REVISTA INVESTIGACION OPERACIONAL VOL. 37, NO. 1, 48-57, 2016

ON SOME GENERALIZED CLASSES OF MINIMUM MEAN SQUARE ERROR ESTIMATORS IN SAMPLE SURVEYS

A.K.P.C.Swain[1]
Former Professor of Statistics,Utkal University
Bhubaneswar-751004, India

ABSTRACT
In this paper some generalized classes of minimum mean square error estimators of the finite population mean in the presence of a single auxiliary variable are proposed and their efficiencies are compared both theoretically and with numerical illustrations.

KEYWORDS:Classes of estimators,Ratio estimator,Regression estimator,Simple random sampling,Bias, Mean square error,Efficiency.

MSC:62D05

RESUMEN
En este paper son propuestas algunas clases de estimadores mínimo cuadráticos de la media de poblaciones finitas, en la presencia de una variable auxiliar, y sus eficiencias son comparadas teóricamente y usando ilustraciones numéricas.

1. INTRODUCTION

In large scale sample surveys the sampler looks for information on certain auxiliary variables , correlated with the main variable under study. One of the main reasons for such endeavour is to formulate estimators of the population mean or total of the study variable with the help of auxiliary information which are more efficient than estimators ,not using such information. Ratio and Regression estimators (Watson,1937, Cochran,1940) are classical estimators to estimate the population mean/total of the study variable using auxiliary information. During last eight decades volumes of research have been undertaken to study the properties of these estimators along with their improved versions. As most of the estimators using auxiliary information are biased and non-linear in nature , it is relevant to discuss some classes of minimum mean square error estimators to be used in survey sampling.

Let there be a finite population U , consisting of N units-$U_1,U_2,....,U_N$, where the ith unit U_i is indexed by paired values $(y_i, x_i), i=1,2,...,N$, corresponding to the study variable y and the correlated auxiliary variable x. Define $\bar{Y}$ and $\bar{X}$ as the population means of y and x respectively; S_y^2 and S_x^2 as the finite population variances of y and x respectively; ρ as the correlation coefficient between y and x.
Further, define $C_y = S_y / \bar{Y}, C_x = S_x / \bar{X}, C_{yx} = \rho(S_y / \bar{Y})(S_x / \bar{X}) = \rho C_Y C_X$.
as the coefficient of variation of y ,coefficient of variation of x , and coefficient of covariation between y and x respectively.
For a simple random sample without replacement s of size n , define $\bar{y}$ and $\bar{x}$ as the sample means of y and x respectively. The sample mean $\bar{y}$ is an unbiased estimator of $\bar{Y}$ with sampling variance

$$V(\bar{y}) = \theta \bar{Y}^2 C_y^2 \qquad (1.1)$$

where $\theta = \frac{1}{n} - \frac{1}{N}$.

[1]akpcs@rediffmail.com

When ρ is positive, the classical ratio estimator (Cochran,1977) of $\bar{Y}$ is

$$\bar{y}_R = \bar{y}(\frac{\bar{X}}{\bar{x}}) \qquad (1.2)$$

with approximate mean square error to order n^{-1}(symbolically written as $O(1/n)$) given by

$$MSE(\bar{y}_R) = \theta \bar{Y}^2 (C_y^2 + C_x^2 - 2C_{yx}) \qquad (1.3)$$

Murthy's(1967) product estimator of $\bar{Y}$, when ρ is negative is

$$\bar{y}_P = \bar{y}(\frac{\bar{x}}{\bar{X}}) \qquad (1.4)$$

with approximate mean square error to $O(1/n)$ given by

$$MSE(\bar{y}_P) = \theta \bar{Y}^2 (C_y^2 + C_x^2 + 2C_{yx}) \qquad (1.5)$$

The linear regression estimator used in sample surveys is given by

$$\bar{y}_{lr} = \bar{y} + b(\bar{X} - \bar{x}), \qquad (1.6)$$

where b is the sample regression coefficient of y on x. The approximate mean square error of $\bar{y}_{lr}$ to order n^{-1} is given by

$$MSE(\bar{y}_{lr}) = \theta \bar{Y}^2 C_y^2 (1 - \rho^2) \qquad (1.7)$$

Srivastava (1971) proposed a general class of estimators to estimate the population mean $\bar{Y}$ of the study variable y with advance knowledge of the population mean of the auxiliary variable x given by

$$t_g = \bar{y} H(u), \qquad (1.8)$$

where $u = \bar{x} / \bar{X}$ and $H(.)$ is a parametric function such that it satisfies the conditions

(i) $H(1) = 1$

(ii) The first, second and higher order partial derivatives of H with respect to single variable u exist and are known constants at a given point $u = 1$.These are the necessary and sufficient conditions for the expansion of $H(u)$ in Taylor's series. .

