A SINGLE SERVER RETRIAL QUEUEING SYSTEM WITH TWO TYPES OF BATCH ARRIVALS AND FINITE NUMBER OF RECURRENT REPEATED CUSTOMERS

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ABSTRACT

A single server retrial queueing system with two types (Type I and Type II) of batch arrivals and K recurrent calls is considered. The two batch arrivals follows compound Poisson processes with two different rates. The Type II customers enter into the orbit, if the server is busy. The retrial orbit consists of transit type II customers and a fixed number K recurrent customers. The Type I customers are waiting in a queue before the server, if the service is not immediate. Service times distributions are identical independent distributions (transit and recurrent) and are different for type I customers, whereas the retrial time for both transit and recurrent customers are independently and exponentially distributed with different rates. For this model, the joint distribution of the number customers of type I, type II and recurrent in closed form has been obtained. Some particular models and operating characteristics are obtained. A numerical study is also carried out.

KEYWORDS: Priority queue, Supplementary variable, Joint-distribution, Probability generating function and Operating characteristics.

MSC: 90B22, 60K25, 60K30.

RESUMEN

Se considera que un solo sistema de gestión de colas nuevo juicio servidor con dos tipos (tipos I y II) de las llegadas de lotes y llamadas recurrentes. Los dos llegadas lotes sigue compuestos procesos de Poisson con dos tasas diferentes . Los clientes de tipo II entran en la órbita , si el servidor está ocupado. La órbita de reintentos consiste en el tipo de tránsito II clientes y un número fijo clientes recurrentes. El tipo I clientes están esperando en una cola antes de que el servidor, si el servicio no es inmediata. Servicio veces distribuciones son distribuciones independientes idénticos (de tránsito y recurrentes) y son diferentes para los clientes de tipo I , mientras que el tiempo nuevo juicio tanto para tránsito y recurrentes clientes se distribuyen de manera independiente y de forma exponencial con diferentes tarifas. Para este modelo, la distribución conjunta de los clientes Número de Tipo I, se ha obtenido de tipo II y recurrente en forma cerrada . Se obtienen Algunos modelos particulares y características de funcionamiento. Un estudio numérico también se lleva a cabo

1.INTRODUCTION

In the last three decades, there has been a significant contribution in the area of retrial queueing theory. For detailed survey one can see Yang and Templeton [15] and Falin [5]. Choi and Park [1] investigated an M/G/1 retrial queue with two types of customers in which the service time distribution for both types of customers are the same. Khailal et al. [13] investigated the above model at a Markovian level in detail. Falin et al. [6] investigated a similar model, in which they assumed different service time distributions for both types of customers. In 1995, Choi et al. [3] studied an M/G/1 retrial queueing system with two types of customers and finite capacity. Choi and Chang [2] investigated an $M_1, M_2/G/1$ retrial queue with recurrent calls in the retrial group, in which they obtained generating function of queue lengths by using the supplementary variable method. Kalyanaraman and Srinivasan [11], considered an M/G/1 queue with two types of arrivals and recurrent repeated arrivals. For this model they obtained the joint distribution of number of calls in the priority queue and in the retrial group in closed form. Kalyanaraman and Srinivasan [12], studied an M/G/1 retrial queue with geometric loss and with type I batch arrivals and type II single arrivals. In 2011, Kalyanaraman and Thillaigovindan [14] analyzed a feedback retrial queueing system with two types of arrivals and the type I arrival being a batch arrival of fixed size K. Kalyanaraman [9] investigated a feedback retrial queue with type I arrival being batch arrivals and the type II arrival single arrivals. The same author [10] analyzed a feedback retrial queueing system with two types of batch arrivals. Ioannis Dimitrious [8] investigated a batch arrival priority queue with recurrent repeated demands, admission control and hybrid failure recovery discipline.

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This article deals a retrial queue with two types of customers, in which both types of customers arrives in batches of variable size. In addition the orbit consists of a fixed number K of recurrent customers. As an example, in a computer communication system, type I customers are identified as incoming messages and transit type II customers are identified as outgoing messages, on the other hand, there are a fixed number of messages in CPU which are taken as K recurrent customers. In section 2, we describe the system mathematically. In section 3, we obtain the joint probability generating function for the number of customers in the priority queue and in the retrial group when the server is busy as well as idle. The expressions for some particular models are deduced in section 4. Some operating characteristics are derived in section 5 and a numerical study is also carried out in section 6. The last section contains a conclusion.

2. THE MODEL

In this article, a single server retrial queueing system with type I batch arrivals, transit type II batch arrivals and K recurrent customers has been considered. The type I customers arrives in batches of size k with probability c_k and type II customers arrive in batches of size k with probability d_k according to two independent Poisson processes with rates $\lambda_1\bar{c} = \lambda_1\sum_{k=1}^{\infty}kc_k$ and $\lambda_2\bar{d} = \lambda_2\sum_{k=1}^{\infty}kd_k$ respectively. When the arriving type I customers are blocked due to server being busy, they are queued in a priority queue of infinite capacity. Otherwise, any one of the customers in the batch being served, the other customers in the batch queued in the priority queue. As soon as the server is free, the customer in the priority queue is served using FCFS rule. If the type II customers, upon arrival finds the server busy, they enter into an orbit of infinite capacity in order to seek service again after a random amount of time. On the other hand, if the type II customers find the server free on their arrival, anyone in the batch occupies the server and leaves the system after service completion, whereas others in the batch enter into the orbit. In addition, the orbit consists of a fixed number of K recurrent customers. The recurrent customers, which finds the server free, occupy and return to the retrial orbit after service completion. The orbit consists of $(j-K)$ transit customers and K recurrent customers. The retrial time for each transit and recurrent customers are exponentially distributed with rates α_1 and α_2 respectively.This retrial rate depends on the number of transit customers and the recurrent customers and is independent of all previous retrial times and all other stochastic processes related to the system.

The customers in the retrial orbit are served, if there are no customers in the priority queue. It is assumed that the type I customers are non-preemptive priority over transit type II customers and recurrent customers.

The service times are independent, identically distributed and are different for both priority customers and the customers in the orbit. The density functions are respectively $b_1(x), b_2(x)$ and the Laplace transformation of the distribution function $B_k(x)$ is $B_k^*(s) = \int_0^{\infty} e^{-sx} b_k(x)dx,\ k = 1,2.$

The inter arrival time, service time and retrial times are mutually independent random variables.

The supplementary variable, residual service time of a customer in service, is used for the analysis of this model.

The Stochastic process related to this model is $\{(\xi(t), N_p(t), N_r(t), S_k(t)) : t \geq 0\}$ where

$N_p(t)$ = number of customers in the priority queue at time t,

$N_r(t)$ = number of customers in the retrial group at time t,

$\xi(t)$ = the server state at time t, is defined as

$$\xi(t) = \begin{cases} 0, \text{ when the server is idle} \\ 1, \text{ when the server is busy with type I customer} \\ 2, \text{ when the server is busy with type II customer} \\ 3, \text{ when the server is busy with recurrent customer} \end{cases}$$

$S_k(t)$ = the residual service time of a type k customer in service at time t, is a Markov process with state space $\{0,1,2,3\}\times\{0,1,2,3,\cdots\}\times(0,\infty)$ and the corresponding stationary process is $\{(\xi, N_p, N_r, S_k)\}$.

The related probabilities are defined as $q_j(t) = Pr\{\xi(t)=0, N_r(t)=j\}$,

$p(k,i,j;x,t)dx = Pr\{\xi(t)=k, N_p(t)=i, N_r(t)=j, S_k(t)\in(x,x+dx)\}$, k=1, 2, 3.

In steady state, the probabilities are defined as $q_j = \lim_{t\to\infty} q_j(t)$, this leads to

$p(k,i,j;x) = \lim_{t\to\infty} p(k,i,j;x,t)$ and the Laplace transformation of $p(k,i,j;x)$ is $p^*(k,i,j;s) = \int_0^\infty e^{-sx} p(k,i,j;x)dx, i=1,2,3,\ j\geq 0.$

It is clear that, $p(k,i,j;0) = \int_0^\infty p(k,i,j;x)dx = Pr\{\xi=k, N_p=i, N_r=j\}$ is the steady state probability that there are i customers in the priority queue, j customers in the retrial group and the server is busy with k th-type customer.

