BAYES ESTIMATION AND PREDICTION OF A THREE COMPONENT MIXTURE OF RAYLEIGH DISTRIBUTION UNDER TYPE-I CENSORING

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ABSTRACT
This paper considers maximum likelihood estimation, Bayes estimation and prediction of a three component mixture of Rayleigh distribution under type-I censoring. By assuming independent priors for the unknown parameters of the Rayleigh mixture model, Bayes estimates are computed under the squared error loss, quadratic loss and general entropy loss functions. Mathematical expressions for the joint posterior, marginal posterior-distribution, Bayes estimators, posterior predictive distribution and Bayes point predictors are given in explicit forms. Detailed Monte Carlo simulations are used to study the performances of the maximum likelihood and Bayes estimators in terms of simulated risks. It is observed that the Bayes estimates under the considered loss functions are more precise than the maximum likelihood estimates. Moreover, the Bayes point predictive estimates of the future observation are also obtained under the squared error loss and general entropy loss functions.

KEYWORDS: Maximum likelihood estimation, Bayes estimators, General entropy loss function, Posterior predictive distribution, Monte Carlo Simulations, Predictive intervals.

MSC: 62F15, 65C60

RESUMEN
Este trabajo considera el uso de máxima verosimilitud, estimación Bayesiana y predicción para una distribución mixta de Raleigh de tres componentes bajo censura del tipo I. Asumiendo a priori independientes de los parámetros desconocidos del modelo mixto a priori de los parámetros desconocidos, estimadores Bayesianos son computados bajo la función de pérdida cuadrática y la función de pérdida de entropía generalizada. Expresiones matemáticas explicitas para la a posteriori conjunta. La marginal posterior, estimadores Bayesianos, distribuciones predictiva y predictores puntales de Bayes. Simulaciones de Monte Carlo detalladas son usadas para estudiar el comportamiento de los estimados de Bayes y de máxima verosimilitud en términos del riesgo simulado. Es observado que e los estimados de Bayesiano diferentes funciones de pérdida son precisas al compararse con los estimados de máxima verosimilitud. Estimados puntuales predictivos de Bayes de futuras observaciones son obtenidas bajo la función de pérdida cuadrática y la de entropía generalizada

1. INTRODUCTION

Lifetime distributions have gained special attention under censoring schemes because of their wide applications in engineering, social sciences, public health, and medicine. In most of the life testing and reliability experiments, the experimenter is unable to obtain the complete information or failure times of all of the experimental units. In many situations, the removal of units prior to failure is preplanned, i.e., units are lost from the test before failure. This loss may be due to time saving, the cost associated with testing or it may be designed in the study. In such type of experiments, the obtained data is so called the censored data (see Balakrishnan et al. (2003)). The two most commonly used censoring schemes are type-I and type-II censoring. Gupta (1952) discriminated between the two types of censored samples, i.e., in type-II censored samples; the experiment is terminated after observing some fixed percentage of observations whereas in type-I censored samples, the censoring takes place at certain fixed points. The prime distinctions between type-I and type-II censoring is that in the former case the number of observations is a random variable, while in the latter case it is fixed in advance. One of the major

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drawbacks with these censoring schemes is that they cannot allow for removal of units at points other than the terminal points of experiment (see Soliman (2005)).

Finite mixtures of lifetime distributions have also got consideration in terms of their practical applications and methodological developments. Sometimes it becomes essential to represent a heterogeneous population by a finite mixture of k components lifetime model. Mendenhall and Hader (1958) divided a failure population into two subpopulations, each representing a different cause or type of failure. Maximum likelihood estimation method was used to estimate the unknown parameters of exponentially distributed failure time distributions by considering censored lifetime data. Chen et al. (1985) suggested a two component mixture model for analysis of cancer survival data by extending the idea of Berkson and Gage (1952). Gordon (1990a, 1990b) assumed that the survival function of treated cancer patients can be modeled by a mixture of two subpopulations. Conditioning on the age of patients at the surgery or initial treatment, Gompertz distribution is used to model the survival time distribution of both subpopulations. Masuyama (1977) applied a mixture of two gamma distributions to the rheumatoid arthritis. Radhakrishna et al. (1992) derived both moment and maximum likelihood estimators of the unknown parameters of two component mixture of the generalized gamma distribution. Ahmed et al. (1997) obtained approximate Bayes estimators for parameters of mixture of two Weibull distributions under type-II censoring. Al-Hussani et al. (2000) applied both maximum likelihood and Bayes estimation methods on a two component mixture of the Gompertz distribution based on type-I and type-II censoring. Al-Hussani et al. (2001) considered the problem of obtaining Bayesian prediction bounds for the future observation from a finite mixture of two Lomax components under type-I censoring. Jaheen (2003) obtained prediction bounds for sth future observation under a mixture of two component Gompertz lifetime model. Jaheen (2005) discussed the problem of estimating the parameters of finite mixture of two exponential distributions based on record statistics. Monte Carlo simulations were used to compare the maximum likelihood and Bayes estimates. Marin et al. (2005) applied Bayesian methods to fit a mixture of Weibull distributions with unknown number of components to heterogeneous rightly censored survival data. Shawky and Bakoban (2009) considered estimation of parameters, reliability, and failure rate functions of finite mixture of two components from the exponentiated gamma distribution by using maximum likelihood and Bayes methods of estimation. Abu-Zindah (2010) used the maximum likelihood and Bayes estimation methods to estimate the parameters, reliability, and hazard functions of a mixture of exponentiated Pareto and exponential distribution under complete and type-II censoring schemes. Erişoğlu et al. (2011) proposed a mixture of two different distributions to model heterogeneous survival data. Ahmed et al. (2011) suggested a mixture of Burr type XII distribution. The Bayes method of estimation is used to the estimate the parameters of the finite mixture of Burr type XII distribution and its reciprocal under type-I censoring.

On similar lines, in this paper, we extend the finite mixture of two components to a finite mixture of three components, which is also useful to model heterogeneous populations. It is here assumed that the lifetime of components follow Rayleigh distribution. Both maximum likelihood and Bayes methods of estimation are used to estimate the unknown parameters of the three component Rayleigh mixture model under type-I censoring. Bayes estimation of unknown parameters is considered under both symmetric and asymmetric loss functions. Mathematical expressions of joint posterior distribution, marginal posterior distribution, and Bayes estimators under each of the loss function are obtained in explicit forms. A detailed Monte Carlo simulation study is conducted to investigate the performances of the maximum likelihood and Bayes estimators. The values of estimated risk functions of both maximum likelihood and Bayes estimators are obtained under each of the considered loss functions. For predicting the future observation, the posterior predictive distribution is derived. The Bayes point predictive estimators are also obtained in explicit forms by considering both symmetric and asymmetric loss functions. Monte Carlo simulations have been used to examine the performance of the Bayes point predictors, for interval estimation of the future observation, and the prediction intervals under each case are also calculated.

The rest of the paper is organized as follows. In Sections 2 and 3, the maximum likelihood and Bayes estimation methods are applied on mixture model, respectively, to estimate the unknown parameters of the three component Rayleigh mixture model. In Sections 4, the posterior predictive distribution is derived. Section 5 provides numerical comparisons of considered estimators using a Monte Carlo study. In Section 6, some concluding remarks are given.

