EFFECT OF MISCLASSIFICATION DUE TO MEASUREMENT ERROR ON THE POWER OF CONTROL CHART FOR PROPORTIONS (ATTRIBUTES)

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ABSTRACT
The *p* chart plays the important role in controlling the proportion of defective items produced. In recent years researchers in various quality control procedures consider the possibility of misclassification errors as an important issue. In the present paper, a method of obtaining the expression of the power of control chart for binomial distribution (proportion of defectives) is being studied by considering approximate expressions for calculating the probabilities of errors of misclassification due to measurement error. Formulae are derived for calculating probabilities of misclassification due to measurement error. The relationship between apparent fraction defective (AFD) and true fraction defective (TFD) has been used to study the power of control chart.

KEYWORDs: Measurement error, misclassification, binomial distribution, power.

MSC: 62P30

RESUMEN
Las p-cartas juegan un rol importante en el control de la proporción de artículos defectuosos producidos. En años recientes investigadores de varios procedimientos de control de la calidad consideran la posibilidad de errores de mala clasificación como un aspecto importante. En el presente trabajo, un método para obtener la expresión de la potencia de la carta de control para la distribución binomial (proporción de defectuosos) ha sido estudiada considerando expresiones aproximadas para calcular las probabilidades de error de mala clasificación debido a errores de medición. Fórmulas son derivadas para calcularlas. La relación entre la aparente fracción de defectuosos (AFD) y la verdadera (TFD) son usadas para estudias la potencia de la carta de control.

1. INTRODUCTION

Statistical techniques now-a-days have been successfully applied to different production processes in industries to achieve desired quality levels of manufactured products with an optimum production cost. It is widely acknowledged within the industrial process, processes produced are often contaminated with measurement error which can introduce serious bias in the derived results. The nature and magnitude of measurement error and its effect on the actual performance of various control charts can be overwhelming and studied by several researchers. For a recent and brief review see Maravelakis (2012). See also Sankle et al. (2012) and Chakraborty and Khurshid (2013 a, b) and references therein.
To employ statistical techniques, inspections are made on the finished products, during the time of production or after the production. In every inspection system, there may be either of two possible types of errors: (i) a good (conforming) item to a specification may be misclassified as defective

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(nonconforming) or (ii) a defective (nonconforming) item may be misclassified as good (conforming). These types of errors are classified as misclassification errors (or inspection errors) and are generally due to chance causes and can be estimated (Sankle and Singh, 2012).

Misclassification is a special case of measurement error. Apparently there is no unified theory that encompasses the key elements of misclassification error which is usually studied separately from measurement error, though there is clearly much overlap. Misclassification errors may significantly alter the performance of (attribute) control charts, as has been investigated by several authors, including Dorris and Foote (1978), Case (1980), Schneider and Tang (1987), Suich (1988) and Johnson et al. (1991). Recently Chen et al. (2011) studied inspection errors in multinomial control charts. More recently Balamurali and Kalyanasundaran (2011) studied the effect of misclassification error on the operating characteristic (OC) curve of analysis of means (ANOM).

The *p* chart plays the vital role in controlling the proportion of defective items produced. Singh et al. (2002) illustrated cumulative sum control charts for proportions under inspection error (see also Singh and Sayyed, 2001 for cumulative sum control charts for Poisson variables under inspection error). In the present paper, the power of control chart for binomial distribution (proportion of defectives) is being studied by considering approximate expressions for calculating the probabilities of errors of misclassification due to measurement error. The relationship between apparent fraction defective (AFD) and true fraction defective (TFD) has been used to study the power of control chart.

2. TERMINOLOGY AND FEW ASSUMPTIONS

Summarizing the following notation to be used throughout this paper, will facilitate further development. The misclassification error may be of two types: P_1 (type I error) is the probability of classifying a good item as a defective one and P_2 (type II error) is the probability of classifying a defective item as good one. Further AFD (apparent fraction defective) is the proportion of defective items if error of misclassification is presented is denoted by π and TFD (true fraction defective) is the proportion of defective items when there is no error of misclassification and is denoted by $P.$ It is obvious that $AFD = TFD$, if the error of misclassification is zero.