For $|Z_1|,|Z_2|\leq 1$, the following probability generating functions

$Q(Z_2) = \sum_{j=K-1}^{\infty} q_j Z_2^j$,

$C(Z_1) = \sum_{j=1}^{\infty} c_j Z_1^j$,

$D(Z_2) = \sum_{j=1}^{\infty} d_j Z_2^j$,

$P^*(k,i,s,Z_2) = \sum_{j=0}^{\infty} p^*(k,i,j,s)Z_2^j; i=0,1,2,...,k=1,2,3,$

$P^*(k,s,Z_1,Z_2) = \sum_{i=0}^{\infty} P^*(k,i,s,Z_2)Z_1^i; k=1,2,3,$

$P(k,i,0,Z_2) = \sum_{j=0}^{\infty} p^*(k,i,j,0)Z_2^j; i=0,1,2,...,k=1,2,3,$

$P(k,0,Z_1,Z_2) = \sum_{i=0}^{\infty} P(k,i,0,Z_2)Z_1^i$ are defined for the analysis.

3. THE ANALYSIS

In this section we present the mathematical analysis of the model defined in the previous section.

Using the mean drift argument of Falin [4], it can be shown that the system is stable if $\rho_1+\rho_2<1$ where $\rho_1 = -\lambda_1 \bar{c} B_1^{*'}(0)$, $\rho_2 = -\lambda_2 \bar{d} B_2^{*'}(0)$.

Usual arguments lead to the following differential-difference equations

For $j\geq 0, x\geq 0, i\geq 0$

$$(\lambda + (j-K)\alpha_1 + K\alpha_2)q_j = p(1,0,j;0) + p(2,0,j;0) + p(3,0,j-1;0) \tag{1}$$

$$-p'(1,0,j;x) = -\lambda p(1,0,j;x) + \lambda_1 b_1(x) q_j + b_1(x) p(1,1,j;0) + \lambda_2 \sum_{k=1}^{j} d_k p(1,0,j-k;x) \tag{2}$$

$$-p'(1,i,j;x) = -\lambda p(1,i,j;x) + b_1(x) p(1,i+1,j;0) + \lambda_1 \sum_{k=1}^{i} c_k p(1,i-k,j;x) + \lambda_2 \sum_{k=1}^{j} d_k p(1,i,j-k;x) \tag{3}$$

$$-p'(2,0,j;x)=-\lambda p(2,0,j;x)+\lambda_2 b_2(x)\sum_{k=0}^{j} d_{k+1}q_{j-k}+\lambda_2\sum_{k=1}^{j} d_k p(2,0,j-k;x)+(j-K+1)\alpha_1 b_2(x)q_{j+1} \quad (4)$$

$$-p'(2,i,j;x)=-\lambda p(2,i,j;x)+\lambda_1\sum_{k=1}^{i} c_k p(2,i-k,j;x)+\lambda_2\sum_{k=1}^{j} d_k p(2,i,j-k;x) \quad (5)$$

$$-p'(3,0,j;x)=-\lambda p(3,0,j;x)+\lambda_2\sum_{k=1}^{j} d_k p(3,0,j-k;x)+K\alpha_2 b_2(x)q_{j+1} \quad (6)$$

$$-p'(3,i,j;x)=-\lambda p(3,i,j;x)+\lambda_1\sum_{k=1}^{i} c_k p(3,i-k,j;x)+\lambda_2\sum_{k=1}^{j} d_k p(3,i,j-k;x) \quad (7)$$

and the normalization condition is,

$$\sum_{i=0}^{\infty}\sum_{j=K-1}^{\infty}\int_0^{\infty}\left[p(1,i,j;x)+p(2,i,j;x)+p(3,i,j-1;x)\right]dx+\sum_{j=K}^{\infty} q_j=1 \quad (8)$$

where $\lambda=\lambda_1+\lambda_2, q_j=0$ and $p(\gamma,i,j;x)=0, i\geq 0, j=K-1, \gamma=1,2,3.$

Multiplying equation (1) by Z_2^j and then summing over j, we get

$$[\lambda-K(\alpha_1-\alpha_2)]Q(Z_2)+\alpha_1 Z_2 Q'(Z_2)=P(1,0;0,Z_2)+P(2,0;0,Z_2)+Z_2 P(3,0;0,Z_2) \quad (9)$$

By taking Laplace transformation on equations (2) to (7) and multiplying by Z_2^j and then summing over j, the following equations can be obtained

$$(s-\lambda+\lambda_2 D(Z_2))P^*(1,0;s,Z_2)=P(1,0;0,Z_2)-\lambda_1 B_1^*(s)Q(Z_2)-B_1^*(s)P(1,1;0,Z_2) \quad (10)$$

$$(s-\lambda+\lambda_2 D(Z_2))P^*(1,i;s,Z_2)=P(1,i;0,Z_2)-B_1^*(s)P(1,i+1;0,Z_2)-\lambda_1\sum_{k=1}^{i} c_k P^*(1,i-k;s,Z_2) \quad (11)$$

$$(s-\lambda+\lambda_2 D(Z_2))P^*(2,0;s,Z_2)=P(2,0;0,Z_2)-\lambda_2 B_2^*(s)\frac{D(Z_2)}{Z_2}Q(Z_2)-\alpha_1 B_2^*(s)Q'(Z_2)+K\alpha_1 B_2^*(s)\frac{Q(Z_2)}{Z_2} \quad (12)$$

$$(s-\lambda+\lambda_2 D(Z_2))P^*(2,i;s,Z_2)=-\lambda_1\sum_{k=1}^{i} c_k P^*(2,i-k;s,Z_2) \quad (13)$$

$$(s-\lambda+\lambda_2 D(Z_2))P^*(3,0;s,Z_2)=P(3,0;0,Z_2)-\frac{K\alpha_2}{Z_2}B_2^*(s)Q(Z_2) \quad (14)$$

$$(s-\lambda+\lambda_2 D(Z_2))P^*(3,i;s,Z_2)=-\lambda_1\sum_{k=1}^{i} c_k P^*(3,i-k;s,Z_2) \quad (15)$$

Multiplying equations (11), (13) and (15) by Z_1^i and adding (10), (12) and (14) and summing over i, which leads to

$$(s-\lambda+\lambda_1 C(Z_1)+\lambda_2 D(Z_2))P^*(1;s,Z_1,Z_2)=\left(1-\frac{B_1^*(s)}{Z_1}\right)P(1;0,Z_1,Z_2)-\lambda_1 B_1^*(s)Q(Z_2)+\frac{B_1^*(s)}{Z_1}P(1;0,0,Z_2) \quad (16)$$

$$(s-\lambda+\lambda_1 C(Z_1)+\lambda_2 D(Z_2))P^*(2;s,Z_1,Z_2)=P(2,0;0,Z_2)-\lambda_2 B_2^*(s)\frac{D(Z_2)}{Z_2}Q(Z_2)-\alpha_1 B_2^*(s)Q'(Z_2)+\frac{K\alpha_1}{Z_2}B_2^*(s)Q(Z_2) \quad (17)$$

$$(s-\lambda+\lambda_1 C(Z_1)+\lambda_2 D(Z_2))P^*(3;s,Z_1,Z_2)=P(3,0;0,Z_2)-\frac{K\alpha_2}{Z_2}B_2^*(s)Q(Z_2) \quad (18)$$