2. MAXIMUM LIKELIHOOD ESTIMATION OF THREE COMPONENT MIXTURE MODEL

The probability density function (pdf) of the Rayleigh distribution is

$$h(x;\theta)=\frac{x}{\theta^2}\exp\left(-\frac{x^2}{2\theta^2}\right), \qquad x>0;\ \theta>0,$$

(1)

where θ is the unknown scale parameter. The cumulative distribution function (cdf) of Rayleigh distribution is $H(x;\theta)=1-\exp\left(-\frac{x^2}{2\theta^2}\right)$. Based on a random sample of size n i.e. $x_1, x_2, \ldots, x_n$, the maximum likelihood estimate of θ is

$$\hat{\theta}_{ML}=\sqrt{\frac{1}{2n}\sum_{i=1}^{n}x_i^2}\ .$$

A heterogeneous population can be described by a finite mixture of k components i.e.

$$f(x)=\sum_{j=1}^{k}p_j h_j(x), \tag{2}$$

where $h_j(x)$ is the jth pdf component and p_j is the mixing proportion, which satisfies $0<p_j<1$ and $\sum_{j=1}^{k}p_j=1$. Similarly, the corresponding cdf is given by

$$F(x)=\sum_{j=1}^{k}p_j H_j(x), \tag{3}$$

where $H_j(x)$ is the jth cdf component.

A mixture of three component densities, i.e., $h_1(x)$, $h_2(x)$ and $h_3(x)$, with mixing weights p_1, p_2 and p_3, is given by

$$f(x)=p_1h_1(x)+p_2h_2(x)+p_3h_3(x), \qquad \sum_{j=1}^{3}p_j=1.$$

(4)

Using (1), we have

$$f(x)=p_1\frac{x}{\theta_1^2}\exp\left(-\frac{x^2}{2\theta_1^2}\right)+p_2\frac{x}{\theta_2^2}\exp\left(-\frac{x^2}{2\theta_2^2}\right)+p_3\frac{x}{\theta_3^2}\exp\left(-\frac{x^2}{2\theta_3^2}\right),\ \theta_j>0 \text{ for } j=1,2,3. \tag{5}$$

Similarly, the corresponding cdf of three component mixture Rayleigh model is

$$F(x)=p_1H_1(x)+p_2H_2(x)+p_3H_3(x), \tag{6}$$

where $H_j(x)=1-\exp\left(-\frac{x^2}{2\theta_j^2}\right)$ for $j=1,2,3$.

Considering n elements from mixture model (4) are used in a life testing experiment with a test termination time T. From the test, it is evaluated that out of n elements, r elements were having the lifetime in the interval $[0,T]$, out of these r elements, r_1, r_2 and r_3 elements are observed from the first, second and third subpopulations respectively, satisfying that $r=\sum_{j=1}^{3}r_j$, and the remaining $n-r$ units are still working after the termination time T i.e. $[T,\infty]$. Let x_{ji} be the failure time of the ith unit belonging to jth subpopulation for $i=1,2,\ldots,r_j$ and $j=1,2,3$. The likelihood function under such conditions may be written in the following form (see Mendenhall and Hader (1958)).

$$L(\theta_1,\theta_2,\theta_3,p_1,p_2;\mathbf{x})\propto\prod_{i=1}^{r_1}\{p_1h_1(x_{1i})\}\prod_{i=1}^{r_2}\{p_2h_2(x_{2i})\}\prod_{i=1}^{r_3}\{p_3h_3(x_{3i})\}\{1-F(t)\}^{n-r}, \tag{7}$$

here, $\mathbf{x}=\left(x_{11},...,x_{1r_1},x_{21},...,x_{2r_2},x_{31},...,x_{3r_3}\right)$ are the non-censored observations of time failures. After some simplification, we have

$$L\left(\theta_1,\theta_2,\theta_3,p_1,p_2;\mathbf{x}\right)\propto\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{\left(n-r\right)_{k_1}\left(k_1\right)_{k_2}}{k_1!k_2!}\,p_1^{n-r_2-r_3-k_1}\,p_2^{r_2+k_1-k_2}\,p_3^{r_3+k_2}\prod_{j=1}^{3}\theta_j^{-2r_j}\exp\left(-\frac{A_{jk}}{2\theta_j^2}\right)\right\}, \tag{8}$$

where

$$\left(n-r\right)_{k_1}=\frac{\left(n-r\right)!}{\left(n-r-k_1\right)!},\ \left(k_1\right)_{k_2}=\frac{k_1!}{\left(k_1-k_2\right)!},\ A_{1k}=\sum_{i=1}^{r_1}x_{1i}^2+t^2\left(n-r-k_1\right),\ A_{2k}=\sum_{i=1}^{r_2}x_{2i}^2+t^2\left(k_1-k_2\right),\text{ and }A_{3k}=\sum_{i=1}^{r_3}x_{3i}^2+t^2k_2.$$

Now, taking log on both sides of (8), we have

$$\ell=\log\left\{L\left(\underline{\theta};\mathbf{x}\right)\right\}=\log\left(C\right)+r_1\log\left(p_1\right)+r_2\log\left(p_2\right)+r_3\log\left(1-p_1-p_2\right)-\sum_{j=1}^{3}\left\{\sum_{i=1}^{r_j}\frac{x_{ji}^2}{2\theta_j^2}+2r_j\log\left(\theta_j\right)\right\}$$
$$+\left(n-r\right)\log\left\{1-F(t)\right\}, \tag{9}$$

where C is a normalizing constant and $\underline{\theta}=(\theta_1,\theta_2,\theta_3,p_1,p_2)$.

Differentiating (9) with respect to θ_1, θ_2, θ_3, p_1 and p_2, we have

$$\frac{\partial\ell}{\partial\theta_j}=\frac{-2r_j}{\theta_j}+\frac{\sum_{i=1}^{r_j}x_{ji}^2}{\theta_j^3}-\frac{p_jt^2\left(n-r\right)\exp\left(-\frac{t^2}{2\theta_j^2}\right)}{\theta_j^3\left\{\sum_{i=1}^{3}p_i\exp\left(-\frac{t^2}{2\theta_i^2}\right)\right\}},\text{ for } j=1,2,3 \tag{10}$$

and

$$\frac{\partial\ell}{\partial p_h}=\frac{r_h}{p_h}-\frac{r_3}{p_3}+\frac{\left(n-r\right)\left\{\exp\left(-\frac{t^2}{2\theta_h^2}\right)-\exp\left(-\frac{t^2}{2\theta_3^2}\right)\right\}}{\left\{\sum_{i=1}^{3}p_i\exp\left(-\frac{t^2}{2\theta_i^2}\right)\right\}},\text{ for } h=1,2. \tag{11}$$

As it is difficult to obtain the exact solution of above equations, therefore, numerical techniques can easily be applied to get the solution from above equations. The obtained estimates $\hat{\underline{\theta}}_{ML}=\left(\hat{\theta}_{1ML},\hat{\theta}_{2ML},\hat{\theta}_{3ML},\hat{p}_{1ML},\hat{p}_{2ML}\right)$ will be the corresponding maximum likelihood estimates of θ_1, θ_2, θ_3, p_1 and p_2, respectively. In order to get the variances of above maximum likelihood estimators, we have asymptotically $\hat{\underline{\theta}}_{ML}\overset{asymp.}{\sim}N\left(\underline{\theta},I^{-1}\left(\underline{\theta}\right)\right)$, $I\left(\underline{\theta}\right)$ is the Fisher information matrix of order 5×5 i.e.,