Thus ,expanding $H(u)$ about the point $u = 1$,in a Taylor's series we have

$$H(u) = H[1 + (u-1]$$
$$= H(1) + (u-1)[\frac{\partial H}{\partial u}]_{u=1} + (u-1)^2 [\frac{1}{2} \frac{\partial^2 H}{\partial u^2}]_{u=1} + ... \qquad (1.9)$$

In this paper we approximate $H(u)$ by first three terms of Taylor's series expansion.

Also, assuming $|u-1| < 1$, the higher order terms may be neglected.

Hence, we write

$$t_g = \bar{y} H(u) = \bar{y} H[1 + (u-1]$$
$$= \bar{y}(1 + (u-1)[\frac{\partial H}{\partial u}]_{u=1} + (u-1)^2 [\frac{1}{2} \frac{\partial^2 H}{\partial u^2}]_{u=1})$$
$$= \bar{y}(1 + (u-1)H_1 + (u-1)^2 H_2), \qquad (1.10)$$

where $H_1 = [\frac{\partial H}{\partial u}]_{u=1}$ and $H_2 = \frac{1}{2}[\frac{\partial^2 H}{\partial u^2}]_{u=1}$.

Thus, $t_g = \bar{Y}(1+e_0)(1+e_1H_1+e_1^2H_2+....)$ (1.11)

where $e_0 = \dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_1 = u-1 = \dfrac{\bar{x}-\bar{X}}{\bar{X}}$.

We have $E(e_0)=E(e_1)=0, E(e_0^2)=\theta C_y^2, E(e_1^2)=\theta C_x^2$ and $E(e_0e_1)=\theta C_{yx}$.

Hence to first order of approximation , that is , to terms of order n^{-1}(symbolically to $O(1/n)$)

$$MSE(t_g)=E(t_g-\bar{Y})^2 = \theta\bar{Y}^2[C_y^2+H_1^2C_x^2+2H_1C_{yx})] \quad (1.12)$$

Also, to $O(1/n)$ the bias of t_g is given by

$$B(t_g)=\theta\bar{Y}[H_2C_x^2+H_1C_{yx})\,] \quad (1.13)$$

The minimum value of $MSE(t_g)$ when minimized with respect to H_1 gives

$$Min.MSE(t_g)=\theta\bar{Y}^2C_y^2(1-\rho^2) \quad (1.14)$$

Thus, Srivastava (1971) concludes that if we attach any function of $u=\bar{x}/\bar{X}$ to $\bar{y}$, the asymptotic mean square error of the resultant estimator can not be lower than that given by (1.5):

Searls (1964) considered an estimator of the population mean $\bar{Y}$ given by

$$\bar{y}_{searls} = \lambda\bar{y} \quad (1.15)$$

where λ is a real constant to be suitably chosen.Under simple random sampling without replacement , the minimum mean square error of the Searls' estimator is given by

$$Min.MSE(\bar{y}_{searls}) = \bar{Y}^2\frac{\theta C_y^2}{1+\theta C_y^2} \quad (1.16)$$

In the following we shall consider certain general classes of minimum mean square error estimators and compare their lower bounds to asymptotic mean square errors along with optimum biases.

2.GENERALIZED CLASSES OF MINIMUM MEAN SQUARE RATIO TYPE ESTIMATORS

Consider the following classes of estimators

(i) $t_{g1} = w_1\bar{y}+w_2H(u)$

(ii) $t_{g2} = \bar{y}[w_1+w_2H(u)]$

(iii) $t_{g3} = w_1\bar{y}H(u)+w_2[1-H(u)]$

where w_1 and w_2 are real constants to be suitably determined and $H(u)$ is as defined by Srivastava(1971):

Class I (C_1):

$$t_{g1} = w_1\bar{y}+w_2H(u) \quad (2.1)$$

Using Taylor's series expansion of $H(u)$ with respect to u , given in (1.9), we have to second degree of approximation

$$t_{g1} = w_1\bar{Y}(1+e_0) + w_2(1+H_1e_1+H_2e_1^2+....)$$

$$t_{g1} - \bar{Y} = \bar{Y}[(w-1)+we_0+w_2^{'}(H_1e_1-e_0+H_2e_1^2)] \quad (2.2)$$

where $w_2^{'} = \frac{w_2}{\bar{Y}}$ and $w = w_1 + w_2^{'}$.