By substituting $s=\lambda-\lambda_1 C(Z_1)-\lambda_2 D(Z_2)$ in (16), (17) and (18), we get

$$P(1,0;0,Z_2)=\lambda_1 z_1 Q(Z_2)-\frac{[Z_1-B_1^*(l)]}{B_1^*(l)}P(1;0,Z_1,Z_2) \tag{19}$$

$$P(2,0;0,Z_2)=B_2^*(l)\left[\left(\lambda_2\frac{D(Z_2)}{Z_2}-\frac{K\alpha_1}{Z_2}\right)Q(Z_2)+\alpha_1 Q^{'}(Z_2)\right] \tag{20}$$

$$Z_2P(3,0;0,Z_2)=B_2^*(l)K\alpha_2 Q(Z_2) \tag{21}$$

where $l=\lambda-\lambda_1 C(Z_1)-\lambda_2 D(Z_2)$

Using equations (19), (20) and (21) in (9) and on simplification one can get the following equation

$$\left[\lambda-\lambda_1 Z_1+K\alpha_2\big(1-B_2^*(l)\big)-K\alpha_1\left(1-\frac{B_2^*(l)}{Z_2}\right)-\lambda_2 B_2^*(l)\frac{D(Z_2)}{Z_2}\right]Q(Z_2)+\alpha_1\left[Z_2-B_2^*(l)\right]Q^{'}(Z_2)$$
$$=\frac{[B_1^*(l)-Z_1]}{B_1^*(l)}P(1;0,Z_1,Z_2) \tag{22}$$

Now we define $f(Z_1,Z_2)=\dfrac{B_1^*(l)-Z_1}{B_1^*(l)}$ for each fixed $Z_2, |Z_2|\le 1,$ by Rouche's theorem, there is a unique solution $Z_1=h(Z_2)$ of the equation $f(Z_1,Z_2)=0,$ now (22) becomes

$$Q^{'}(Z_2)=\left\{\frac{\lambda-\lambda_1 h(Z_2)+K\alpha_2(1-U(Z_2))-\lambda_2 U(Z_2)\dfrac{D(Z_2)}{Z_2}}{\alpha_1(U(Z_2)-Z_2)}+\frac{K}{Z_2}\right\}Q(Z_2) \tag{23}$$

where $h(Z_2)$ is the root of the equations $Z_1=B_1^*\big(\lambda-\lambda_1 C(Z_1)-\lambda_2 D(Z_2)\big)$ and $U(Z_2)=B_2^*\big(\lambda-\lambda_1 C(Z_1)-\lambda_2 D(Z_2)\big)$

Using equation (23) in (22), it can be seen that

$$P(1;0,Z_1,Z_2)=\frac{\{L[Z_2-B_2^*(l)]+R[U(Z_2)-Z_2]\}B_1^*(l)}{[B_1^*(l)-Z_1][U(Z_2)-Z_2]}Q(Z_2) \tag{24}$$

where $L=\lambda-\lambda_1 h(Z_2)-K\alpha_1\left(1-\dfrac{U(Z_2)}{Z_2}\right)+K\alpha_2(1-U(Z_2))-\lambda_2 U(Z_2)\dfrac{D(Z_2)}{Z_2}$ and

$$R=\lambda-\lambda_1 Z_1-K\alpha_1\left(1-\frac{B_2^*(l)}{Z_2}\right)+K\alpha_2(1-B_2^*(l))-\lambda_2 B_2^*(l)\frac{D(Z_2)}{Z_2}$$

Using equation (24) in (19), leads to

$$P(1;0,0,Z_2)=\frac{[\lambda_1 Z_1+R][U(Z_2)-Z_2]+L[Z_2-B_2^*(l)]}{[U(Z_2)-Z_2]}Q(Z_2) \tag{25}$$

Using equation (23) in (20), leads to

$$P(2;0,0,Z_2)=\frac{\left\{\left(\lambda_2\dfrac{D(Z_2)}{Z_2}-\dfrac{K\alpha_1}{Z_2}\right)[U(Z_2)-Z_2]+L\right\}B_2^*(l)}{[U(Z_2)-Z_2]}Q(Z_2) \tag{26}$$

The general solution of the differential equation (23) is

$$Q(Z_2)=Q(1)exp\left\{\frac{-1}{\alpha_1}\int_{Z_2}^{1}\frac{T}{U(x)-x}dx\right\} \tag{27}$$

where $T = \lambda - \lambda_1 h(x) - K\alpha_1\left(1 - \frac{U(x)}{x}\right) + K\alpha_2(1 - U(x)) - \lambda_2 U(x)\frac{D(x)}{x}$ and $Q(1)$ is a constant, which is the probability that the server is idle.

Putting $s = 0$ in equations (10) and (11) and summing over i, which leads to

$$\lambda_2 (D(Z_2) - 1)\sum_{i=0}^{\infty} P^*(1,i;0,Z_2) = P(1,0;0,Z_2) - \lambda_1 Q(Z_2) \tag{28}$$

Putting $s = 0$ in equations (12) and (13) and summing over i, which leads to

$$\lambda_2 (D(Z_2) - 1)\sum_{i=0}^{\infty} P^*(2,i;0,Z_2) = P(2,0;0,Z_2) - [\lambda_2 D(Z_2) - K\alpha_1]\frac{Q(Z_2)}{Z_2} - \alpha_1 Q'(Z_2) \tag{29}$$

Putting $s = 0$ in equations (14) and (15) and summing over i, which leads to

$$\lambda_2 Z_2 (D(Z_2) - 1)\sum_{i=0}^{\infty} P^*(3,i;0,Z_2) = Z_2 P(3,0;0,Z_2) - K\alpha_2 Q(Z_2) \tag{30}$$

Adding equations (28)-(30) and using (8), which leads to

$$\lambda_2 (D(Z_2) - 1)\sum_{i=0}^{\infty}\left[\sum_{k=1}^{2} P^*(k,i;0,Z_2) + Z_2 \sum_{i=0}^{\infty} P^*(3,i;0,Z_2)\right] = \left[\lambda_2\left(1 - \frac{D(Z_2)}{Z_2}\right) - \frac{K\alpha_1}{Z_2}(Z_2 - 1)\right]Q(Z_2) + \alpha_1 (Z_2 - 1)Q'(Z_2) \tag{31}$$

Evaluating at $Z_2 = 1$ and using the normalization condition, the above equation becomes

$$Q'(1) = \frac{1}{\alpha_1}[\lambda_2 \bar{d} - Q(1)(\lambda_2 - K\alpha_1)] \tag{32}$$

Putting $Z_2 = 1$ in equation (23), we get

$$Q'(1) = \frac{\left\{\lambda_2 \bar{d}\rho_1 + \rho_2 \bar{c}(\lambda_2 + K\alpha_2) + K\alpha_1 \bar{c}(1 - \rho_1 - \rho_2) + \lambda_2 \bar{c}(\bar{d} - 1)(1 - \rho_1)\right\}Q(1)}{\alpha_1 \bar{c}(1 - \rho_1 - \rho_2)} \tag{33}$$

From equation (32) and (33), we get

$$P_I = Q(1) = \frac{\lambda_2 \bar{d}\bar{c}(1 - \rho_1 - \rho_2)}{[\lambda_2 \bar{d}(\bar{c} + \rho_1 - \rho_1 \bar{c}) + K\alpha_2 \rho_2 \bar{c}]} \tag{34}$$

is the probability that the server is idle. In steady state, the probability generating function of the number of customers in the orbit, when the server is idle is obtained from equations (34) and (27).