It is assumed that the measurements have been taken only to classify the production items into acceptable and rejectable units with certain specifications that can be expressed in terms of mean and standard deviation of the measurable quality characteristics.

The quality characteristic x is normally distributed with mean μ and standard deviation σ_p

$$f(x)\,dx = \frac{1}{\sigma_p\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma_p}\right)^2\right]dx.$$

The variable v is normally distributed with mean x and standard deviation σ_e

$$f(v)\,dv = \frac{1}{\sigma_e\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{v-x}{\sigma_e}\right)^2\right]dv.$$

The units beyond $x = \mu \pm K\sigma_p$ are defective and the units within $x = \mu \pm K\sigma_p$ are non-defective.

3. EVALUATING PROBABILITIES OF MISCLASSIFICATION

Here we have classified the production process, after measurement into one of the two categories. They are either conforming (good) or non-conforming (defective) units. If P_1 is the probability of misclassification of a conforming unit and P_2 is the probability of misclassification of a non-conforming unit, then owing Singh (1964) with the above mentioned assumptions (Section 2), P_1 and P_2 can be evaluated as

$$P_1 = \int_{-K\sigma_p}^{K\sigma_p} f(x)\,dx\,[1 - \int_{-K\sigma_p}^{K\sigma_p} f(v)\,dv]$$

(1)

and

$$P_2 = \int_{K\sigma_p}^{\infty} f(x)\,dx \int_{-K\sigma_p}^{K\sigma_p} f(v)\,dv + \int_{-\infty}^{-K\sigma_p} f(x)\,dx \int_{-K\sigma_p}^{K\sigma_p} f(v)\,dv .$$

(2)

In fact P_1 and P_2 are the inspection risks, which are the type I and type II errors and take the values between 0 and 1.

The approximate expressions for P_1 and P_2 (Singh, 1964) are:

$$P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\}$$

(3)

and

$$P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\}$$

(4)

where

$$a = \sigma_e / \sigma_p \;,\; h = \frac{K\sigma_p}{\sqrt{\sigma_e^2 + \sigma_p^2}} \;,\; \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}v^2\right] dv$$

and

$$T(h,a) = \frac{1}{\sqrt{2\pi}} \int_0^a \frac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2}\,dx .$$

Here, $1.5 \le K \le 3$ and $(\sigma_e/\sigma_p) \le 0.5$ hold good for finding P_1 and P_2. Singh (1964) studied measurement error in acceptance sampling plan and calculated P_1 and P_2 based on the graphic representation of the probabilities of misclassification data for different values of K and $a = \sigma_e/\sigma_p$.

It has been shown by Lavin (1946) that due to misclassification error, the probability of acceptance of the lot will be obtained by replacing true fraction defective (TFD) P by the apparent fraction defective (AFD) π where

$$\pi = P(1 - P_2) + P_1(1 - P).$$

π yields a random variable X whose binomial distribution has parameter π instead of P. See also Collins and Case (1976), Johnson et al. (1991) and Mittag and Rinne (1993) for published material based on Lavin equation.

4. POWER OF CONTROL CHART FOR PROPORTIONS DATA

The data can often represented by a binomial distribution if it consists number or proportion of units having a specific attribute. In this section we develop the expression for the power of control chart for binomial (p-chart) under misclassification due to measurement error. Recently Khoo (2013) presented power functions for Shewhart control chart.

Following Kanazuka (1986), the power of detecting the change of process for the control chart is given by

$$P_d = P\left\{X \geq UCL\right\} + P\left\{X \leq LCL\right\}$$

where UCL and LCL are upper and lower control limits respectively.