$$I\left(\underline{\theta}\right)=\begin{bmatrix}
-E\left(\frac{\partial^2l}{\partial\theta_1^2}\right) & -E\left(\frac{\partial^2l}{\partial\theta_1\partial\theta_2}\right) & -E\left(\frac{\partial^2l}{\partial\theta_1\partial\theta_3}\right) & -E\left(\frac{\partial^2l}{\partial\theta_1\partial p_1}\right) & -E\left(\frac{\partial^2l}{\partial\theta_1\partial p_2}\right)\\
-E\left(\frac{\partial^2l}{\partial\theta_2\partial\theta_1}\right) & -E\left(\frac{\partial^2l}{\partial\theta_2^2}\right) & -E\left(\frac{\partial^2l}{\partial\theta_2\partial\theta_3}\right) & -E\left(\frac{\partial^2l}{\partial\theta_2\partial p_1}\right) & -E\left(\frac{\partial^2l}{\partial\theta_2\partial p_2}\right)\\
-E\left(\frac{\partial^2l}{\partial\theta_3\partial\theta_1}\right) & -E\left(\frac{\partial^2l}{\partial\theta_3\partial\theta_2}\right) & -E\left(\frac{\partial^2l}{\partial\theta_3^2}\right) & -E\left(\frac{\partial^2l}{\partial\theta_3\partial p_1}\right) & -E\left(\frac{\partial^2l}{\partial\theta_3\partial p_1}\right)\\
-E\left(\frac{\partial^2l}{\partial p_1\partial\theta_1}\right) & -E\left(\frac{\partial^2l}{\partial p_1\partial\theta_2}\right) & -E\left(\frac{\partial^2l}{\partial p_1\partial\theta_3}\right) & -E\left(\frac{\partial^2l}{\partial p_1^2}\right) & -E\left(\frac{\partial^2l}{\partial p_1\partial p_2}\right)\\
-E\left(\frac{\partial^2l}{\partial p_2\partial\theta_1}\right) & -E\left(\frac{\partial^2l}{\partial p_2\partial\theta_2}\right) & -E\left(\frac{\partial^2l}{\partial p_2\partial\theta_3}\right) & -E\left(\frac{\partial^2l}{\partial p_2\partial p_1}\right) & -E\left(\frac{\partial^2l}{\partial p_2^2}\right)
\end{bmatrix}_{5\times5}.$$

The inverse of the above symmetric matrix will yield the variances of the corresponding maximum likelihood estimators in the diagonal. The elements of above matrix can be obtained by differentiating (10) and (11) with respect to corresponding parameters and then by taking the expectation to get the results.

3. BAYES ESTIMATION OF THREE COMPONENT MIXTURE MODEL

In Bayesian estimation, we need to specify the prior distributions for the unknown parameters of mixture model (5) to quantify the available prior information or in case of lack of prior information; it is conventional to take non-informative priors. Here, three independent priors i.e., square inverted gamma distributions are assumed for θ_1 , θ_2 and θ_3 , and Dirichlet distribution is assumed as a joint prior distribution for unknown mixing weights p_1 and p_2 . The pdf of square root inverted gamma distribution is

$$g_j\left(\theta_j;a_j,b_j\right)=\frac{2b_j^{a_j}}{\Gamma\left(a_j\right)}\theta_j^{-\left(2a_j+1\right)}\exp\left(-\frac{b_j}{\theta_j^2}\right),\ \theta_j>0,\ a_j>0,\ b_j>0,\ \text{for } j=1,2,3. \quad (12)$$

In density kernel notation, (12) can be written as

$$g_j\left(\theta_j;a_j,b_j\right)\propto\theta_j^{-\left(2a_j+1\right)}\exp\left(-\frac{b_j}{\theta_j^2}\right),\ \text{for } j=1,2,3. \quad (13)$$

The pdf of Dirichlet distribution is

$$g_4\left(p_1,p_2;a_4,a_5,a_6\right)=\frac{\Gamma(a_4)\Gamma(a_5)\Gamma(a_6)}{\Gamma\left(a_4+a_5+a_6\right)}p_1^{a_4-1}p_2^{a_5-1}p_3^{a_6-1},\qquad \sum_{j=1}^{3}p_j=1,\qquad a_{3+j}>0,\qquad \text{for}\qquad j=1,2,3. \quad (14)$$

The density kernel of (14) is

$$g_4\left(p_1,p_2\right)\propto p_1^{a_4-1}p_2^{a_5-1}p_3^{a_6-1}. \quad (15)$$

Now, the joint prior of $\underline{\theta}=(\theta_1,\theta_2,\theta_3,p_1,p_2)$ can be written as

$$g\left(\underline{\theta}\right)=\left\{\prod_{j=1}^{3}g_j\left(\theta_j;a_j,b_j\right)\right\}g_4\left(p_1,p_2;a_4,a_5,a_6\right), \quad (16)$$

or

$$g\left(\underline{\theta}\right)\propto\prod_{j=1}^{3}p_j^{a_{3+j}-1}\theta_j^{-\left(2a_j+1\right)}\exp\left(-\frac{b_j}{\theta_j^2}\right). \quad (17)$$

By combining the likelihood function given in (8) and the joint prior (17), density kernel of the joint posterior distribution of $\underline{\theta}$ given data is given by

$$g\left(\underline{\theta}\,|\,\mathbf{x}\right)\propto\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}\left(k_1\right)_{k_2}}{k_1!k_2!}\prod_{j=1}^{3}p_j^{\alpha_j-1}\theta_j^{-\left\{2\left(a_j+r_j\right)+1\right\}}\exp\left(-\frac{A_{jk}+2b_j}{2\theta_j^2}\right)\right\}, \quad (18)$$

where $\alpha_1=n+a_4-r_2-r_3-k_1$, $\alpha_2=a_5+r_2+k_1-k_2$ and $\alpha_3=a_6+r_3+k_2$.

The complete joint posterior distribution of $\underline{\theta}$ given data is

$$g\left(\underline{\theta}\,|\,\mathbf{x}\right)=\frac{\displaystyle\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left[\frac{(n-r)_{k_1}\left(k_1\right)_{k_2}}{k_1!k_2!}\prod_{j=1}^{3}\left\{p_j^{\alpha_j-1}\theta_j^{-\left\{2\left(a_j+r_j\right)+1\right\}}\exp\left(\frac{A_{jk}+2b_j}{2\theta_j^2}\right)\right\}\right]}{\displaystyle\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}\left(k_1\right)_{k_2}}{k_1!k_2!\Gamma\left(\alpha_1+\alpha_2+\alpha_3\right)}\prod_{j=1}^{3}\frac{\Gamma\left(\alpha_j\right)\Gamma\left(a_j+r_j\right)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}, \quad (19)$$

where Γ(.) is the gamma function. In order to get the Bayes estimator under the suitable loss functions, marginal posterior distributions of θ_1, θ_2, θ_3, p_1 and p_2 given data, obtained from (19), are given below.

$$g_i\left(\theta_i \mid \mathbf{x}\right)=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\,\Gamma(\alpha_i)}{k_1!k_2!\theta_i^{2(a_i+r_i)+1}}\exp\left(-\frac{A_{ik}+2b_i}{2\theta_i^2}\right)\prod_{\substack{j=1\\ j\neq i}}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}},\ \text{for } i=1,2,3, \tag{20}$$

$$g_4\left(p_1 \mid \mathbf{x}\right)=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\,p_1^{\alpha_1-1}(1-p_1)^{\alpha_2+\alpha_3-1}\Gamma(\alpha_2)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}, \tag{21}$$

and

$$g_5\left(p_2 \mid \mathbf{x}\right)=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\,p_2^{\alpha_2-1}(1-p_2)^{\alpha_1+\alpha_3-1}\Gamma(\alpha_1)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_1+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}. \tag{22}$$

3.1. Bayes estimators under squared error loss function

The most commonly used symmetric loss function is squared error loss function (SELF) and most of the studies use SELF to measure the performance of estimators. The SELF is given by

$$l_S\left(\hat{\lambda},\lambda\right)=\left(\hat{\lambda}-\lambda\right)^2, \tag{23}$$

where $\hat{\lambda}$ is the estimator of parameter λ. The Bayes estimator under the above loss function is the mean of posterior distribution $\hat{\lambda}_B=E_p\left(\lambda\right)$, where $E_p(\cdot)$ is the expectation with respect to posterior distribution of λ. Bayes estimators of θ_1, θ_2, θ_3, p_1 and p_2, obtained from their respective posterior distributions, under SELF are given below.