Thus to first order of approximation

$$B(t_{g1}) = Bias(t_{g1}) = \bar{Y}[(w-1)+w_2^{'}\theta H_2 C_x^2] \quad (2.3)$$

$$MSE(t_{g1}) = \bar{Y}^2[(w-1)^2 + w^2A_1 + w_2^{'2}A_2 + 2ww_2^{'}A_{12}] \quad (2.4)$$

where $A_1 = \theta C_y^2$,

$$A_2 = \theta[H_1^2C_x^2 + C_y^2 - 2H_1C_{yx}],\ A_{12} = \theta[H_1C_{yx} - C_y^2]$$

Minimizing (2.4) with respect to w and $w_2^{'}$ we have

$$w = \frac{1}{1+A_1-(A_{12}^2/A_2)} \quad \text{and} \quad w_2^{'} = -w\frac{A_{12}}{A_2}$$

$$Min.MSE(T_{g1}) = \frac{A_1-(A^2{}_{12}/A_2)}{1+A_1-(A_{12}^2/A_2)}$$

$$= \bar{Y}^2[\frac{\theta\frac{H_1^2C_y^2C_x^2(1-\rho^2)}{H_1^2C_x^2+C_y^2-2H_1C_{yx}}}{1+\theta\frac{H_1^2C_y^2C_x^2(1-\rho^2)}{H_1^2C_x^2+C_y^2-2H_1C_{yx}}}] \quad (2.5)$$

$$B(t_{g1}) = \bar{Y}[\frac{(A_{12}^2 - A_1A_2) - \theta H_2C_x^2A_{12}}{A_2[1+A_1-(A_{12}^2/A_2)]}] \quad (2.6)$$

$$= \theta\bar{Y}\left[\frac{H_1^2(C_{yx}^2 - C_y^2C_x^2) - H_2C_x^2(H_1C_{yx}-C_y^2)}{(H_1^2C_x^2+C_y^2-2H_1C_{yx}) - \theta H_1^2C_x^2(\frac{C_{yx}^2}{C_x^2}-C_y^2)}\right] \quad (2.7)$$

Class II (C_2):

$$t_{g2} = \bar{y}[w_1 + w_2H(u)] \quad (2.8)$$

Write

$$t_{g2} = \bar{Y}(1+e_0)[w_1+w_2(1+H_1e_1+H_2e_1^2)] \quad (2.9)$$

$$= \bar{Y}[w_1+w_2+w_2H_1e_1+w_2H_2e_1^2+w_1e_0+w_2e_0+w_2H_1e_0e_1+w_2H_2e_0e_1^2] \quad (2.10)$$

To terms of order n^{-1},

$$B(t_{g2}) = Bias(t_{g2}) = \bar{Y}[(w-1)+w_2\theta(H_2C_x^2+H_1C_{yx})] \quad (2.11)$$

and $$MSE(t_{g2}) = \bar{Y}^2[(w-1)^2 + w^2\theta C_y^2 + w_2^2H_1^2\theta C_x^2 + 2ww_2\theta H_1C_{yx}], \quad (2.12)$$

where $w = w_1 + w_2$.

Minimizing (2.12) with respect to w and w_2 we have

$$w_2 = -w\frac{B12}{B_2} \qquad w = \frac{1}{1+B_1-\dfrac{B_{12}^2}{B_2}}$$

,

where $B_1 = \theta C_y^2$, $B_2 = \theta H_1^2 C_x^2$ and $B_{12} = \theta H_1 C_{yx}$.

Substituting optimum values of w and w_2 in (2.12) we have

$$Min.MSE(t_{g2}) = \bar{Y}^2 \frac{\theta C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{2.13}$$

and $$B(t_{g2}) = \bar{Y}[\frac{(\dfrac{B_{12}^2}{B_2} - B_1) - \theta(H_2 C_x^2 + H_1 C_{yx})(B_{12}/B_2)}{1+B_1-\dfrac{B_{12}^2}{B_2}}] \tag{2.14}$$

$$= -\theta\bar{Y}\left[\frac{C_y^2 + \dfrac{H_2}{H_1}C_{yx}}{1-\theta(\dfrac{C_{yx}^2}{C_x^2} - C_y^2)}\right] \tag{2.15}$$

Class III (C_3)

$$t_{g3} = w_1 \bar{y} H(u) + w_2[1-H(u)] \tag{2.16}$$

Expanding t_{g3} in Taylor's series and keeping terms up to second degree, we have

$$t_{g3} = w_1\bar{Y}(1+e_0)(1+H_1 e_1 + H_2 e_1^2) - w_2(H_1 e_1 + H_2 e_1^2)$$

$$= \bar{Y}[w_1(1+e_0)(1+H_1 e_1 + H_2 e_1^2) - w_2^{'}(H_1 e_1 + H_2 e_1^2)] \tag{2.17}$$

where $w_2^{'} = w_2/\bar{Y}$.