Thus under misclassification the control limits for binomial (p-chart) are $\pi \pm K\sqrt{\frac{\pi(1-\pi)}{n}}$ and centre line CL is π. Hence, the power of the control chart under misclassification is

$$P_d = \left[1 - \sum_{x_e=0}^{UCL-1}\binom{n}{x_e}\pi^{x_e}(1-\pi)^{n-x_e}\right] + \sum_{x_e=0}^{LCL}\binom{n}{x_e}\pi^{x_e}(1-\pi)^{n-x_e}. \quad (5)$$

The operating characteristic (OC) curve, under misclassification, which illustrates the probability that a sample fraction defective x_e/n, will fall within control limits as a function of the error process fraction defective π is given by

$$P_e(\pi) = \sum_{x_e=LCL}^{UCL}\binom{n}{x_e}\pi^{x_e}(1-\pi)^{n-x_e}. \quad (6)$$

5. EXAMPLE AND ILLUSTRATION

Consider the data for fraction defective (p -chart), where 4 samples, each of size 15 were inspected and number of defectives along with proportions of defectives are obtained as follows:

Sample #	Number of defects (d_i)	Fraction defectives $P_i = (d_i/n)$
1	1	0.07
2	4	0.27
3	2	0.13
4	5	0.33

Here overall sample proportion of defectives is $p = \overline{p} = 0.2$ and its standard deviation is $\sigma_P = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.10.$ For our analysis we have kept $p = \overline{p} = 0.2$, the overall sample proportion of defective fixed and the values of n being changed in different situations to see the effect of the size of the sample on the power of control chart.

6. CALCULATIONS AND CONCLUSIONS

To obtain the power of control chart (P_d) and operating characteristic (OC) curve ($P_e(\pi)$) under misclassification error we have to first find $\pi = P(1-P_2)+P_1(1-P)$ based on the approximate expressions for P_1 and P_2 (Equations 3 and 4).

Table 1 gives the values of $h = \frac{K}{\sqrt{a^2+1}}$ for different combinations of $a = \sigma_e / \sigma_p$ and $T(h,a)$. Here we have used Monte Carlo simulation to find $T(h,a)$. True values of fraction defective P can be obtained from the normal probability table for different values of K. The values of P_1 and P_2 for different combinations of $T(h,a)$ and $\Phi(h)$ for fixed K have been tabulated in Table 1. It has been observed from Table 1 (A-G) that for fixed K, the values of P_1 and P_2 show a decreasing trend if the measurement error $a = \sigma_e / \sigma_p$ decreases. On the other hand, we also observe that for fixed $a = \sigma_e / \sigma_p$ the values of P_1 is greater than P_2 and when $h \cong K$ then $P_1 = P_2$.

Table 1: Values of $T(h,a) = \frac{1}{\sqrt{2\pi}} \int_0^a \frac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2} dx$, $\Phi(h)$,

$P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\}$ and $P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\}$

Table 1-A: When $K = 1.5$ and $\Phi(K) = 0.9332$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.5	1.34	0.07039360	0.9099	0.16408720	0.11748720
0.4	1.39	0.05503907	0.9177	0.12557814	0.09457814
0.3	1.44	0.04001047	0.9251	0.08812094	0.07192094
0.25	1.46	0.03294319	0.9279	0.07118638	0.06058638
0.20	1.47	0.02635462	0.9292	0.05670924	0.04870924
0.15	1.48	0.01970635	0.9306	0.04201270	0.03681270
0.10	1.49	0.01305494	0.9319	0.02740988	0.02480988
0.05	1.50	0.00646451	0.9332	0.01292902	0.01292902

Table 1-B: When $K = 1.75$ and $\Phi(K) = 0.9599$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.5	1.57	0.04915861	0.9418	0.11641722	0.08021722
0.4	1.63	0.03763664	0.9484	0.08677328	0.06377328
0.3	1.68	0.02722368	0.9535	0.06084736	0.04804736
0.25	1.70	0.02237561	0.9554	0.04925122	0.04025122
0.20	1.72	0.01759671	0.9573	0.03779342	0.03259342
0.15	1.73	0.01315372	0.9582	0.02800744	0.02460744
0.10	1.74	0.008706727	0.9591	0.018213454	0.016613454
0.05	1.75	0.004304809	0.9599	0.008609618	0.008609618