Substituting $s = 0$ in equation (16)

$$P^*(1;0,Z_1,Z_2) = \frac{P(1;0,Z_1,Z_2)(1 - Z_1) + \lambda_1 Z_1 Q(Z_2) - P(1,0,0;Z_2)}{lZ_1} \tag{35}$$

(35) together with equations (24) and (25) yields the joint probability generating function of the number of customers in the priority queue and in the orbit when the server is busy with a type I customer and is

$$P^*(1;0,Z_1,Z_2) = \frac{[1 - B_1^*(l)]\left\{L[Z_2 - B_2^*(l)] + R[U(Z_2) - Z_2]\right\}}{l[B_1^*(l) - Z_1][U(Z_2) - Z_2]}Q(Z_2) \tag{36}$$

Putting $s = 0$ in equation (17),

$$P^*(2;0,Z_1,Z_2) = \frac{\left[\frac{\lambda_2 D(Z_2) - K\alpha_1}{Z_2}\right]Q(Z_2) + \alpha_1 Q'(Z_2) - P(2;0,0,Z_2)}{l} \tag{37}$$

(37) together with equations (23) and (26) yields the joint probability generating function of the number of customers in the priority queue and in the orbit when the server is busy with a type II customer and is

$$P^*(2;0,Z_1,Z_2)=\frac{[1-B_2^*(l)]\{\lambda-\lambda_1 h(Z_2)+K\alpha_2(1-U(Z_2))-\lambda_2 D(Z_2)\}}{l[U(Z_2)-Z_2]}Q(Z_2) \tag{38}$$

Putting $s=0$ in equation (18),

$$P^*(3;0,Z_1,Z_2)=\frac{1}{l}\left\{\frac{K\alpha_2}{Z_2}Q(Z_2)-P(3;0,0,Z_2)\right\} \tag{39}$$

(39) together with equation (21) yields the joint probability generating function of the number of customers in the priority queue and in the orbit when the server is busy with a K recurrent customers and is

$$P^*(3;0,Z_1,Z_2)=\frac{K\alpha_2[1-B_2^*(l)]}{lZ_2}Q(Z_2) \tag{40}$$

Thus we have the following theorem.

Theorem 1: The stationary distribution of $\{(\xi,N_p,N_r,S_k)\}$ has the following generating functions

$$Q(Z_2)=\frac{\lambda_2\bar{d}\bar{c}(1-\rho_1-\rho_2)}{[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}exp\left\{\frac{1}{\alpha_1}\int_1^{Z_2}\frac{T}{U(x)-x}dx\right\}$$

$$P^*(1;0,Z_1,Z_2)=\frac{[1-B_1^*(l)]\{L[Z_2-B_2^*(l)]+R[U(Z_2)-Z_2]\}Q(Z_2)}{l[B_1^*(l)-Z_1][U(Z_2)-Z_2]}$$

$$P^*(2;0,Z_1,Z_2)=\frac{[1-B_2^*(l)]\{\lambda-\lambda_1 h(Z_2)+K\alpha_2(1-U(Z_2))-\lambda_2 D(Z_2)\}Q(Z_2)}{l[U(Z_2)-Z_2]}$$

$$P^*(3;0,Z_1,Z_2)=\frac{K\alpha_2[1-B_2^*(l)]Q(Z_2)}{lZ_2}$$

Corollary 1:

The probability that the server is busy

$$\begin{aligned}P_B &= P^*(1;0,1,1)+P^*(2;0,1,1)+P^*(3;0,1,1)\\ &=\frac{[\lambda_2\bar{d}(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]}{[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\end{aligned}$$

4. PARTICULAR MODELS

By taking particular values of some parameters of the above model, the following models can be obtained, which are already existing:

(i) When $K=0, d_i=0=c_i, i\neq 1,$ and $B_1(x)=B_2(x)=B(x)$ the system coincides with that of Choi and Park [1].

(ii) When $K=0, d_i=0=c_i, i\neq 1,$ the above results coincide with the results of Falin et al. [6].

(iii) When $d_i=0=c_i, i\neq 1,$ and $B_1(x)=B_2(x)=B(x)$ the system coincides with that of Kalyanaraman and Srinivasan [11].

5. OPERATING CHARACTERISTICS

In this section, we present some operating characteristics related to the model analyzed in section 3.

Using straightforward calculations, the operating characteristics like the mean number of customers in the priority queue, the mean number of customers in the orbit, mean busy period, mean waiting time of a tagged type I customer in the priority queue and mean waiting time of a tagged type II customer in the orbit have been calculated. Putting $Z_2=1$ in equations (36), (38) and (40), after the differential coefficient with respect to Z_1 has been obtained and then taking $Z_1=1$.

$$\lim_{Z_1\to 1} P^{*'}(1;0,Z_1,1) = \frac{1}{2\bar{c}(1-\rho_1)[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\{\rho_1 c_2[\lambda_2\bar{d}(\rho_1+\rho_2\bar{c}) + K\alpha_2\rho_2\bar{c}]+\lambda_1^2\bar{c}^3\beta_1[\lambda_2\bar{d}(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\lambda_1\lambda_2\rho_1\beta_2\bar{d}\bar{c}^2 \times[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]\} \quad (41)$$

$$\lim_{Z_1\to 1} P^{*'}(2;0,Z_1,1) = \frac{\lambda_1\lambda_2\bar{c}\bar{d}\beta_2}{2} \quad (42)$$

$$\lim_{Z_1\to 1} P^{*'}(3;0,Z_1,1) = \frac{\lambda_1\lambda_2\bar{d}\bar{c}^2 K\alpha_2\beta_2(1-\rho_1-\rho_2)}{2[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \quad (43)$$

Putting $Z_1 = 1$ in equations (36), (38) and (40), after the differential coefficient with respect to Z_2 has been obtained and then taking $Z_2 = 1$.

$$\lim_{Z_2\to 1} P^{*'}(1;0,1,Z_2) = \frac{\lambda_2(D_1+\bar{d}^2(1-\rho_2)D_2)}{2\lambda_1\bar{d}\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} + \frac{\rho_1}{\alpha_1}\left\{\frac{1}{\bar{c}(1-\rho_1-\rho_2)} - \frac{(\lambda_2-K\alpha_1)}{[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\right\} \times[\lambda_2\bar{d}(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}] + \frac{d_2\rho_1\rho_2}{2\bar{d}(1-\rho_1-\rho_2)} \quad (44)$$

where $D_1 = \lambda_1\lambda_2\bar{c}^2\bar{d}^3\beta_2\rho_1(2-\rho_1-\rho_2)[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]$ and

$$D_2 = \lambda_1^2\bar{c}^3\beta_1[\lambda_2\bar{d}(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\rho_1^2 c_2[\lambda_2\bar{d}(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]$$

$$\lim_{Z_2\to 1} P^{*'}(2;0,1,Z_2) = \frac{\rho_2[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}-\bar{c}(1-\rho_1-\rho_2)(\lambda_2-K\alpha_1)]}{\alpha_1\bar{c}(1-\rho_1-\rho_2)} + \frac{\lambda_2^2\bar{d}^2\beta_2}{2} + \frac{\lambda_2\bar{d}^2\rho_2}{2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \times\left\{\frac{D_2}{\lambda_1\bar{d}\bar{c}^2}+\lambda_2\beta_2[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]\right\} + \frac{\rho_2 d_2(1-\rho_1)}{2\bar{d}(1-\rho_1-\rho_2)} \quad (45)$$

$$\lim_{Z_2\to 1} P^{*'}(3;0,1,Z_2) = \frac{K\alpha_2\bar{c}(1-\rho_1-\rho_2)[\lambda_2^2\bar{d}^2\alpha_1\beta_2+2\rho_2(\alpha_1(K-1)-\lambda_2)]}{2\alpha_1[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} + \frac{K\alpha_2\rho_2}{\alpha_1} \quad (46)$$

From (32) and (33)

$$Q'(1) = \frac{\lambda_2\bar{d}[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}-\bar{c}(1-\rho_1-\rho_2)(\lambda_2-K\alpha_1)]}{\alpha_1[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \quad (47)$$

Theorem: 2

The marginal means of the two dimensional random variables (N_p, N_r) has been obtained as:

Mean number of customers in the priority queue is

$$E(N_p) = \frac{1}{2\bar{c}(1-\rho_1)[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\{\rho_1 c_2[\lambda_2\bar{d}(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}] + \lambda_1^2\bar{c}^3\beta_1[\lambda_2\bar{d}(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\lambda_1\lambda_2\beta_2\bar{d}\bar{c}^2[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]\} \quad (48)$$

