RECOVERY OF THE CONSUMER MULTIATTRIBUTIVE UTILITY MAXIMIZATION PROBLEM

Evgeniy M. Ozhegov[1]
National Research University Higher School of Economics
38 Studencheskaya ulitsa, Perm 614000, Russia

ABSTRACT
This article contains a prediction model for the demand for the consumer commodities. I consider the classical model of the consumer utility function maximization in a given budget constraint where there are two products: the first one for which the demand is being estimated, and the rest of the consumption bundle which is the second product. The utility function is introduced as multiattributive utility function with an unidentified number of attributes. An approach to estimate the exact number of attributes and the parameters of the model in a given class of utility functions for each attribute was proposed. The estimation is derived through the optimization of corrected Akaike information criterion, where the parameters of the utility function are continuous and the number of attributes is integer and positive. This model was tested on the prediction of the homogenous product demand with the Giffen effect.

KEYWORDS: Demand estimation; multiattributive utility function; unidentified number of attributes

MSC: 90B50, 90C29, 91B42

RESUMEN
Este trabajo presenta un modelo de predicción para la demanda del consumidor de bienes. Considero el clásico modelo de maximización de la función de utilidad del consumidor, dada una restricción de presupuesto donde hay dos productos: el primero para el cual la demanda está siendo estimada, y el resto el paquete de consumo es el segundo producto. La función de utilidad es introducida como una función de utilidad multiatributo con un número no identificado de atributos. Se propone un enfoque para estimar el número exacto de atributos y los parámetros del modelo en una clase dada de funciones de utilidades para cada atributo. La estimación es derivada a través del la optimización del criterio de información corregida de Akaike, donde los parámetros de la función de utilidad son continuos y el número de atributos es entero y positivo. Este modelo fue probado para hacer la predicción de la demanda del producto homogéneas con el efecto de Giffen.

1. INTRODUCTION

The traditional approach to the consumer preference modeling is the use the concept of preference as a binary relation on the set of products. If one of the two alternatives from the feasible set is preferable than the second one then a consumer will choose the first alternative. The most common theoretical concept is the concept of classical rational preference. A consumer makes rational choice if he has strict preference that is coherent, irreflexive, transitive and asymmetric. The consumer choice and, respectively, the demand function can always be described as the most preferred set of products in each budget set.

However, this approach to the consumer behavior modeling is favorable theoretically only. Empirically consumer's preferences do not satisfy the requirements of classical rationality. In particular, experiments with consumers indicate the following problems: 1) inability to evaluate equivalence of sets of products due to the lack of information on the products; 2) lack of preferences transitivity; 3) dependence of preferences on the external consumption conditions, such as fashion, outside air temperature, consumer mood, and also variability of preferences over time.

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[1] tos600@gmail.com

So, all these special cases of consumer behavior, which are not mentioned in the theory of rational preference, were used by the number of authors in the modeling of set consumer choice situations. The generalized utility function [Shafer, 1974] was introduced to model non-transitive preferences, the variability of preferences over time was formalized in models of intertemporal choice with discounted utility [Shane, Loewenstein, O'Donoghue, 2002] and demand switching models [Hutchinson, 1986].

The traditional approach to demand estimation assumes that the product has attributes [McFadden, 1973], [Berry, 1994], [Berry, Levihnson and Pakes, 1995], [Nevo, 2000], [Dyer, 2005]. We always can say that the product has a fixed set of attributes which are important for the consumer at the decision-making moment. Buying a refrigerator, consumer pays attention to the volume of the refrigerator, energy usage, noise level, freezer presence and its capacity, country of origin. Differences in values of attributes generate demand for differentiated products which can be modeled as the choice from the discrete set of heterogenenous products (discrete choice models). This paper will study the nonlinear dependence of demand on price for some product. The positive reaction of demand on price on some price intervals called «Giffen behavior» was also studied by demand-for-heterogeneous-products models. In [Jensen and Miller, 2008] the set of cheap products that differentiated by price and nutrition was considered. Authors found strong evidence of Giffen behavior in two China provinces that means that demand function has positive price parameter estimate.