Table 1-C: When $K = 2.0$ and $\Phi(K) = 0.9772$

$a = \sigma_e / \sigma_p$	$h = \dfrac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	1.79	0.03308721	0.9633	0.08007442	0.05227442
0.40	1.86	0.02471443	0.9686	0.05802886	0.04082886
0.30	1.92	0.01745997	0.9726	0.03951994	0.03031994
0.25	1.94	0.01433163	0.9738	0.03206326	0.02526326
0.20	1.96	0.01125022	0.9750	0.02470044	0.02030044
0.15	1.98	0.008244447	0.9761	0.017588894	0.015388894
0.10	1.99	0.00545368	0.9767	0.01140736	0.01040736
0.05	2.00	0.002692772	0.9772	0.005385544	0.005385544

Table 1-D: When $K = 2.25$ and $\Phi(K) = 0.9878$

$a = \sigma_e / \sigma_p$	$h = \dfrac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.02	0.020711060	0.9783	0.050922120	0.031922120
0.40	2.09	0.015359960	0.9817	0.036819920	0.024619920
0.30	2.16	0.010555770	0.9846	0.024311540	0.017911540
0.25	2.18	0.008656291	0.9854	0.019712582	0.014912582
0.20	2.20	0.006785243	0.9861	0.015270486	0.011870486
0.15	2.23	0.004852139	0.9871	0.010404278	0.009004278
0.10	2.24	0.003208399	0.9875	0.006716798	0.006116798
0.05	2.25	0.001582424	0.9878	0.003164848	0.003164848

Table 1-E: When $K = 2.50$ and $\Phi(K) = 0.9938$

$a = \sigma_e / \sigma_p$	$h = \dfrac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.24	0.012561410	0.9875	0.031422820	0.018822820
0.40	2.33	0.008819303	0.9901	0.021338606	0.013938606
0.30	2.40	0.006015980	0.9918	0.014031960	0.010031960
0.25	2.43	0.004810231	0.9925	0.010920462	0.008320462
0.20	2.45	0.003766157	0.9929	0.008432314	0.006632314
0.15	2.48	0.002681432	0.9934	0.005762864	0.004962864
0.10	2.49	0.001772875	0.9936	0.003745750	0.003345750
0.05	2.50	0.008734041	0.9938	0.001746808	0.001746808

Table 1-F: When $K = 2.75$ and $\Phi(K) = 0.9970$

$a = \sigma_e / \sigma_p$	$h = \dfrac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.47	0.0070560530	0.9932	0.017912106	0.010312106
0.40	2.56	0.0049003870	0.9948	0.012000774	0.007600774
0.30	2.64	0.0032323650	0.9959	0.007564730	0.005364730
0.25	2.67	0.0025776230	0.9962	0.005955246	0.004355246
0.20	2.70	0.0019622790	0.9965	0.004424558	0.003424558
0.15	2.72	0.0014301680	0.9967	0.003160336	0.002560336
0.10	2.74	0.0009200657	0.9969	0.001940131	0.001740131
0.05	2.75	0.0004528698	0.9970	0.000905740	0.000905740

Table 1-G: When $K = 3.00$ and $\Phi(K) = 0.9987$

$a = \sigma_e / \sigma_p$	$h = \dfrac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.69	0.003860856	0.9964	0.010021712	0.005421712
0.40	2.79	0.002578258	0.9974	0.006456516	0.003856516
0.30	2.88	0.001637422	0.9980	0.003974844	0.002574844
0.25	2.91	0.001302675	0.9982	0.003105350	0.002105350
0.20	2.94	0.000988815	0.9984	0.002277630	0.001677630
0.15	2.97	0.000698636	0.9985	0.001597271	0.001197271

0.10	2.99	0.000448474	0.9986	0.000996947	0.000796947
0.05	3.00	0.000220578	0.9987	0.000441156	0.000441156

Note: The function $T(h,a)$ has been tabulated by Owen (1956) and Smirnov and Bolsev (1962). Interested readers may obtain a simple QBASIC program from the first author.

The relationship between apparent fraction defective (AFD) and true fraction defective (TFD) is shown in Table 2 and Figure 1.