$$\hat{\theta}_{iS} \mid \mathbf{x}=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\,\Gamma(\alpha_i)\Gamma(a_i+r_i-0.5)2^{a_i+r_i-1.5}}{k_1!k_2!(A_{ik}+2b_i)^{a_i+r_i-0.5}}\prod_{\substack{j=1\\ j\neq i}}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}},\ \text{for } i=1,2,3, \tag{24}$$

$$\hat{p}_{1S} \mid \mathbf{x}=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\,\Gamma(\alpha_1+1)\Gamma(\alpha_2)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3+1)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}, \tag{25}$$

and

$$\hat{p}_{2S}\mid\mathbf{x}=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\Gamma(\alpha_1)\Gamma(\alpha_2+1)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3+1)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}. \tag{26}$$

3.2. Bayes estimators under quadratic loss function

The second symmetric loss function considered here is quadratic loss function (QLF). The QLF is given by

$$l_Q\left(\hat{\lambda},\lambda\right)=\lambda^{-2}\left(\hat{\lambda}-\lambda\right)^2, \tag{27}$$

where $\hat{\lambda}$ is the estimator of parameter λ. The corresponding Bayes estimator under QLF is $\hat{\lambda}_B=E_p\left(\lambda^{-1}\right)/E_p\left(\lambda^{-2}\right)$. Now, the Bayes estimators of θ_1, θ_2, θ_3, p_1 and p_2, obtained from their respective posterior distributions, under QLF are given below.

$$\hat{\theta}_{iQ}\mid\mathbf{x}=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\Gamma(\alpha_i)\Gamma(a_i+r_i+0.5)2^{a_i+r_i-0.5}}{k_1!k_2!(A_{ik}+2b_i)^{a_i+r_i-0.5}}\prod_{\substack{j=1\\j\neq i}}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\Gamma(\alpha_i)\Gamma(a_i+r_i+1)2^{a_i+r_i}}{k_1!k_2!(A_{ik}+2b_i)^{a_i+r_i-0.5}}\prod_{\substack{j=1\\j\neq i}}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}, \text{ for } i=1,2,3,$$

(28)

$$\hat{p}_{1Q}\mid\mathbf{x}=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\Gamma(\alpha_1-1)\Gamma(\alpha_2)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3-1)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\Gamma(\alpha_1-2)\Gamma(\alpha_2)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3-2)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}, \tag{29}$$

and

$$\hat{p}_{2Q}\mid\mathbf{x}=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\Gamma(\alpha_1)\Gamma(\alpha_2-1)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3-1)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}{\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}\Gamma(\alpha_1)\Gamma(\alpha_2-2)\Gamma(\alpha_3)}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3-2)}\prod_{j=1}^{3}\frac{\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}}\right\}}. \tag{30}$$

3.3. Bayes estimators under general entropy loss function

Due to symmetry, SELF and QLF give equal weight to under estimation as well as over estimation. It is quite well known that the use of symmetric loss functions may be inappropriate in many circumstances, particularly when positive and negative errors have different consequences. Overestimation consequences in loss of human life are much more serious than the consequences of underestimation; in such case an asymmetric loss function is very much appropriate. In overcome this difficulty associated with SELF, a lot of asymmetric loss functions are proposed for use but one of most popular asymmetric loss function is linear-exponential loss (LINEX) proposed by Varian (1975) and Zellner (1986). Regardless of popularity and flexibility of LINEX loss function for location parameter estimation, it appears to be unsuitable for scale parameter and other quantities (see Basu and Ebrahimi (1991), Singh et al. (2008)). Basu and Ebrahimi (1991) proposed a modified LINEX loss function and a suitable alternative to modified loss function is the general entropy loss function (GELF) proposed by Calabria and Pulcini (1994), given by

$$l_G\left(\hat{\lambda},\lambda\right) \propto \left(\frac{\hat{\lambda}}{\lambda}\right)^c - c\ln\left(\frac{\hat{\lambda}}{\lambda}\right) - 1, \qquad c \neq 0,$$

where positive (negative) value of c is used when overestimation is more (less) serious than under estimation. For $c=-1$, Bayes estimator under GELF coincides with the Bayes estimator under SELF. The Bayes estimator under GELF is $\hat{\lambda}_G = \left\{E_p\left(\lambda^{-c}\right)\right\}^{-\frac{1}{c}}$. Now, the Bayes estimators of θ_1, θ_2, θ_3, p_1 and p_2, obtained from their respective posterior distributions, under GELF are given below.

$$\hat{\theta}_{iG} \mid \mathbf{x} = \left(\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{ \frac{(n-r)_{k_1}(k_1)_{k_2}\,\Gamma(\alpha_i)\Gamma(a_i+r_i+0.5c)\,2^{a_i+r_i+0.5c-1}}{k_1!k_2!\left(A_{ik}+2b_i\right)^{a_i+r_i-0.5}} \prod_{\substack{j=1\\ j\neq i}}^{3} \frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)\,2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}} \right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{ \frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!} \prod_{j=1}^{3} \frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)\,2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}} \right\}} \right)^{-\frac{1}{c}}, \quad \text{for } i=1,2,3, \tag{31}$$

$$\hat{p}_{1G} \mid \mathbf{x} = \left(\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{ \frac{(n-r)_{k_1}(k_1)_{k_2}\,\Gamma(\alpha_1-c)\Gamma(\alpha_2)\Gamma(\alpha_3)}{k_1!k_2!\,\Gamma(\alpha_1+\alpha_2+\alpha_3-c)} \prod_{j=1}^{3} \frac{\Gamma(a_j+r_j)\,2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}} \right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{ \frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\,\Gamma(\alpha_1+\alpha_2+\alpha_3)} \prod_{j=1}^{3} \frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)\,2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}} \right\}} \right)^{-\frac{1}{c}}, \tag{32}$$

and

$$\hat{p}_{2G} \mid \mathbf{x} = \left(\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{ \frac{(n-r)_{k_1}(k_1)_{k_2}\,\Gamma(\alpha_1)\Gamma(\alpha_2-c)\Gamma(\alpha_3)}{k_1!k_2!\,\Gamma(\alpha_1+\alpha_2+\alpha_3-c)} \prod_{j=1}^{3} \frac{\Gamma(a_j+r_j)\,2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}} \right\}}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{ \frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\,\Gamma(\alpha_1+\alpha_2+\alpha_3)} \prod_{j=1}^{3} \frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)\,2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}} \right\}} \right)^{-\frac{1}{c}}. \tag{33}$$

4. DERIVATION OF POSTERIOR PREDICTIVE DISTRIBUTION

In many of statistical problems, the results based on previous data are useful in predicting the future data coming from the same population. One possibility is to construct an interval based on predictive distribution with some specified probability or it is possible to get point estimate by assuming any suitable loss function. In Bayesian context, posterior predictive distribution is mostly used for prediction of future observations. The posterior predictive distribution of Y given $\mathbf{x}$ is defined as

$$h(y \mid \mathbf{x}) = \int_0^1\int_0^{1-p_2}\int_0^\infty\int_0^\infty\int_0^\infty g(\underline{\theta} \mid \mathbf{x})\, f(y \mid \underline{\theta})\; \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}\theta_3 \mathrm{d}p_1 \mathrm{d}p_2, \qquad y>0, \tag{34}$$

where $f(y \mid \underline{\theta}) = p_1 \frac{y}{\theta_1^2}\exp\left(-\frac{y^2}{2\theta_1^2}\right) + p_2 \frac{y}{\theta_2^2}\exp\left(-\frac{y^2}{2\theta_2^2}\right) + p_3 \frac{y}{\theta_3^2}\exp\left(-\frac{y^2}{2\theta_3^2}\right)$.