Mean number of customers in the orbit is

$$E(N_r) = \frac{\left\{\lambda_1\lambda_2^2\bar{c}^2\bar{d}^2\beta_2[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]+\lambda_2\bar{d}D_2\right\}}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}+K$$
$$+\frac{\left\{\lambda_2[\rho_1\bar{d}+\rho_2\bar{c}+\bar{c}(\bar{d}-1)(1-\rho_1)]+K\alpha_2\rho_2\bar{c}\right\}}{\alpha_1\bar{c}(1-\rho_1-\rho_2)}+\frac{d_2\rho_2}{2\bar{d}(1-\rho_1-\rho_2)} - \frac{K\alpha_2\rho_2\bar{c}(1-\rho_1-\rho_2)}{[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \quad (49)$$

Mean busy period is

$$E(T_b) = \frac{[\lambda_2\bar{d}(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]}{\lambda\lambda_2\bar{d}\bar{c}(1-\rho_1-\rho_2)} \quad (50)$$

Mean waiting time of a tagged type I customer in the priority queue is

$$E(W_p) = \frac{1}{2\lambda_1\bar{c}^2(1-\rho_1)[\lambda_2\bar{d}(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\{\rho_1c_2[\lambda_2\bar{d}(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]$$
$$+\lambda_1^2\bar{c}^3\beta_1[\lambda_2\bar{d}(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\lambda_1\lambda_2\beta_2\bar{d}\bar{c}^2[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]\}$$

(51)

Mean waiting time of a tagged type II customer in the orbit is

$$E(W_r) = \frac{\left\{\lambda_1\lambda_2\bar{c}^2\bar{d}\beta_2[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]+D_2\right\}}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}+\frac{K}{\lambda_2\bar{d}}$$
$$+\frac{\left\{\lambda_2[\rho_1\bar{d}+\rho_2\bar{c}+\bar{c}(\bar{d}-1)(1-\rho_1)]+K\alpha_2\rho_2\bar{c}\right\}}{\lambda_2\bar{d}\alpha_1\bar{c}(1-\rho_1-\rho_2)}+\frac{d_2\rho_2}{2\lambda_2\bar{d}^2(1-\rho_1-\rho_2)}$$
$$-\frac{K\alpha_2\rho_2\bar{c}(1-\rho_1-\rho_2)}{\lambda_2\bar{d}[\lambda_2\bar{d}(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \quad (52)$$

Proof:

Equations (48) is obtained by adding (41), (42) and (43), equations (49) is obtained by adding (44), (45), (46) and (47).

Busy period T_b is the length of the time interval that keeps the server busy continuously and this continues till the instant the server becomes free again and let T_0 be the length of the idle period. For this model, T_b and T_0 generates an alternating renewal process and therefore

$$\frac{E(T_b)}{E(T_0)} = \frac{Pr\{T_b\}}{1-Pr\{T_b\}} = \frac{P_B}{1-P_B}$$

But $E(T_0) = \frac{1}{\lambda}$

$$E(T_b) = \frac{P_B}{\lambda(1-P_B)} \quad (53)$$

Using corollary on equation (53), we get the equation (50). Equation (51) is obtained using $\frac{E(N_p)}{\lambda_1\bar{c}}$ and Equation (52) obtained by using $\frac{E(N_r)}{\lambda_2\bar{d}}$.

6. NUMERICAL STUDY

In this section, some numerical examples related to the model analyzed in this article are given. For the sake of convenience, it has been assumed that the type I and type II service times are exponentially distributed random variables with mean $\frac{1}{\mu_1}$ and $\frac{1}{\mu_2}$ and the batch size distributions are geometrically distributed with parameters θ_1 and θ_2. In order to see the effect of the parameters type I service rate μ_1, type II service rate μ_2, type I arrival rate λ_1, type II arrival rate λ_2 on the mean number of customers in the priority queue, the mean number of customers in the orbit, the mean busy period, the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit, the probability that the server is idle and the probability that the server is busy of the model discussed in this paper by fixing the values of the retrial rates α_1, α_2 and K, θ_1, θ_2. Some numerical results are obtained. The results are presented in graphs and tables. The figures 1 to 3 (4 to 6) represents the surface of type I arrival rate, type I service rate (type II service rate) and the mean number of customers in the priority queue, and the mean number of customers in the orbit and the mean busy period respectively. The figures 7 to 9 (10 to 12) represents the surface of type II arrival rate, type I service rate (type II service rate)and the mean number of customers in the priority queue, the mean number of customers in the orbit, and the mean busy period respectively. The figures 13 and 14 (19 and 20) represent the surface of type I arrival rate, type I service rate (type II arrival rate, type II service rate) and the mean waiting time of a tagged type I customer in the priority queue, mean waiting time of tagged type II customer in the orbit respectively (mean waiting time of a tagged type I customer in the priority queue, mean waiting time of type II customer in the orbit respectively). Figures 1, 4 shows that as a type I arrival rate increases, the mean number of customers in the priority queue increases steadily for small value of type I service rate, that is, 4 whereas the increment is too small for comparatively big values, that is, like 8. The same situation has been encountered in the case of type II service rate. Figure 7 represents the surface of type II arrival rate, type I service rate and the mean number of customers in the priority queue is a convex surface for increasing the values of type I service rate, type II service rate and the mean number of customers in the priority queue is convex with respect to type II service rate but increases with respect to type II arrival rate. The surface of the mean number of customers in the priority queue, type I arrival rate (type II arrival rate) and type I service rate in the figures 2 (8) is almost a convex surface at the comparatively large value of type I arrival rate and smallest value of type I service rate (is a flat and increasing surface for increasing value of the type II arrival rate). Figures 5 and 11 represent the surface area of the mean number of customers in the orbit versus type I arrival rate versus type II service rate and the mean number of customers in the orbit versus type II arrival rate versus type II service rate respectively. The first surface is convex with respect to type II service rate but slightly increases for the increasing value of type I arrival rate, whereas the second surface is again convex with respect to type II service rate and increases with respect to type II arrival rate for smallest values of type II service rate. Figures 3, 6, 9 and 12 represent the surface of the mean busy period versus type I arrival rate or type II arrival rate versus type I service rate or type II service rate. All the surfaces are convex with respect to service rate and are increased for increasing values of arrival rate. Figures 13 and 14 shows that as the type I arrival rate increases the mean waiting time of a tagged type I customer in the priority queue and the mean waiting time of a tagged type II customer in the orbit increases, whereas with respect type I service rate, the figures are concave for large value of type I arrival rate. Figures 15 and 16 shows with respect to type I arrival rate, the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit are increasing function but with respect to type II service rate their concave in nature. Figures 17 and 18 shows the figures the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit are increasing with respect to type II arrival rate but concave with respect to type I service rate. Finally figures 19 and 20 respects the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit are increasing with respect to type II arrival rate and concave with respect to type I service rate. Tables 1-4 shows the probabilities that the server is idle and the server is busy for the various values of $\lambda_1, \lambda_2, \mu_1$ and μ_2. From tables it can be seen that, for increasing values of type I arrival rate and type II arrival rate, for fixed values of type I service rate and type II service rate the idle probabilities decreases whereas the busy probability increases as expected.