Thus, to terms of order n^{-1},

$$B(t_{g3}) = Bias(t_{g3}) = \bar{Y}[(w_1-1) + w_1\theta H_1 C_{yx} - w_2^{'}\theta H_2 C_x^2 + w_1\theta H_2 C_x^2] \tag{2.18}$$

$$MSE(t_{g3}) = \bar{Y}^2[(w_1-1)^2 + w_1^2\theta(C_y^2 + H_1^2 C_x^2 + 2H_1 C_{yx}) + w_2^{'2}\theta H_1^2 C_x^2 - 2w_1 w_2^{'}\theta(H_1 C_{yx} + H_1^2 C_x^2)]$$

$$= \bar{Y}^2[(w_1-1)^2 + w_1^2 D_1 + w_2^{'2} D_2 - 2w_1 w_2^{'} D_{12}], \tag{2.19}$$

where $D_1 = \theta[C_y^2 + H_1^2 C_x^2 + 2H_1 C_{yx}]$, $D_2 = \theta H_1^2 C_x^2$

$D_{12} = \theta[H_1 C_{yx} + H_1^2 C_x^2]$

Minimizing (2.19) with respect to w_1 and $w_2^{'}$ gives the optimum values of w_1 and $w_2^{'}$ as and $w_1 = \dfrac{1}{1+D_1-\dfrac{D_{12}^2}{D_2}}$ and $w_2^{'} = w_1 \dfrac{D_{12}}{D_2}$

Substituting the optimum values of w_1 and $w_2^{'}$ in (2.19) , we have

$$Min.MSE(t_{g3}) = \bar{Y}^2\theta \frac{C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{2.20}$$

$$B(t_{g3}) = \bar{Y}\,\frac{(\frac{D_{12}^2}{D_2}-D_1)+\theta(H_1C_{yx}+H_2C_x^2)-\theta H_2C_x^2(D_{12}/D_2)}{1+D_1-\frac{D_{12}^2}{D_2}} \tag{2.21}$$

$$= -\theta\bar{Y}\left[\frac{(C_y^2-\frac{C_{yx}^2}{C_x^2})+C_{yx}(\frac{H_2}{H_1}-H_1)}{1-\theta(\frac{C_{yx}^2}{C_x^2}-C_y^2)}\right] \tag{2.22}$$

3.COMPARISON OF BIASES AND MEAN SQUARE ERRORS OF t_{g1}, t_{g2} AND t_{g3}

Now,

$$Min.MSE(t_{g1}) = \bar{Y}^2[\frac{\theta\frac{H_1^2C_y^2C_x^2(1-\rho^2)}{H_1^2C_x^2+C_y^2-2H_1C_{yx}}}{1+\theta\frac{H_1^2C_y^2C_x^2(1-\rho^2)}{H_1^2C_x^2+C_y^2-2H_1C_{yx}}}] \tag{3.1}$$

$$Min.MSE(t_{g2}) = Min.MSE(t_{g3}) = \bar{Y}^2\frac{\theta C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{3.2}$$

Thus, t_{g1} will be more efficient than both t_{g2} and t_{g3} if

$$C_y^2 - 2H_1C_{yx} > 0 \tag{3.3}$$

4.**SOME SPECIAL CASES OF GENERALIZED CLASSES OF ESTIMATORS**

Defining $H(u)$ differently , we can generate special cases of the proposed generalized classes of minimum mean square error estimators. In the following we discuss some of these minimum mean square error estimators relating to ratio estimator and product estimator(Murthy,1967)and compare them as regards their large sample biases and mean square errors.