By replacing the values of $g(\underline{\theta} \mid \mathbf{x})$ and $f(y \mid \underline{\theta})$ in (27), and after some simplification, we have

$$h(y\mid\mathbf{x})=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left[\frac{(n-r)_{k_1}(k_1)_{k_2}\,y}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3+1)}\sum_{i=1}^{3}\left\{\frac{\Gamma(\alpha_i+1)\Gamma(a_i+r_i+1)}{\left(A_{ik}+2b_i+y^2\right)^{a_i+r_i+1}2^{-a_i-r_i}}\prod_{\substack{j=1\\ j\neq i}}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)}{\left(A_{jk}+2b_j\right)^{a_j+r_j}2^{-a_j-r_j+1}}\right\}\right]}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}},\ \text{for } y>0. \qquad (35)$$

4.1. Bayes point predictor under squared error loss function

By assuming SELF, the mean of the posterior predictive distribution will be the Bayes point predictor of $Y\mid\mathbf{x}$. Therefore, the mean of posterior predictive distribution is

$$\hat{Y}_S\mid\mathbf{x}=E_{pp}(Y\mid\mathbf{x})=\int_0^\infty y h(y\mid\mathbf{x})\,\mathrm{d}y. \qquad (36)$$

$$\hat{Y}_S\mid\mathbf{x}=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left[\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3+1)}\sum_{i=1}^{3}\left\{\frac{\sqrt{\pi}\,\Gamma(\alpha_i+1)\Gamma(a_i+r_i-0.5)}{\left(A_{ik}+2b_i\right)^{a_i+r_i-0.5}2^{-a_i-r_i+2}}\prod_{\substack{j=1\\ j\neq i}}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)}{\left(A_{jk}+2b_j\right)^{a_j+r_j}2^{-a_j-r_j+1}}\right\}\right]}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}. \qquad (37)$$

Similarly, we can find

$$E_{pp}(Y^2\mid\mathbf{x})=\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left[\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3+1)}\sum_{i=1}^{3}\left\{\frac{\Gamma(\alpha_i+1)\Gamma(a_i+r_i-1)}{\left(A_{ik}+2b_i\right)^{a_i+r_i-1}2^{-a_i-r_i+1}}\prod_{\substack{j=1\\ j\neq i}}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)}{\left(A_{jk}+2b_j\right)^{a_j+r_j}2^{-a_j-r_j+1}}\right\}\right]}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1}\left\{\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)}\prod_{j=1}^{3}\frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{\left(A_{jk}+2b_j\right)^{a_j+r_j}}\right\}}.$$

(38)

The variance of posterior predictive distribution is

$$V_{pp}(Y\mid\mathbf{x})=E_{pp}(Y^2\mid\mathbf{x})-\left\{E_{pp}(Y\mid\mathbf{x})\right\}^2, \qquad (39)$$

where $E_{pp}(\cdot)$ and $V_{pp}(\cdot)$ represents expectation and variance with respect to posterior predictive distribution.

4.2. Bayes point predictor under general entropy loss function

By assuming GELF, the Bayes point predictor of $Y\mid\mathbf{x}$ based on the mean of posterior predictive distribution is

$$\hat{Y}_G\mid\mathbf{x}=\left\{E\left(Y^{-c}\mid\mathbf{x}\right)\right\}^{-\frac{1}{c}}=\left(\int_0^\infty y^{-c}h(y\mid\mathbf{x})\,\mathrm{d}y\right)^{-\frac{1}{c}}. \qquad (40)$$

$$\hat{Y}_G \mid \mathbf{x} = \left(\frac{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1} \left[\frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3+1)} \sum_{i=1}^{3} \left\{ \frac{\Gamma(\alpha_i+1)\Gamma(a_i+r_i+0.5c)}{(A_{ik}+2b_i)^{a_i+r_i+0.5c}\, 2^{-a_i-r_i+1}} \prod_{\substack{j=1\\ j\neq i}}^{3} \frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)}{(A_{jk}+2b_j)^{a_j+r_j}\, 2^{-a_j-r_j+1}} \right\} \right]}{\sum_{k_1=0}^{n-r}\sum_{k_2=0}^{k_1} \left\{ \frac{(n-r)_{k_1}(k_1)_{k_2}}{k_1!k_2!\Gamma(\alpha_1+\alpha_2+\alpha_3)} \prod_{j=1}^{3} \frac{\Gamma(\alpha_j)\Gamma(a_j+r_j)2^{a_j+r_j-1}}{(A_{jk}+2b_j)^{a_j+r_j}} \right\}} \right)^{-\frac{1}{c}} . \qquad (41)$$

A $100\gamma\%$ prediction interval for $Y \mid \mathbf{x}$ is given by

$$p(LPL < Y \mid \mathbf{x} < UPL) = \gamma , \qquad (42)$$

where lower prediction limit (*LPL*) and upper prediction limit (*UPL*) are obtained by solving the following two equations, respectively.

$$p(Y \mid \mathbf{x} > LPL) = \int_{LPL}^{\infty} h(y \mid \mathbf{x})\mathrm{d}y = \frac{1+\gamma}{2} \text{ and } p(Y \mid \mathbf{x} > UPL) = \int_{UPL}^{\infty} h(y \mid \mathbf{x})\mathrm{d}y = \frac{1-\gamma}{2} . \qquad (43)$$

5. SIMULATION STUDY

As it is difficult to compare maximum likelihood and Bayes estimators theoretically, therefore, we use Monte Carlo simulations to compare both methods of estimation using different parameters values and for different censoring times T. As in mixture model (5), we have five unknown parameters, i.e., θ_1, θ_2, θ_3, p_1 and p_2. For θ_1, θ_2, and θ_3, we have assumed three independent square root inverted gamma priors with hyper parameters (a_1,b_1), (a_2,b_2) and (a_3,b_3) respectively, and Dirichlet prior having hyper parameters a_4, a_5 and a_6 is assumed as a joint prior for p_1 and p_2.

TABLE 1: Maximum likelihood, Bayes estimates and estimated risks with $a_j = 10$ and $b_j = 30$ for $j = 1,2,3$, $n = 30$, $c = 1.20$, and $p_1 = p_2 = 0.4$.