Figure.1.Mean number of customers in priority Queue for Different Type I arrival, Type I service rate

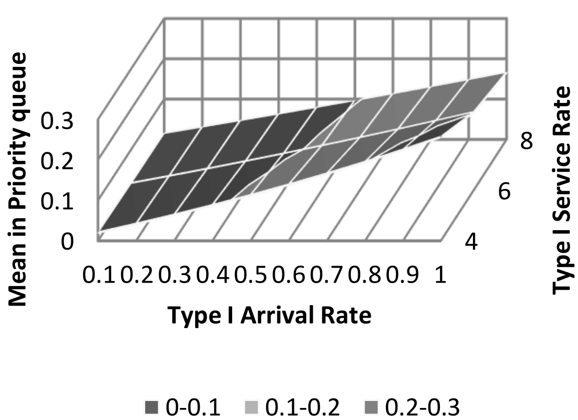

Figure.2.Mean number of customers in orbit for Different Type I arrival , Type I service rate

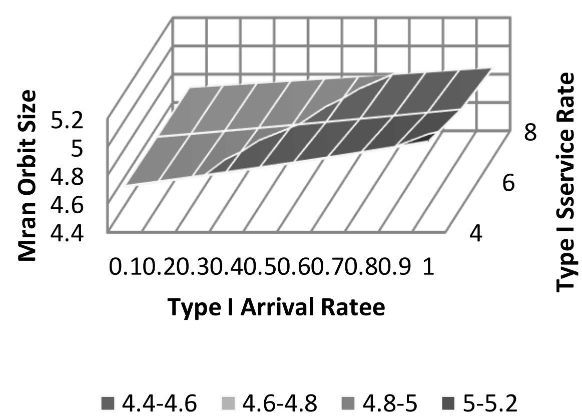

Figure.3.Mean Busy period for Different Type I arrival, Type I service rate

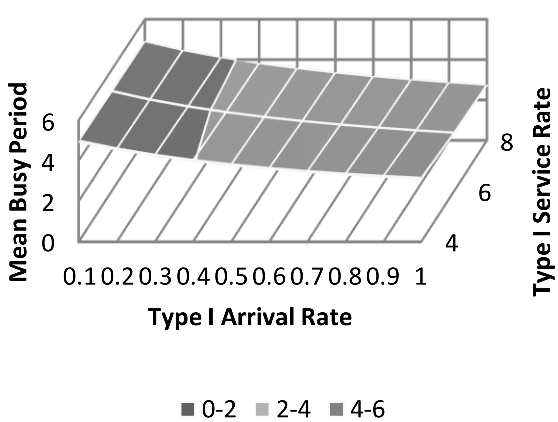

Figure.4.Mean number of customers in priority queue for Different Type I arrival, Type II service rate

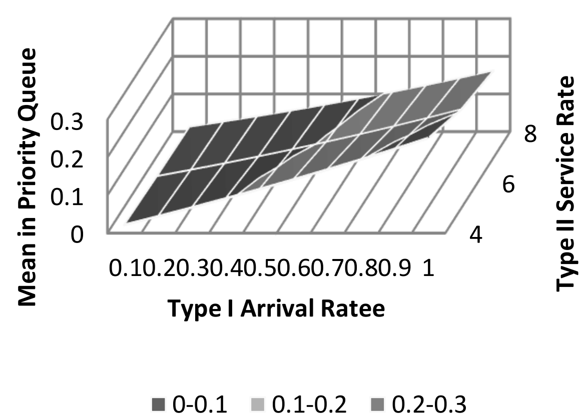

Figure.5.Mean number of customers in the orbit for Different Type I arrival, Type II service rate

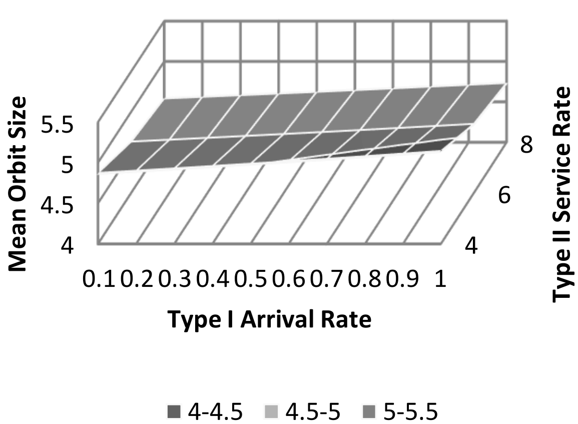

Figure.6.Mean busy periodfor Different Type I arrival, Type II service rate

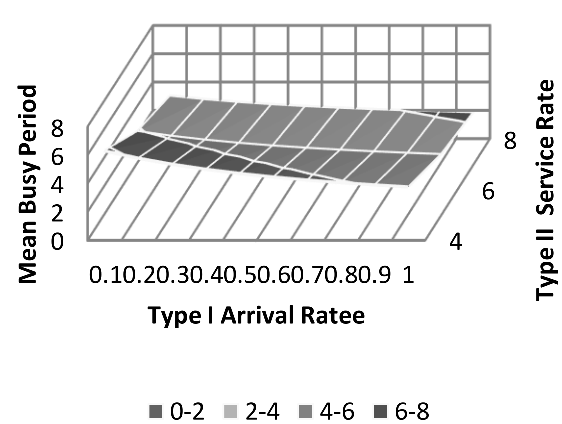

Figure.7.ean number of customers in priority Queue for Different Type II arrival, Type I service rate

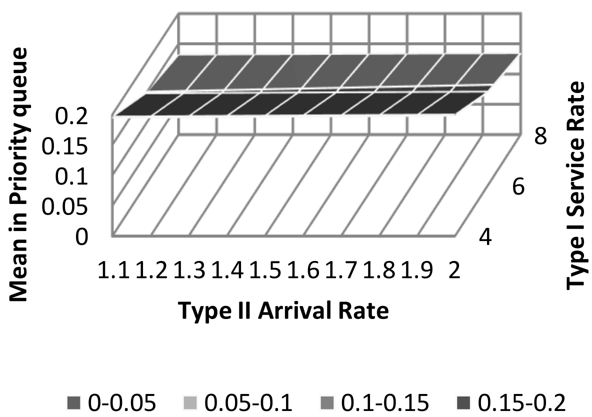

Figure.8.Mean number of customers in orbit for Different Type II arrival ,Type I service rate

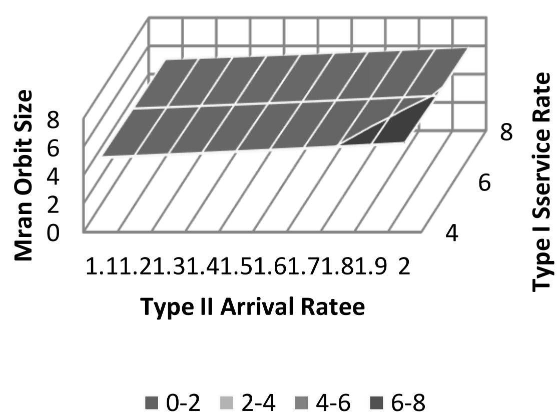

Figure.9.Mean Busy period for Different Type I arrival, Type I service rate

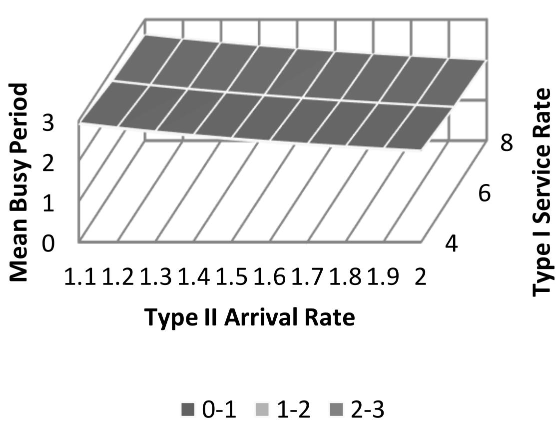

Figure.10.Mean number of customers in priority queue for Different Type IIarrival ,Type II service rate

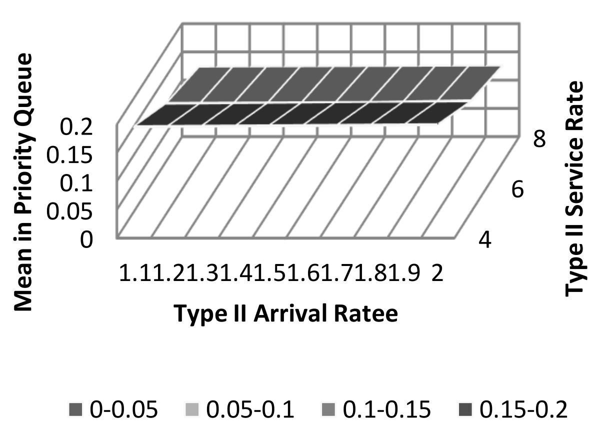

Figure.11.Mean number of customers in the orbit for Different Type I arrival ,Type II service rate