(i)When y and x are positively correlated , define the minimum mean square error estimators as

$$t_{R1} = w_1\bar{y} + w_2(\frac{\bar{X}}{\bar{x}})$$

$$t_{R2} = \bar{y}[w_1 + w_2(\frac{\bar{X}}{\bar{x}})]$$

$$t_{R3} = w_1\bar{y}(\frac{\bar{X}}{\bar{x}}) + w_2(1-\frac{\bar{X}}{\bar{x}})$$

where w_1 and w_2 are so chosen to make the mean square errors to terms of order n^{-1} minimum. Thus the optimum biases and mean square errors to $O(1/n)$ are found as

$$B(t_{R1}) = \theta\bar{Y}\frac{C_{yx}(C_{yx}+C_x^2)}{(C_y^2+C_x^2+2C_{yx})+\theta C_x^2C_y^2(1-\rho^2)} \tag{4.1}$$

$$B(t_{R2}) = -\theta\bar{Y}\frac{(C_y^2-C_{yx})}{1+\theta(C_y^2-\frac{C_{yx}^2}{C_x^2})} = -\theta\bar{Y}\frac{(C_y^2-C_{yx})}{1+\theta C_y^2(1-\rho^2)} \tag{4.2}$$

$$B(t_{R3}) = -\theta\bar{Y}\frac{(C_y^2-\frac{C_{yx}^2}{C_x^2})}{1+\theta(C_y^2-\frac{C_{yx}^2}{C_x^2})} = -\theta\bar{Y}\frac{C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{4.3}$$

$$MSE(t_{R1}) = \theta\bar{Y}^2\frac{C_y^2C_x^2(1-\rho^2)}{(C_y^2+C_x^2+2C_{yx})}/(1+\theta\frac{C_y^2C_x^2(1-\rho^2)}{(C_y^2+C_x^2+2C_{yx})}) \tag{4.4}$$

$$MSE(t_{R2}) = \theta\bar{Y}^2\frac{C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{4.5}$$

$$MSE(t_{R3}) = \theta\bar{Y}^2\frac{C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{4.6}$$

(ii)When y and x are negatively correlated, define the minimum mean square estimators as

$$t_{P1} = w_1\bar{y} + w_2(\frac{\bar{x}}{\bar{X}})$$

$$t_{P2} = \bar{y}[w_1 + w_2(\frac{\bar{x}}{\bar{X}})]$$

$$t_{P3} = w_1\bar{y}(\frac{\bar{x}}{\bar{X}}) + w_2(1-\frac{\bar{x}}{\bar{X}})$$

where w_1 and w_2 are so chosen to make the minimum mean square errors to order n^{-1} minimum. The optimum biases and mean square errors to $O(1/n)$ are found as

$$B(t_{P1}) = -\theta\bar{Y}\frac{C_x^2C_y^2(1-\rho^2)}{(C_y^2+C_x^2-2C_{yx})+\theta C_x^2C_y^2(1-\rho^2)} \tag{4.7}$$

$$B(t_{P2}) = -\theta\bar{Y}\frac{C_y^2}{1+\theta C_y^2(1-\rho^2)} \tag{4.8}$$

$$B(t_{P3}) = -\theta\bar{Y}\frac{C_y^2(1-\rho^2)-C_{yx}}{1+\theta C_y^2(1-\rho^2)} \tag{4.9}$$

$$MSE(t_{P1}) = \theta\bar{Y}^2\frac{C_y^2C_x^2(1-\rho^2)}{(C_y^2+C_x^2-2C_{yx})}/(1+\theta\frac{C_y^2C_x^2(1-\rho^2)}{(C_y^2+C_x^2-2C_{yx})}) \tag{4.10}$$

$$MSE(t_{P2}) = \theta\bar{Y}^2\frac{C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{4.11}$$

$$MSE(t_{P3}) = \theta \bar{Y}^2 \frac{C_y^2(1-\rho^2)}{1+\theta C_y^2(1-\rho^2)} \tag{4.12}$$

Note: For optimum values of W_1 and W_2 ,

(i) t_{R1} is more efficient than both t_{R2} and t_{R3},which are equally efficient.,

(ii) t_{P1} is more efficient than both t_{P2} and t_{P3},which are equally efficient,

In practice the w_1 and w_2 are to be substituted by their consistent estimates from the sample.

5. NUMERICAL ILLUSTRATIONS

Consider the following natural populations given in Table 1.