	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$
			$T=4$					$T=5$					$T=6$		
$\underline{\hat{\theta}}_{ML}$	1.76105	1.75856	1.84574	0.39633	0.39672	1.77186	1.76366	1.79890	0.39910	0.39868	1.77835	1.77151	1.78549	0.39971	0.39955
$r_S(\underline{\hat{\theta}}_{ML})$	0.09975	0.09901	0.22687	0.00145	0.00142	0.07515	0.07702	0.17692	0.00027	0.00027	0.07074	0.06812	0.14244	5.5E-05	5.2E-05
$r_Q(\underline{\hat{\theta}}_{ML})$	0.03077	0.03054	0.06998	0.00908	0.00889	0.02318	0.02376	0.05457	0.00171	0.00166	0.02182	0.02101	0.04393	0.00034	0.00033
$r_G(\underline{\hat{\theta}}_{ML})$	0.02295	0.02279	0.04541	0.00675	0.00653	0.01723	0.01774	0.03408	0.00125	0.00123	0.01619	0.01573	0.03263	0.00025	0.00024
$\underline{\hat{\theta}}_{S}$	1.79545	1.79443	1.80001	0.39928	0.39974	1.79326	1.79039	1.79862	0.39983	0.39971	1.79517	1.79109	1.79863	0.39997	0.39981
$r_S(\underline{\hat{\theta}}_{S})$	0.01893	0.01847	0.01341	0.00086	0.00088	0.0198	0.02051	0.01758	0.00019	0.00019	0.02165	0.02056	0.01965	3.7E-05	3.6E-05
$r_Q(\underline{\hat{\theta}}_{S})$	0.00584	0.00570	0.00414	0.00538	0.00548	0.00611	0.00633	0.00542	0.00119	0.00118	0.00668	0.00634	0.00606	0.00023	0.00023
$r_G(\underline{\hat{\theta}}_{S})$	0.00419	0.00409	0.00296	0.00395	0.00401	0.0044	0.00456	0.00385	0.00087	0.00086	0.00479	0.00458	0.00430	0.00017	0.00017
$\underline{\hat{\theta}}_{Q}$	1.74183	1.74088	1.73004	0.35066	0.35118	1.74783	1.74499	1.7369	0.35591	0.35578	1.75308	1.74906	1.74072	0.35695	0.35678
$r_S(\underline{\hat{\theta}}_{Q})$	0.0204	0.02011	0.01664	0.00345	0.00342	0.02107	0.02194	0.01992	0.00216	0.00218	0.02272	0.02201	0.02172	0.0019	0.00191
$r_Q(\underline{\hat{\theta}}_{Q})$	0.00629	0.0062	0.00513	0.02158	0.02137	0.0065	0.00677	0.00614	0.01353	0.01361	0.00701	0.00679	0.00670	0.01185	0.01194

$r_G(\hat{\underline{\theta}}_Q)$	0.00475	0.00468	0.0039	0.01790	0.01773	0.00489	0.00509	0.00463	0.01063	0.01070	0.00524	0.0051	0.00503	0.00917	0.00924
$\hat{\underline{\theta}}_G$	1.76547	1.76449	1.76054	0.37339	0.37388	1.76787	1.76502	1.76388	0.37639	0.37626	1.77166	1.76761	1.76608	0.37700	0.37683
$r_S(\hat{\underline{\theta}}_G)$	0.01902	0.01865	0.01399	0.00165	0.00164	0.01999	0.02079	0.01793	0.00076	0.00077	0.0218	0.02093	0.01997	0.00057	0.00058
$r_Q(\hat{\underline{\theta}}_G)$	0.00587	0.00575	0.00432	0.01029	0.01024	0.00616	0.00641	0.00553	0.00477	0.00481	0.00673	0.00645	0.00616	0.00356	0.0036
$r_G(\hat{\underline{\theta}}_G)$	0.00434	0.00426	0.00321	0.00824	0.00819	0.00456	0.00474	0.00407	0.00365	0.00368	0.00494	0.00477	0.00452	0.00267	0.00270

The results given in Table 1 can be obtained as:

1. The assumed values of hyper parameters are $a_j = 10$ and $b_j = 30$ for $j = 1,2,3$. The values of θ_1, θ_2, and θ_3 are the means of three independent square root inverted gamma priors, with hyperparameters already defined. For Dirichlet prior, we considered $a_4 = a_5 = a_6 = 1$. The mixing proportions are assumed to be $p_1 = p_2 = 0.40$.
2. For a given sample of size n; np_1, np_2 and $n(1-p_1-p_2)$ observations are drawn from Rayleigh distribution with parameters θ_1, θ_2, and θ_3, respectively.
3. In each case, the sample observations that exceed *T* are censored, such that we have r_1, r_2 and r_3 observations from the three Rayleigh distributions with parameters θ_1, θ_2, and θ_3, respectively. Note that $n-r$ observations were left uncensored and $r = \sum_{j=1}^{3} r_j$.
4. The maximum likelihood estimates are computed by solving non-linear equations given in (10) and (11). Similarly, Bayes estimates of θ_1, θ_2, θ_3, p_1 and p_2 are calculated from equations given in Sections 3.1, 3.2 and 3.3, respectively.

TABLE 2: Maximum likelihood, Bayes estimates and estimated risks with $a_j = 10$ and $b_j = 30$ for $j = 1,2,3$, $n = 50$, $c = 1.20$, and $p_1 = p_2 = 0.4$.

	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$
			$T = 4$					$T = 5$					$T = 6$		
$\hat{\underline{\theta}}_{ML}$	1.76751	1.76920	1.82967	0.39734	0.39759	1.77832	1.78232	1.80193	0.39912	0.39926	1.78970	1.78399	1.78780	0.39979	0.39985
$r_S(\hat{\underline{\theta}}_{ML})$	0.06734	0.06575	0.16339	0.0009	0.00092	0.04911	0.05056	0.11084	0.00018	0.00018	0.04216	0.04348	0.09081	3.1E-05	3E-05
$r_Q(\hat{\underline{\theta}}_{ML})$	0.02077	0.02028	0.0504	0.00564	0.00572	0.01515	0.01560	0.03419	0.00112	0.00111	0.01300	0.01341	0.02801	0.00019	0.00019
$r_G(\hat{\underline{\theta}}_{ML})$	0.01534	0.01502	0.03227	0.00412	0.00419	0.01114	0.01134	0.02320	0.00082	0.00080	0.00951	0.00984	0.01990	0.00014	0.00014
$\hat{\underline{\theta}}_S$	1.79139	1.79308	1.79862	0.39945	0.39973	1.79198	1.79417	1.7993	0.39967	0.39974	1.79662	1.79311	1.79731	0.39989	0.39998
$r_S(\hat{\underline{\theta}}_S)$	0.01871	0.01874	0.01679	0.00053	0.00054	0.01905	0.01952	0.02021	0.00012	0.00013	0.01840	0.01908	0.02096	2.4E-05	2.3E-05
$r_Q(\hat{\underline{\theta}}_S)$	0.00577	0.00578	0.00518	0.00329	0.00338	0.00588	0.00602	0.00623	0.00078	0.00081	0.00568	0.00589	0.00646	0.00015	0.00014
$r_G(\hat{\underline{\theta}}_S)$	0.00417	0.00417	0.00371	0.00240	0.00245	0.00425	0.00432	0.00444	0.00057	0.00058	0.00409	0.00425	0.00462	0.00011	0.00010
$\hat{\underline{\theta}}_Q$	1.74913	1.75081	1.73771	0.37101	0.37130	1.75757	1.7597	1.74776	0.37404	0.37410	1.76534	1.7619	1.75037	0.37479	0.37488
$r_S(\hat{\underline{\theta}}_Q)$	0.01982	0.01972	0.01886	0.00141	0.00141	0.01979	0.02006	0.02120	0.00081	0.00081	0.01884	0.01972	0.02214	0.00066	0.00066
$r_Q(\hat{\underline{\theta}}_Q)$	0.00611	0.00608	0.00582	0.00884	0.00883	0.0061	0.00619	0.00654	0.00506	0.00507	0.00581	0.00608	0.00683	0.00414	0.00409

$r_G(\hat{\underline{\theta}}_Q)$	0.00460	0.00457	0.00441	0.00696	0.00696	0.00456	0.0046	0.00490	0.00385	0.00386	0.00432	0.00453	0.00511	0.00311	0.00308
$\hat{\underline{\theta}}_G$	1.76785	1.76954	1.76438	0.38410	0.38438	1.77283	1.77498	1.7704	0.38582	0.38588	1.77922	1.77575	1.77104	0.38632	0.38642
$r_S(\hat{\underline{\theta}}_G)$	0.01887	0.01883	0.01701	0.0008	0.00081	0.01916	0.01952	0.02009	0.00033	0.00033	0.0184	0.01919	0.02107	0.00021	0.00021
$r_Q(\hat{\underline{\theta}}_G)$	0.00582	0.00581	0.00525	0.00502	0.00506	0.00591	0.00602	0.0062	0.00207	0.00209	0.00568	0.00592	0.0065	0.00133	0.00130
$r_G(\hat{\underline{\theta}}_G)$	0.00431	0.00429	0.00389	0.00387	0.00389	0.00436	0.00441	0.00455	0.00156	0.00157	0.00417	0.00435	0.00477	0.00098	0.00096