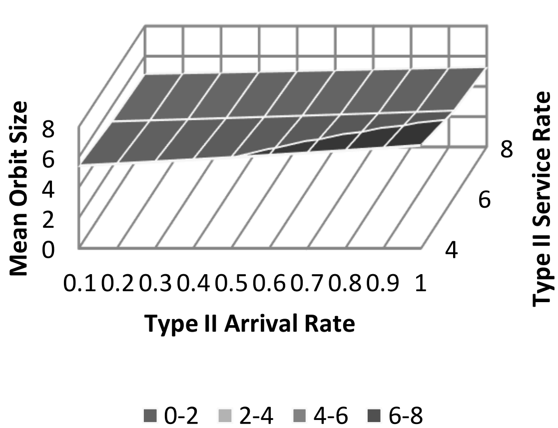

Figure.12.Mean busy periodfor Different Type II arrival, Type II service rate

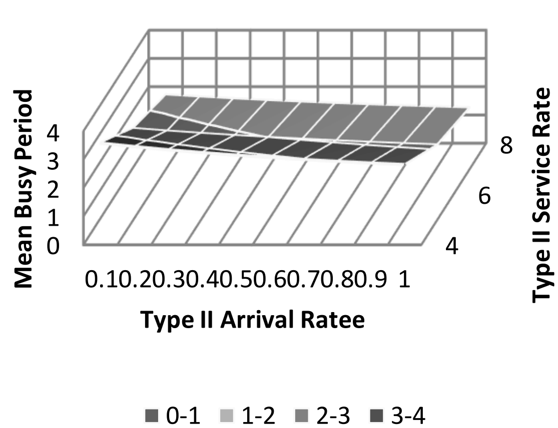

Figure.13.Mean waiting time of a tagged type I customer in the priority queue for Different Type I arrival, Type I service rate

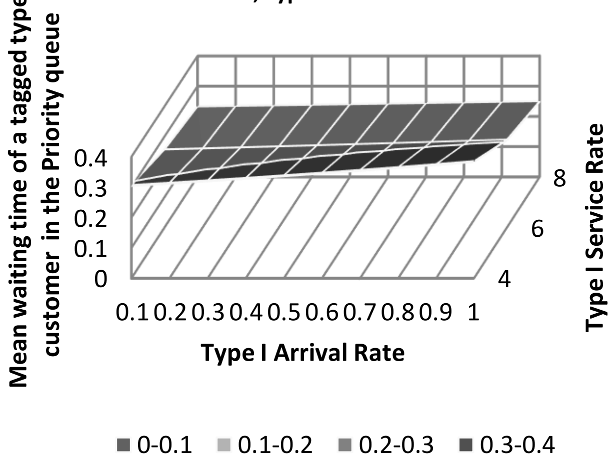

Figure.14.Mean waiting time of a tagged type II customer in the orbit for Different Type I arrival, Type I service rate

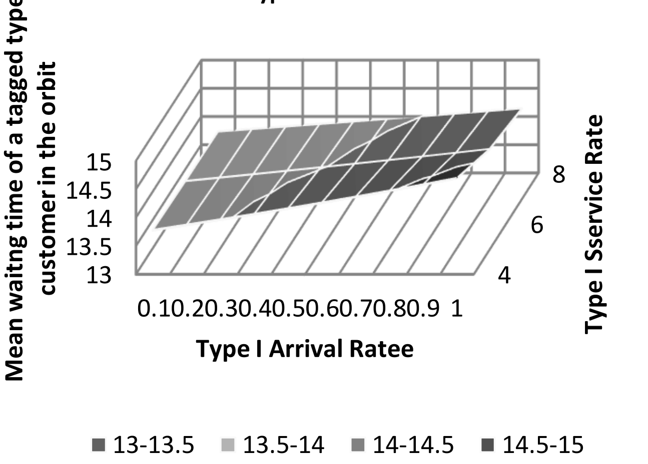

Figure.15.Mean waiting time of a tagged type I customer in the priority queue for Different Type I arrival, Type II service rate

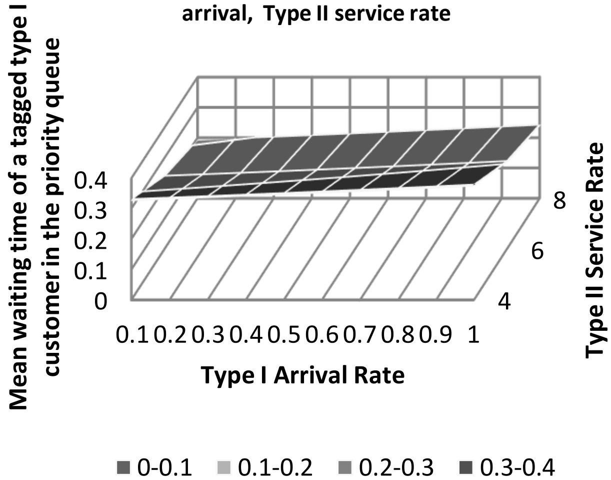

Figure.16.Mean waiting time of a tagged type II customers in the orbit for Different Type I arrival, Type II service rate

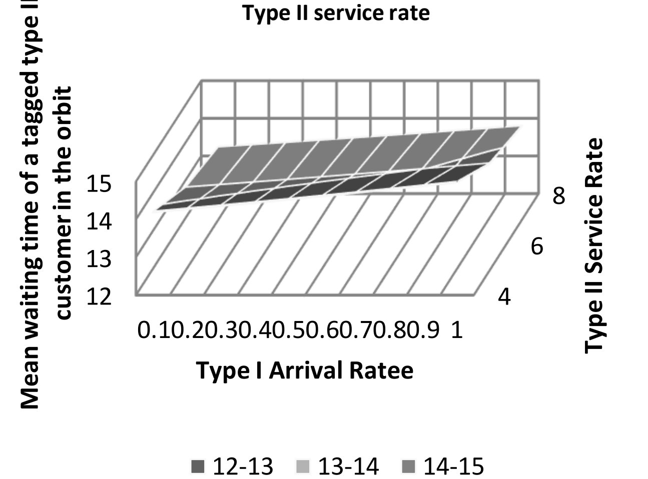

Figure.17.Mean waiting time of a tagged type I customers in the priority queue for Different Type II arrival ,Type I service rate

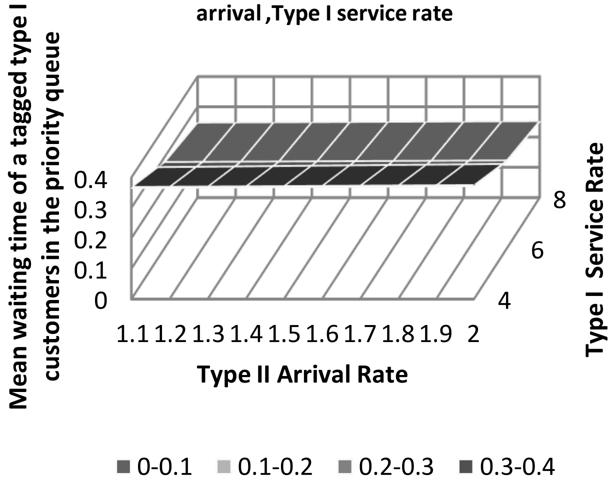

Figure.18.Mean waiting time of a tagged type II customers in the orbit for Different Type II arrival, Type I service rate

