	15.0	0.28	2400	560.74
α	500	0.10	500	474.75
	750	0.20	1000	665.71
	1250	0.28	2400	792.29
	1500	0.28	2400	1024.65
A	25.0	0.20	1000	750.61
	37.5	0.20	1000	792.75
	112.5	0.28	2400	1284.68
	225.0	0.28	2400	1482.39
i	0.15	0.28	2400	3258.18
	0.25	0.28	2400	1630.89
	0.35	0.20	1000	1018.17
	0.45	0.20	1000	710.47
β	0.05	0.20	1000	518.41
	0.15	0.28	2400	892.04
	0.20	0.28	2400	915.16
	0.25	0.28	2400	962.80

The close look on Table 4 gives following managerial insights :

(1) The retailer will find the optimal special order quantity by determining the advantage of the price discount compared to additional holding cost, he will have to incur. For example, for α = 500 or β = 5 %, the retailor adopts the regular order policy. Also, for the optimal order quantity, the retailer not only maximizes his total cost savings but also the price discount rate.
(2) Increase in scale parameter of demand; α and ordering cost: A, increases total cost savings. This suggest that when demand and ordering cost are likely to increase, it is advantageous for the retailer to take the advantage of special order quantity at the discounted rate.
(3) Increase in holding charge fraction decreases special order quantity and total cost savings. The opposite effect is observed when the stock-dependent parameter is considered.

4. CONCLUSIONS

The retailer's ordering policy is analyzed when a supplier offers a temporary price discount linked to order quantity and demand is stock-dependent. The optimal policy of special order is determined to maximize the total cost savings. A decision making algorithm is proposed to find the optimal solution. The theoretical results are validated by numerical examples. A sensitivity analysis is carried out to determine most critical inventory parameters. Eventually, the results divulge that (1) the special order quantity should be calculated by taking the difference between the total cost when the retailer avails / not avails of a temporary price discount, (2) retailer should keep remnant inventory as low as possible, (3) to avail of the offer of a temporary price discount is advantageous when the unit price, market demand and ordering cost are likely to increase. Thus decision policy provides a building block to the retailer in order to survive in the competitive market.

The developed model can be studied to compare various promotional schemes offered by the supplier. The model can be analyzed when retailer passes the part od price discount to his customers. The model can also be studied for different demand functions.

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