By repeating the steps two to three 5000 times, estimated risks of both maximum likelihood and Bayes estimators are calculated under both symmetric and asymmetric loss functions. With different values of censoring time T and for different sample sizes n, the results of estimated risks have been computed and are given in Tables 1-4. Note that $r_{q_1}(\hat{\underline{\theta}}_{q_2})$ represents the values of estimated risk functions of estimator q_2 under q_1 loss function i.e., $q_2 = \hat{\underline{\theta}}_{ML}$, $\hat{\underline{\theta}}_S$, $\hat{\underline{\theta}}_Q$, $\hat{\underline{\theta}}_G$, and $q_1 = S$ for SELF, Q for QLF, and G for GELF. In Tables 1 and 2, we consider the case when overestimation is more serious than under estimation by taking $c = 1.20$ and in Tables 3 and 5, the case when underestimation is more serious than over estimation by taking $c = -1.20$ under GELF.

5.1. Overestimation is more serious than underestimation

It is observed from the results given in Tables 1 and 2 that the estimated risks of Bayes estimators are precise as compared to the estimated risks of maximum likelihood estimators under both symmetric and asymmetric loss functions. As obvious, with an increase in sample size from $n = 30$ to $n = 50$, the estimated risks of each estimator decreases and vise versa. Further, the Bayes estimators under SELF and GELF are having smaller estimated risks as compared to the other estimators.

TABLE 3: Maximum likelihood, Bayes estimates and estimated risks with $a_j = 10$ and $b_j = 30$ for $j = 1,2,3$, $n = 30$, $c = -1.20$, and $p_1 = p_2 = 0.4$

	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$
	$T = 4$					$T = 5$					$T = 6$				
$\hat{\underline{\theta}}_{ML}$	1.7604	1.75174	1.8545	0.39648	0.39568	1.77291	1.76949	1.79661	0.39894	0.39889	1.77539	1.77577	1.77231	0.3997	0.39974
$r_S(\hat{\underline{\theta}}_{ML})$	0.09898	0.09885	0.23115	0.00136	0.00138	0.07404	0.07694	0.18569	0.00027	0.00027	0.06654	0.06787	0.14459	4.7E-05	4.6E-05
$r_Q(\hat{\underline{\theta}}_{ML})$	0.03053	0.03049	0.07130	0.00852	0.00865	0.02284	0.02373	0.05727	0.00171	0.00171	0.02052	0.02093	0.0446	0.00029	0.00029
$r_G(\hat{\underline{\theta}}_{ML})$	0.02524	0.02557	0.04530	0.00665	0.00693	0.01844	0.01824	0.03123	0.00128	0.00131	0.01697	0.01606	0.03834	0.00022	0.00021
$\hat{\underline{\theta}}_S$	1.79461	1.79228	1.80055	0.39935	0.39914	1.79409	1.79263	1.79975	0.39971	0.39968	1.79385	1.79278	1.79341	0.40005	0.39991
$r_S(\hat{\underline{\theta}}_S)$	0.01843	0.01879	0.01329	0.0009	0.00091	0.01992	0.01947	0.01764	0.0002	0.0002	0.02033	0.02008	0.0196	3.4E-05	3.3E-05
$r_Q(\hat{\underline{\theta}}_S)$	0.00568	0.00580	0.0041	0.00562	0.00566	0.00614	0.00600	0.00544	0.00125	0.00122	0.00627	0.00619	0.00604	0.00022	0.00021
$r_G(\hat{\underline{\theta}}_S)$	0.00412	0.00420	0.00292	0.00427	0.00437	0.00444	0.00439	0.00386	0.00093	0.00092	0.00459	0.00454	0.00428	0.00016	0.00015
$\hat{\underline{\theta}}_Q$	1.74108	1.73879	1.73070	0.35077	0.35055	1.74857	1.74717	1.73789	0.35577	0.35575	1.75181	1.75076	1.73570	0.35704	0.35689
$r_S(\hat{\underline{\theta}}_Q)$	0.02001	0.02058	0.01649	0.00348	0.00351	0.02110	0.02087	0.01982	0.00219	0.00219	0.02154	0.02140	0.02222	0.00189	0.0019

$r_Q(\hat{\underline{\theta}}_Q)$	0.00617	0.00635	0.00509	0.02173	0.02197	0.00651	0.00644	0.00611	0.01367	0.01366	0.00664	0.00660	0.00685	0.01178	0.01185
$r_G(\hat{\underline{\theta}}_Q)$	0.00500	0.00515	0.00416	0.02206	0.02253	0.00520	0.00519	0.00494	0.01214	0.01216	0.00533	0.00530	0.0055	0.01003	0.01009
$\hat{\underline{\theta}}_G$	1.79741	1.79508	1.80429	0.40154	0.40133	1.79646	1.79501	1.80305	0.40170	0.40168	1.79604	1.79497	1.79647	0.40201	0.40187
$r_S(\hat{\underline{\theta}}_G)$	0.01851	0.01886	0.01340	0.00089	0.0009	0.01997	0.01951	0.01774	0.0002	0.0002	0.02037	0.0201	0.01965	3.8E-05	3.6E-05
$r_Q(\hat{\underline{\theta}}_G)$	0.00571	0.00582	0.00413	0.00559	0.00562	0.00616	0.00602	0.00547	0.00126	0.00123	0.00628	0.0062	0.00606	0.00024	0.00023
$r_G(\hat{\underline{\theta}}_G)$	0.00411	0.00419	0.00292	0.00414	0.00424	0.00443	0.00437	0.00385	0.00091	0.00091	0.00458	0.00453	0.00426	0.00017	0.00017

5.2. Underestimation is more serious than overestimation

It is clearly observable from the results given in Tables 3 and 4 that Bayes estimators under SELF and GELF are efficient as compared to the Bayes estimator under QLF and maximum likelihood estimators. With an increase in sample size from $n=30$ to $n=50$, the estimated risks of each estimator decreases and vise versa. It is worth mentioning that the Bayes estimates under both symmetric and asymmetric loss functions are efficient as compared to maximum likelihood estimates.

TABLE 4: Maximum likelihood, Bayes estimates and estimated risks with $a_j=10$ and $b_j=30$ for $j=1,2,3$, $n=50$, $c=-1.20$, and $p_1=p_2=0.4$.