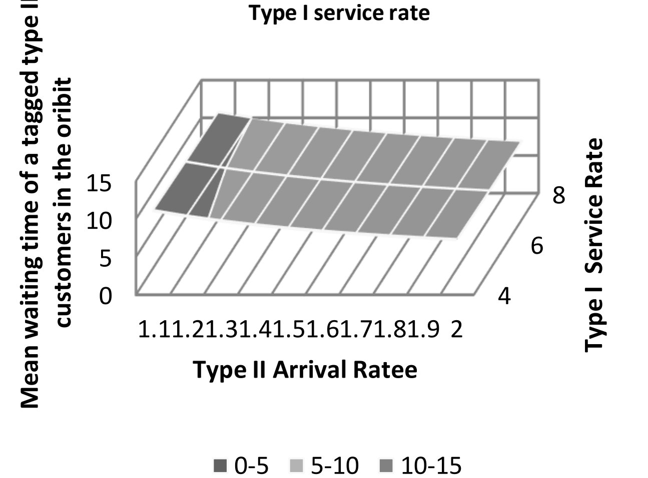

Figure.19.Mean waiting time of a tagged type I customer in the priority queue for Different Type II arrival, Type II service rate

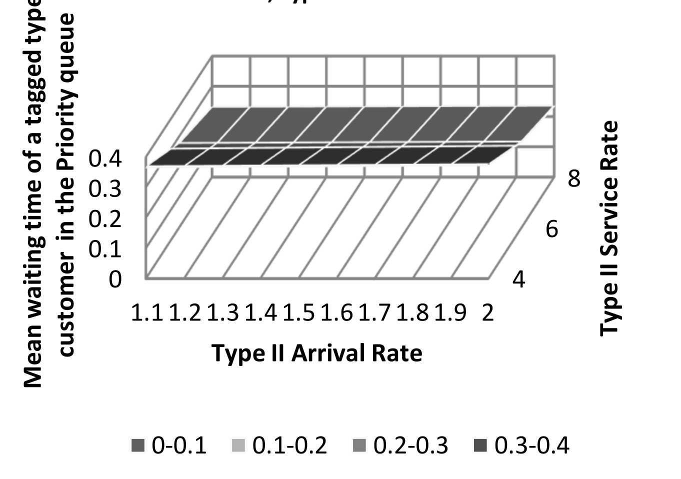

Figure.20.Mean waiting time of a tagged type II customer in the orbit for Different Type II arrival, Type II service rate

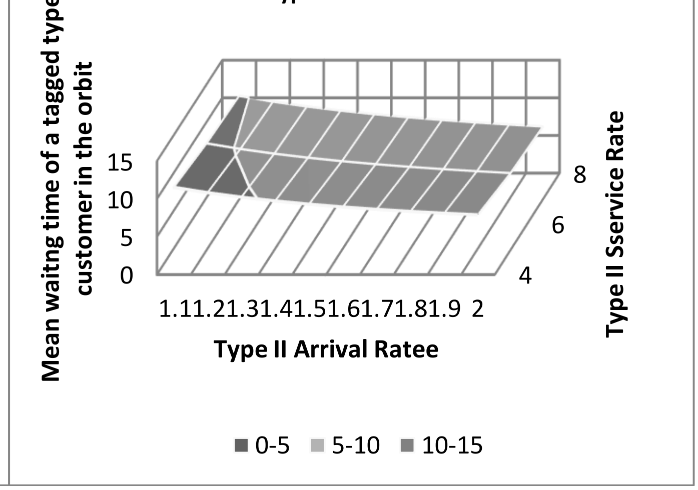

Table 1: The probabilities P_I & P_B

	$\alpha_1 = 3.0, \alpha_2 = 4.0, \lambda_2 = 0.8, \mu_2 = 5.0, \theta_1 = 0.4, \theta_2 = 0.3$					
λ_1	$\mu_1 = 4.0$		$\mu_1 = 6.0$		$\mu_1 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.1826	0.8174	0.1839	0.8161	0.1845	0.8155
0.2	0.1790	0.8210	0.1814	0.8186	0.1826	0.8174
0.3	0.1754	0.8246	0.1790	0.8210	0.1808	0.8192
0.4	0.1718	0.8282	0.1766	0.8234	0.1790	0.8210
0.5	0.1682	0.8318	0.1742	0.8258	0.1772	0.8228
0.6	0.1646	0.8354	0.1718	0.8282	0.1754	0.8246
0.7	0.1611	0.8389	0.1694	0.8306	0.1736	0.8264
0.8	0.1575	0.8425	0.1670	0.8330	0.1718	0.8282
0.9	0.1540	0.8460	0.1646	0.8354	0.1700	0.8300
1.0	0.1504	0.8496	0.1623	0.8377	0.1682	0.8318

Table 2: The probabilities P_I & P_B

	$\alpha_1 = 3.0, \alpha_2 = 4.0, \lambda_1 = 0.8, \mu_1 = 5.0, \theta_1 = 0.4, \theta_2 = 0.3$					
λ_1	$\mu_1 = 4.0$		$\mu_1 = 6.0$		$\mu_1 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.0150	0.8500	0.2142	0.7858	0.2691	0.7309
0.2	0.1476	0.8524	0.2108	0.7892	0.2648	0.7352
0.3	0.1452	0.8548	0.2074	0.7926	0.2606	0.7394
0.4	0.1429	0.8571	0.2040	0.7960	0.2563	0.7437
0.5	0.1409	0.8595	0.2007	0.7993	0.2520	0.7480
0.6	0.1381	0.8619	0.1973	0.8027	0.2478	0.7522

0.7	0.1358	0.8642	0.1940	0.8060	0.2436	0.7564
0.8	0.1334	0.8666	0.1906	0.8094	0.2393	0.7607
0.9	0.1311	0.8689	0.1873	0.8127	0.2352	0.7648
1.0	0.1287	0.8713	0.1840	0.8160	0.2310	0.7690

Table 3: The probabilities P_I & P_B

	$\alpha_1 = 3.0, \alpha_2 = 4.0,\ \lambda_1 = 0.8,\ \mu_1 = 5.0,\ \theta_1 = 0.4,\ \theta_2 = 0.3$					
λ_2	$\mu_1 = 4.0$		$\mu_1 = 6.0$		$\mu_1 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.1524	0.8476	0.1619	0.8381	0.1667	0.8333
0.2	0.1508	0.8492	0.1602	0.8398	0.1650	0.8350
0.3	0.1491	0.8509	0.1585	0.8415	0.1633	0.8367
0.4	0.1474	0.8526	0.1568	0.8432	0.1616	0.8384
0.5	0.1457	0.8543	0.1551	0.8449	0.1599	0.8401
0.6	0.1440	0.8560	0.1534	0.8466	0.1582	0.8418
0.7	0.1423	0.8577	0.1517	0.8483	0.1565	0.8435
0.8	0.1406	0.8594	0.1500	0.8500	0.1548	0.8452
0.9	0.1389	0.8611	0.1483	0.8517	0.1531	0.8469
1.0	0.1372	0.8628	0.1466	0.8534	0.1514	0.8486

Table 4: The probabilities P_I & P_B

	$\alpha_1 = 3.0, \alpha_2 = 4.0,\ \lambda_1 = 0.8,\ \mu_1 = 5.0,\ \theta_1 = 0.4,\ \theta_2 = 0.3$					
λ_2	$\mu_2 = 4.0$		$\mu_2 = 6.0$		$\mu_2 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.1281	0.8719	0.1857	0.8143	0.2348	0.7652
0.2	0.1263	0.8737	0.1841	0.8159	0.2333	0.7667
0.3	0.1246	0.8754	0.1825	0.8175	0.2318	0.7682
0.4	0.1228	0.8772	0.1809	0.8191	0.2303	0.7697
0.5	0.1210	0.8790	0.1792	0.8208	0.2288	0.7712
0.6	0.1193	0.8807	0.1776	0.8224	0.2273	0.7727
0.7	0.1175	0.8825	0.1760	0.8240	0.2258	0.7742
0.8	0.1157	0.8843	0.1743	0.8257	0.2243	0.7757
0.9	0.1139	0.8861	0.1727	0.8273	0.2228	0.7772
1.0	0.1122	0.8878	0.1711	0.8289	0.2213	0.7787

7. CONCLUSION

In the foregoing analysis, a $M/G/1$ queue with retrial customers, with two types of batch arrivals and with a finite number recurrent repeated customers are considered. We obtain the queue length distribution and mean queue length. Extensive numerical work has been carried out to observe the trends of the operating characters of the system.

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