And what can we say when we talk about homogeneous commodity products? What does motivate consumers when they buy a bottle of water? He is certainly interested in such attribute as thirst quenching. The importance of this attribute is not constant, because at some moments the need of thirst quenching increases. As a result, in fixed prices and consumer income, the volume of consumed water increases, and, thus, the demand for other goods declines. Generally, we do not know exactly how the consumer makes decision and which number of attributes he is ruled by. We can only observe the level of demand changing over time. Sample selection models introduced by [Heckman, 1979] and later studied by [Sartori, 2003] and [Winship and Mare, 1992] favored research on demand for the homogenous consumer commodities with corner solutions. [D'Souza and Jolliffe, 2010] used linear regression for panel data approach to study Giffen behavior in demand for wheat in Afghanistan during price shocks in 2007/2008. The authors estimated elasticity of demand for wheat on own price controlled by cross-price elasticities of other products and household and province characteristics.

However, this approaches examined only the dependence of consumption level of some product on the consumer characteristics, but not on the attributes of homogenous product. This paper provides the technique for the estimation of the number of attributes of the homogenous product that was not recently used in previous researches.

2. THE MODEL

Let us consider a model which describes consumer's decision-making process of buying some commodity. It's possible to take as consumers both individuals and households.

Let the x_1 be a product demand for which is being estimated.

Let the x_2 be the rest of the consumption bundle. Let us consider an aggregated amount of bought products except x_1 as a consumption bundle.

p_1, p_2 are the prices for x_1 and weight-average price for the products from the consumption bundle, respectively.

I is the consumer expenditures for commodities x_1 and x_2.

Let us assume that products x_1 and x_2 have equal and unidentified set of attributes. Each attribute has its own utility which can be described with utility function.

$u_j = u_j(x_1, x_2, \theta_j)$ is an utility function of j-th attribute, θ_j is a j-th attribute utility function vector of parameters.

An integral weighted utility of a set of products can be described with utility function

$$U = \sum_{j=1}^{m} \lambda_j u_j(x_1, x_2, \theta_j) \Big/ \sum_{j=1}^{m} \lambda_j \qquad (1)$$

$\lambda_j > 0, \forall j \in 1..m$

where λ_j is a weight of j-th attribute utility and m is a number of attributes.

λ_j can be functions from external for products conditions. In the case with water, λ_j, the weight of the thirst quenching attribute of products, may be (non)linear function from outside temperature. And the other λ_j should be constants if the research is aimed to estimate how demand for water depends on the temperature variation.

Budget set may be written as

$$B(p, I) = \{(x_1, x_2) | x_1 \geq 0, x_2 \geq 0, p_1 x_1 + p_2 x_2 \leq I\} \tag{2}$$

Thus, the consumer choice problem may be described as the problem of utility function maximization in fixed budget set at each time moment.

$$U = \sum_{j=1}^{m} \lambda_j u_j(x_1, x_2, \theta_j) / \sum_{j=1}^{m} \lambda_j \rightarrow max \tag{3}$$

$$(x_1, x_2) \in B(p, I) \tag{4}$$

So, the demand function is the solution of problem (3-4):

$$(x_1^*, x_2^*) = arg \max_{(x_1, x_2) \in B(p, I)} \sum_{j=1}^{m} \lambda_j u_j(x_1, x_2, \theta_j) / \sum_{j=1}^{m} \lambda_j \tag{5}$$

3. THE MODEL ESTIMATION

Empirical model estimation is faced with some challenges.

Firstly, the utility function value is non-measurable. None of consumers can say the exact utility value of each product set.

Secondly, there is unidentified amount of attributes that has some importance for consumer when decision-making.

And thirdly, specifications of attributes' utility functions are unknown.

The last fact can be avoided only by fixing utility function class with some known characteristics. Two other facts can be avoided with the following estimation algorithm.

The consumer makes rational decisions at each time moment, which means that he is aimed to maximize consumed utility. However, the attribute utility function is monotonously increasing with increase in consumption of each product with the fixed amount of all other products. Thus, the following assumption arises: the consumer spends the whole income available for consumption on buying products x_1 and x_2.

Let us write down a Lagrangian and first-order conditions for the problem (3-4):

$$\begin{cases} L(x_1, x_2, p, I, \mu, m, \theta^j, \lambda_j) = U - \mu(p_1 x_1 + p_2 x_2 - I) \rightarrow max \\ \dfrac{\partial L}{\partial x} = 0 \\ \dfrac{\partial L}{\partial \lambda} = 0 \end{cases} \tag{6}$$

Condition $\frac{\partial L}{\partial \lambda} = 0$ allows to rewrite the demand for x_2 as $x_2 = \frac{I - p_1 x_1}{p_2}$.