	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{p}_1$	$\hat{p}_2$
	$T=4$					$T=5$					$T=6$				
$\hat{\underline{\theta}}_{ML}$	1.76810	1.76543	1.82811	0.39747	0.39770	1.77933	1.77818	1.80572	0.39919	0.39915	1.78381	1.78729	1.7902	0.39958	0.39991
$r_S(\hat{\underline{\theta}}_{ML})$	0.06858	0.06802	0.1538	0.00093	0.00095	0.04862	0.05036	0.10742	0.00018	0.00018	0.04349	0.04229	0.08861	3.2E-05	3.1E-05
$r_Q(\hat{\underline{\theta}}_{ML})$	0.02115	0.02098	0.04744	0.00581	0.00593	0.01500	0.01553	0.03313	0.0011	0.00112	0.01342	0.01304	0.02733	0.0002	0.00019
$r_G(\hat{\underline{\theta}}_{ML})$	0.01583	0.01671	0.03292	0.00439	0.00449	0.01166	0.01102	0.02324	0.00081	0.00083	0.01038	0.01008	0.0203	0.00015	0.00014
$\hat{\underline{\theta}}_S$	1.79241	1.78989	1.79811	0.39953	0.39971	1.79239	1.79249	1.79993	0.3997	0.39976	1.79313	1.79499	1.79829	0.39973	0.40001
$r_S(\hat{\underline{\theta}}_S)$	0.01855	0.01920	0.01628	0.00056	0.00056	0.01862	0.01893	0.01993	0.00012	0.00013	0.01900	0.01846	0.02055	2.5E-05	2.4E-05
$r_Q(\hat{\underline{\theta}}_S)$	0.00572	0.00592	0.00502	0.00352	0.00349	0.00574	0.00584	0.00615	0.00078	0.0008	0.00586	0.00569	0.00634	0.00016	0.00015
$r_G(\hat{\underline{\theta}}_S)$	0.00420	0.00434	0.00361	0.00261	0.00257	0.00422	0.00428	0.00441	0.00057	0.00059	0.00430	0.00418	0.00454	0.00012	0.00011
$\hat{\underline{\theta}}_Q$	1.75015	1.74769	1.73724	0.3711	0.37129	1.75804	1.75811	1.74851	0.37408	0.37413	1.76195	1.76379	1.75136	0.37462	0.37491
$r_S(\hat{\underline{\theta}}_Q)$	0.01961	0.02034	0.01845	0.00145	0.00143	0.01937	0.01961	0.02093	0.00081	0.00081	0.01967	0.01902	0.02164	0.00067	0.00065
$r_Q(\hat{\underline{\theta}}_Q)$	0.00605	0.00627	0.00569	0.00905	0.00893	0.00597	0.00605	0.00645	0.00505	0.00505	0.00607	0.00587	0.00668	0.00419	0.00409
$r_G(\hat{\underline{\theta}}_Q)$	0.00486	0.00504	0.00464	0.0081	0.00797	0.00473	0.00478	0.00518	0.00415	0.00416	0.00477	0.00462	0.0053	0.00334	0.00325
$\hat{\underline{\theta}}_G$	1.79459	1.79207	1.80134	0.40087	0.40105	1.79416	1.79426	1.80264	0.40092	0.40097	1.79473	1.7966	1.80075	0.40091	0.40119
$r_S(\hat{\underline{\theta}}_G)$	0.01859	0.01924	0.01638	0.00056	0.00056	0.01865	0.01896	0.02003	0.00012	0.00013	0.01902	0.01849	0.02062	2.5E-05	2.5E-05
$r_Q(\hat{\underline{\theta}}_G)$	0.00573	0.00593	0.00505	0.00351	0.00348	0.00575	0.00585	0.00618	0.00078	0.0008	0.00587	0.0057	0.00636	0.00016	0.00016
$r_G(\hat{\underline{\theta}}_G)$	0.00419	0.00433	0.00361	0.00256	0.00253	0.00421	0.00427	0.0044	0.00057	0.00058	0.00429	0.00417	0.00453	0.00012	0.00011

In case of prediction, Bayes point predictive estimates i.e. $\hat{Y}_S \mid \mathbf{x}$ and $\hat{Y}_G \mid \mathbf{x}$ along with their variances and prediction intervals for different sample sizes and censoring times T are given in Table 5. We have computed 99% prediction intervals for $Y \mid \mathbf{x}$. By following the steps one to three, $\hat{Y}_S \mid \mathbf{x}$ and $\hat{Y}_G \mid \mathbf{x}$, $\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$, $\hat{V}\left(\hat{Y}_G \mid \mathbf{x}\right)$, *LPL* and *UPL* are computed by using (37), (40) and (43), respectively. This process is repeated 5000 times.

The averages of these estimates have been computed and are given in Table 5. It is observed that generally with an increase in sample size n, the posterior predictive variances decreases and vice versa. Similarly, for a given sample size n, with an increase in censoring time T also reduces the posterior predictive variances and the length of prediction intervals.

TABLE 5: Average predictive estimates, predictive estimator variances and prediction intervals for $Y \mid \mathbf{x}$ when $a_j = 10$ and $b_j = 30$ for $j = 1,2,3$, $p_1 = p_2 = 0.4$ and $\gamma = 0.99$.

			$c = 1.20$		$c = -1.20$			
$n = 30$								
T	$\hat{Y}_S \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	*LPL*	*UPL*
4	2.26515	0.0139080	1.44784	0.0125092	1.36945	0.0131845	0.1755497	6.305918
4.5	2.25564	0.0127303	1.44185	0.0099300	1.36351	0.0101690	0.1753481	6.266589
5	2.25020	0.0123185	1.43957	0.0086688	1.36119	0.0087668	0.1754751	6.247670
5.5	2.25333	0.0120623	1.43826	0.0075987	1.36046	0.0075948	0.1751870	6.226495
6	2.25355	0.0118680	1.43835	0.0069793	1.36054	0.0069187	0.1754348	6.225449
$n = 40$								
T	$\hat{Y}_S \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	*LPL*	*UPL*
4	2.26059	0.0131664	1.44069	0.0116657	1.36553	0.0120361	0.1760323	6.2757454
4.5	2.25733	0.0123904	1.43626	0.0096491	1.36173	0.0097665	0.1758270	6.2376015
5	2.25362	0.0123984	1.43356	0.0085325	1.35930	0.0085572	0.1757140	6.2028782
5.5	2.25326	0.0109090	1.43379	0.0072398	1.35961	0.0071478	0.1757360	6.1891145
6	2.25167	0.0116872	1.43084	0.0069482	1.35684	0.0069137	0.1756250	6.1704238
$n = 50$								
T	$\hat{Y}_S \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	*LPL*	*UPL*
4	2.25969	0.0120152	1.43446	0.0113703	1.36261	0.0116202	0.1763362	6.244614
4.5	2.25710	0.0119297	1.43350	0.0096854	1.36223	0.0097642	0.1761888	6.201388
5	2.25369	0.0110347	1.42815	0.0075495	1.35726	0.0075325	0.1761465	6.175324
5.5	2.25210	0.0112499	1.42851	0.0070327	1.35762	0.0070294	0.1761795	6.157978
6	2.24830	0.0110071	1.42488	0.0065086	1.35397	0.0064613	0.1760797	6.139777
$n = 70$								
T	$\hat{Y}_S \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	$\hat{Y}_G \mid \mathbf{x}$	$\hat{V}\left(\hat{Y}_S \mid \mathbf{x}\right)$	*LPL*	*UPL*
4	2.25966	0.0103670	1.42795	0.0103652	1.36133	0.1040730	0.1767588	6.2050698

4.5	2.25594	0.0101833	1.42498	0.0082893	1.35895	0.0082657	0.1765813	6.1543476
5	2.25216	0.0093936	1.42017	0.0069489	1.35460	0.0068759	0.1765268	6.1220325
5.5	2.25069	0.0091417	1.42123	0.0058379	1.35562	0.0057762	0.1768650	6.1116730
6	2.25142	0.0095628	1.42103	0.0056676	1.35564	0.0056372	0.1767510	6.0932675

6. CONCLUSION

In this paper, a three component mixture model based on Rayleigh distributions has been proposed. Maximum likelihood and Bayes methods of estimation have been used to estimate the parameters of the mixture model. Posterior distribution, Bayes estimates and posterior predictive distributions have been derived in explicit forms **-** that can be used for further analysis. It is observed from Tables 1-4 that the Bayes estimates are precise than the maximum likelihood estimates. The Bayes point predictive estimates and prediction intervals have also been computed. Finally, for precise estimation of the unknown parameters of Rayleigh mixture model, Bayes method of estimation is preferable over maximum likelihood estimation especially given that the suitable prior information on the unknown parameters are available. This work can easily be extended to other lifetime distributions in order to control variation in the heterogeneous populations.

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