Table 1. Description of Populations

Sl.no.	Source and Description	N	n	ρ	C_y^2	C_x^2
1	Sukhatme and Chand(1977) y:Apple bearing trees x:Bushels of apples harvested,1959	200	20	0.93	2.5528	4.0250
2	Cochran(1977) y:No. of placebo children x:No. of paralytic cases	34	10	0.7326	1.0248	1.5175
3	Murthy(1967) y:area under wheat 1964 x:Area under wheat,1963	34	7	0.9801	0.5673	0.5191
4	Murthy(1967) y :output x:: fixed capital	80	10	0.9413	0.1238	0.5564
5	Steel and Torrie(1960) y : log of leaf burn in seconds x :chlorine %	50	10	-0.4996	0.2307	0.5614
6	Singh(2003) y.:duration of sleep x :age of subjects	30	8	-0.8552	0.0243	0.0188

Table 2. Comparison of Mean square Errors excluding the constant multiplier

Population	t_{g1}	t_{g2}	t_{g3}	$\bar{y}_R / \bar{y}_P$	$\bar{y}$
1	0.0050 (2298)	0.0153 (751)	0.0153 (751)	0.0277 (415)	0.1149 (100)
2	0.0115 (629)	0.0324 (223)	0.0324 (223)	0.0500 (145)	0.0723 (100)

3	0.0006 (10717)	0.0025 (2572)	0.0025 (2572)	0.0026 (2473)	0.0643 (100)
4	0.0006 (1800)	0.0012 (900)	0.0012 (900)	0.0164 (66)	0.0108 (100)
5	0.0067 (275)	0.0137 (134)	0.0137 (134)	0.0346 (53)	0.0184 (100)
6	0.0001 (2200)	0.0006 (366)	0.0006 (366)	0.0006 (366)	0.0022 (100)

Note: The figures inside the brackets in Table 2 indicate the percent relative efficiency compared to that of the simple mean per unit estimator $\bar{y}$. For populations (1-4) the mean square errors are computed with $t_{g1} = t_{R1}, t_{g2} = t_{R2}$ and $t_{g3} = t_{R3}$ and for populations (5-6) the mean square errors are computed with $t_{g1} = t_{P1}, t_{g2} = t_{P2}$ and $t_{g3} = t_{P3}$.

Comments: The computations with natural populations show that (i)t_{R1} is highly efficient compared to t_{R2} , t_{R3}, $\bar{y}_R$,and $\bar{y}$ (ii)t_{P1} is highly efficient compared to t_{P2} , t_{P3} , $\bar{y}_P$, and $\bar{y}$

6. **CONCLUSIONS**

1. To terms of order n^{-1}, t_{g1} is more efficient than both t_{g2} and t_{g3} if

$$C_y^2 - 2H_1 C_{yx} > 0$$

2.To terms order n^{-1}, t_{g2} and t_{g3} have the same mean square error independent of the choice of $H(u)$,satisfying the stated conditions,.
3.The technique of constructing different classes of minimum mean square estimators using single auxiliary variable can be extended to the cases using more than one auxiliary variable and this problem will be discussed in a later paper.
4.To first order of approximation t_{R1} is more efficient than both t_{R2} and t_{R3} and so also t_{P1} compared to t_{P2} and t_{P3}.
5.The generalized classes of minimum mean square error estimators suggested in this paper are not exhaustive and the researchers may also work with other generalized classes of estimators.

Acknowledgement
The author is thankful to the esteemed referee for his valuable comments which helped the improvement and presentation of findings in this paper.

RECEIVED DECEMBER, 2014
REVISED MARCH, 2015

REFERENCES

[1]COCHRAN,W.G.(1940):The estimation of the yields of cereal experiments by sampling for the ratio of grains to total produce.**Journ**. **Ag**. .,30,262-275.
[2]COCHRAN,W.G.(1977):**SamplingTechniques**.JohnWiley and Sons, New York.
[3]MURTHY, M,N,(1967): **Sampling**: **TheoryandMethods**. Statistical Publishing Society, Calcutta, India.
[4]SEARLS,T.(1964):The utilization of known coefficient of variation in the estimation procedure. **Journ**.**Amer**.**Stat**.**Assoc**.59,1125-1126.
[5]SINGH,S.(2003):**AdvancedsamplingTheorywithApplications**,Kluwer Publishers, London.
[6]SRIVASTAVA,S.K.(1971):A generalized estimator for the mean of a finite population using multiauxiliary information.**Journ**. **Amer**. **Stat**. **Assoc**.,66,404-407.
[7]STEEL , R. G. D. and TORRIE,J.H.(1960): **PrinciplesandProceduresofStatistics**.Mc Graw Hill,New York.
[8]SUKHATME , B.V. and CHAND, L. (1977): **Multivariate ratio-type estimators**. Proceedings, Social Statistics Section,American Statistical Association,927-931.
[9]WATSON,D.J.(1937):The estimation of leaf area in field crops.**Journ**. **Ag**. **Sc**.,30,262-275.