When $p_1 x_1 + p_2 x_2$ is equal to I and, respectively, λ is non-zero, first-order condition, i.e. consumer rationality condition, can be rewritten as

$$\frac{\partial L}{\partial x_1} = \frac{\partial U(x_1, p, I, m, \theta^j, \lambda_j)}{\partial x_1} = 0 \tag{7}$$

It must be noted that utility function U generally is nonlinear. So it is necessary to write second-order condition for utility function maximization.

$$\frac{\partial^2 U(x_1, p, I, m, \theta^j, \lambda_j)}{\partial^2 x_1} \leq 0 \tag{8}$$

For an empirical estimation it is indispensable to have a data of x_1 purchasing amount, prices of x_1 and weighted-average price of consumption bundle and consumer expenditures.

Then it is necessary to introduce additional conditions for consumer rationality at each time period and for normally distributed deviation from rationality with mean equal to zero and constant standard deviation. This means that the consumer is rational and her decision is i.i.d.

$$\frac{\partial U(x_1, p, I, m, \theta^j, \lambda_j)}{\partial x_1} \sim N(0, \sigma) \tag{9}$$

It is possible to recover the utility maximization problem with the following procedure.
Let us write down the likelihood function for n independent observations over consumer's decisions.

$$\mathcal{L}(x_1, p, I) = \prod_{t=1}^{n} \varphi\left(\frac{\partial U(x_1(t), p(t), I(t), m, \theta^j, \lambda_j)}{\partial x_1(t)}\right) \tag{10}$$

Then this is necessary to claim a maximum of likelihood function, herewith to charge a penalty for increase of the attributes' amount and, respectively, the amount of model parameters. Let us introduce the $k(m)$ function as dependence of the amount of utility function parameters on the attributes' amount. The penalty for increase in the number of parameters is implemented in many criteria for selecting optimal econometric model specification. The corrected Akaike informational criterion [Hurvich, Tsai, 1989] was used in this research. Also it is necessary to impose constraints on utility function parameters set, as well as on monotonicity of attributive utility functions, integer positive value of attribute amount and satisfaction of second-order rationality condition (8) at each time moment. Let

$$\Theta = \{(m, \theta^j, \lambda_j) | m \in Z_+, \frac{\partial u_j(x, \theta^j)}{\partial x} \geq 0, \ \frac{\partial^2 U(x_1, p, I, m, \theta^j, \lambda_j)}{\partial^2 x_1} \leq 0\}$$

be the feasible set of model parameters values.
The model parameters estimation problem now may be written down as finding such estimations which maximize corrected informational Akaike criterion in the feasible set of parameter values.

$$(\widehat{m}, \widehat{\theta^j}, \widehat{\lambda_j}) = arg \max_{(m, \theta^j, \lambda_j) \in \Theta} ln\mathcal{L} - k(m) - \frac{2k(m)(k(m)+1)}{n - m - 1} \tag{11}$$

4. THE APPLICATION FOR FIXING CONSUMPTION EFFECTS

As it was mentioned above, the model allows to predict the dependence of demand on the external conditions. In this paper the model will be tested on the ability to predict the Giffen effect in the demand for buckwheat on the retail market within summer 2010. Thus, within a short time period the price of this product rose from 45 rubles per kilogram to 140 rubles. As soon as we do not have the data of the individual consumer purchases, this data of total consumer expenditures and the purchased buckwheat amount was simulated as the i.i.d. In this case, the volume of purchases of the product was growing along with the price. This effect is called the Giffen effect. Standard approaches to estimation of the demand with the Giffen effect imply a demand function for the product with a positive first derivative at a price for the Giffen product, such as linear demand function with a positive coefficient preceding price variable. However, this approach does not explain the presence of normal properties of the product within the periods with small price fluctuations. Thus, with small fluctuations in prices of buckwheat and substitute products, there is negative demand reaction on the increase of the product's price and the decrease of substitute products' price. It means that buckwheat has properties of normal good.
In order to estimate the demand the purchases data of 8 consumers was simulated for the period of 111 days from 25/04/2010 to 13/08/2010. The data contains information of buckwheat purchases per each consumer, total expenditures on the commodities purchasing, the average weighted price of the products in the consumption bundle, excepting the buckwheat. The average daily price per kilogram of buckwheat on the retail market was collected from the groceries.
The following assumptions were made.
The consumer's product set consists of two products: the buckwheat and the rest of the consumption bundle.
The products have unknown number of attributes which are important for the consumer to make a choice. The utility of each attribute can be represented as a power utility function $u_{jt} =$

$(a_{1j}x_{1jt}^{b_{1j}} + a_{2j}x_{2jt}^{b_{2j}}(t))^{\alpha_j}$. This type of utility function implies the following: 1) increase in the consumption of each product leads to the increase of utility; 2) marginal utility is decreasing; 3) zero consumption of both goods leads to zero utility; 4) interdependence of the utility of both product, such as substitution or complementarity.

It is indispensable to test the hypothesis that the demand for the buckwheat varies with the changes in external conditions of consumption. Let the "external conditions" basically mean the change of the buckwheat price.

In order to test this hypothesis, it was necessary to compare the predictive ability of the utility maximization model with unknown number of attributes and fixed over the time values of attribute weights with the utility maximization model with an unknown number of attributes, where the importance weight of one attribute is a function of the external conditions.

The attribute utility weights of the first model was written as $\lambda_{jt} = c_j / \sum_{s=1}^{m} c_s$.

In the second model the first attribute utility weight was defined as

$$f(p_{1t}, \Delta p_{1t}, \theta) = \theta_0 + \theta_1 p_{1t} + \theta_2 \Delta p_{1t},$$
$$\lambda_{1t} = f(p_{1t}, \Delta p_{1t}, \theta)/(f(p_{1t}, \Delta p_{1t}, \theta) + \sum_{s=2}^{m} c_s).$$

And the rest of the attribute weights were defined as

$$\lambda_{jt} = c_j/(f(p_{1t}, \Delta p_{1t}, \theta) + \sum_{s=2}^{m} c_s), \ j = 2..m.$$

The weights c and f at each time moment must be non-negative.

For each model the problem of maximization of the corrected Akaike criterion (11) with a constraint on the parameter values set was solved. Thus, there were constraints: 1) on the values of the attribute utility weights; 2) on the monotonicity of the utility function of each attribute; 3) the second-order consumer rationality condition.

For each model the estimates of the number of attributes which precisely describes the consumer's decision-making process, as well as, the parameters of the attributes' utility functions and the attributes' importance weights were computed. Thus, the maximum predictive power has two attributes' model, where the first attribute is a function of the Giffen product price and its price change $\lambda_{1t} = f(p_{1t}, \Delta p_{1t}, \theta)/(f(p_{1t}, \Delta p_{1t}, \theta) + c_2)$. The second attribute weight was $\lambda_{2t} = c_2/(f(p_{1t}, \Delta p_{1t}, \theta) + c_2)$.

Parameters values were estimated as the following:

Table 1. Model parameters values.

Parameter	a_1	a_2	b_1	b_2	α_j	θ_0	θ_1	θ_2	c_2
Attribute 1	1.261*	0.721*	0.023*	0.012*	0.036*	-0.137*	0.011*	0.014*	-
	(0.248)	(0.112)	(0.000)	(0.001)	(0.011)	(0.06)	(0.003)	(0.004)	
Attribute 2	0.367	0.812*	0.021*	0.013*	0.372	-	-	-	0.056
	(0.290)	(0.138)	(0.000)	(0.001)	(0.257)				(0.097)

The standard errors are in parenthesis
* is significant at the 5% level

The estimations of the model with maximum predictive power allow to make the following conclusions.

It is sufficient to include the two attributes in the model of consumer choice description with Giffen behavior. The first attribute specifies the Giffen effect and the second one defines the normal characteristics of the products.

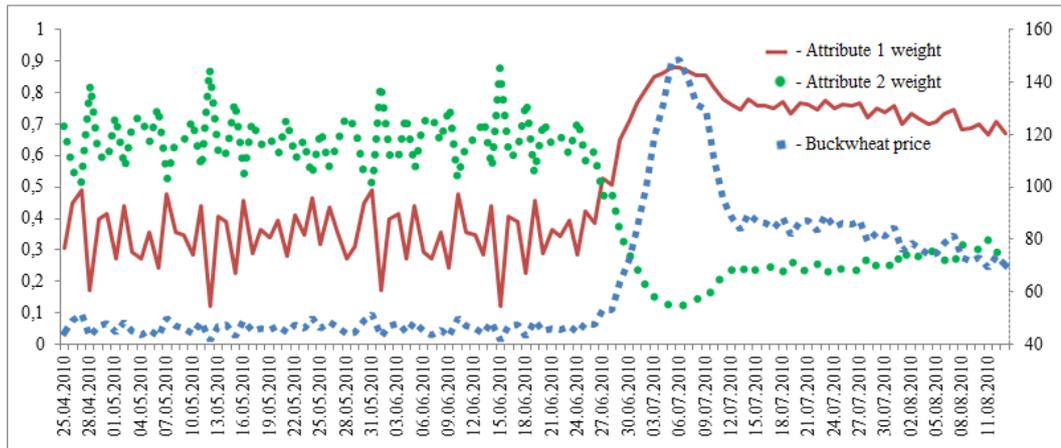As the buckwheat price rises, the importance of the Giffen effect attribute increases.

Figure 1. Weights of attributes.

Thus, the buckwheat relative utility increases, because the parameter values a_1 and b_1 of the first attribute utility function are higher. There is the buckwheat price interval $p_1 \epsilon [0; 112.3]$, where the first derivative of the buckwheat demand function at the price is positive. On this price interval there is the Giffen effect on the buckwheat demand. The further increase of the price provokes the decrease of demand.

With the average consumer's individual demand function, it is possible now to predict the aggregate demand for the product at the fixed price and the distribution of the consumer's expenditures and purchases.
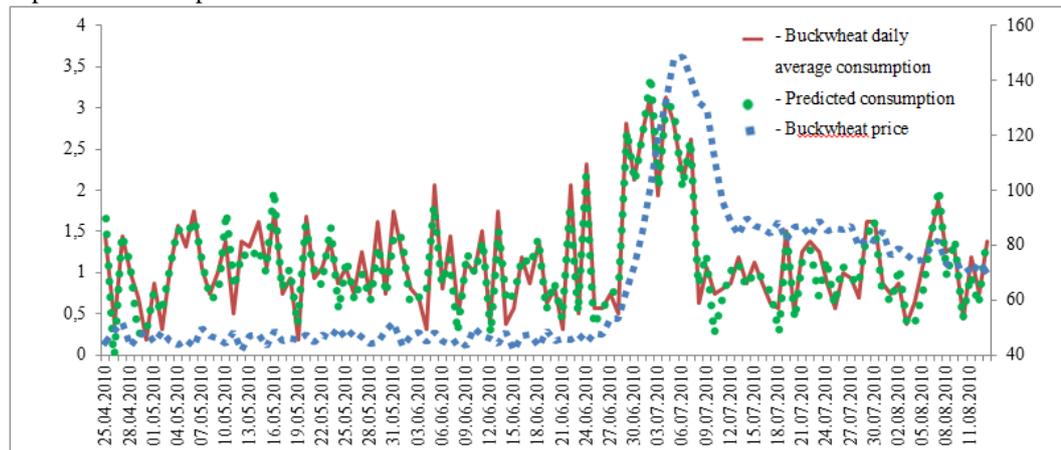

Figure 2. Model prediction.

5. CONCLUSIONS

The proposed model allows to estimate the consumer's demand for homogenous product and the demand dependency on different external factors for the product. The model allows to identify precise amount of the product attributes, when it is impossible to determine them.

The proposed algorithm not only allows to estimate the already specified demand function, but also to test the hypothesis of rationality of the consumer's decision-making and to recover the utility maximizing problem, when the value of utility function can not be measured and identified. The model and estimation method are quite useful for fixing the consumption effects when there is the dependency of demand on the external factors. To do that the researcher should specify the weights of some attributes as the functions for external factors and test the hypothesis whether it is necessary to include this attributes in the utility function or not.

With this model the Giffen behavior for buckwheat demand was fixed in the simulated consumer data. The structure of utility function is the following. There are the two main attributes, which are important for the consumer's choice. One of them generates normal properties for products, and the second one generates the Giffen effect. As the buckwheat price rises, the importance weight of

the Giffen attribute increases. This is the reason why the relative utility of the buckwheat is increasing and the Giffen effect is being observed on some price interval.

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