Table 2: Relationship between $TFD(=P)$ and $AFD(=\pi)$ when $K=2.0$

P	π	π	π
	$a=0.5$	$a=0.10$	$a=0.15$
0	0.005385	0.011407	0.017589
0.01	0.015280	0.021119	0.027259
0.02	0.025170	0.030971	0.036929
0.03	0.035116	0.040753	0.046600
0.04	0.044955	0.050535	0.056270
0.05	0.054846	0.060317	0.065940

It is observed that for fixed K and $a=\sigma_e/\sigma_p$ as the values of the true fraction defective (P) increase, the values of π i.e., apparent (observed) fraction defective also increase and also for fixed P, as the values of measurement error $a=\sigma_e/\sigma_p$ increase, there is considerable increase in the values of π.

Figure 1: Relationship between apparent fraction defective (AFD) and true fraction defective (TFD)

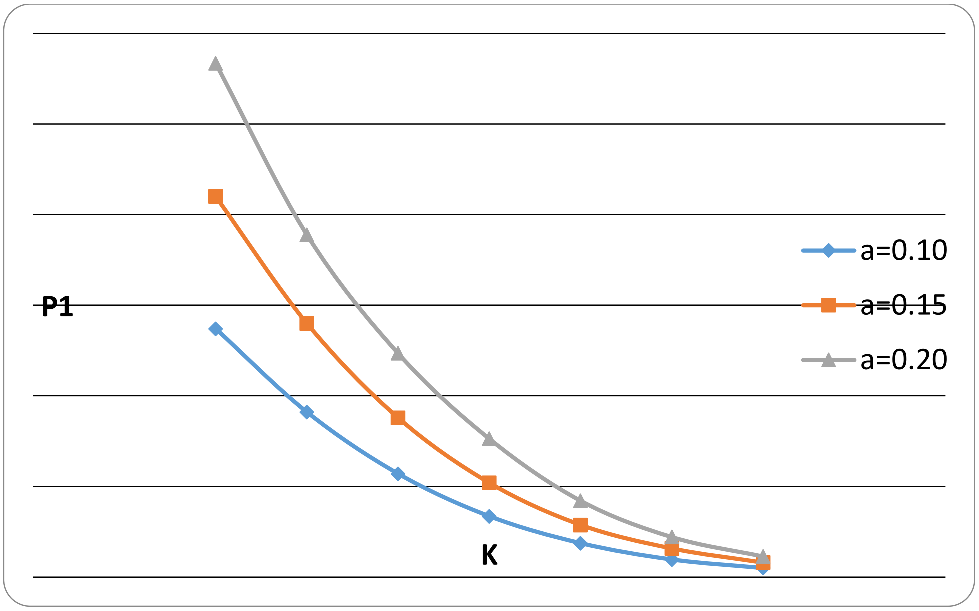

Table 3 and Figure 2 (A and B) depict the effect of K on probabilities of misclassification of conforming units (P_1) and non-conforming units (P_2). For fixed $a=\sigma_e/\sigma_p$, if we increase the values of K, there is a decreasing trend for P_1 but for fixed K, the values of P_1 increase as $a=\sigma_e/\sigma_p$ is increased. The same trend being observed for Table 3B. shows the graphic representation between K and probabilities of misclassification. One can also calculate P_1 and P_2 from the graphs (Figure 2) by knowing the standard deviation σ_e of measurement error (which assumes same for all the values of K).

Table 3: Probabilities of misclassification of conforming units (P_1) and non conforming units (P_2) for different values of K and $a = \sigma_e / \sigma_p$.

K	$a = 0.10$		$a = 0.15$		$a = 0.20$	
	P_1	P_2	P_1	P_2	P_1	P_2
1.50	0.027409880	0.024809880	0.042012700	0.036812700	0.056709240	0.048709240
1.75	0.018213454	0.016613454	0.028007440	0.024607440	0.037793420	0.032593420
2.00	0.011407360	0.010407360	0.017588894	0.015388894	0.024700440	0.020300440
2.25	0.006716798	0.006116798	0.010404278	0.009004278	0.015270486	0.011870486
2.50	0.003745750	0.003345750	0.005762864	0.004962864	0.008432314	0.006632314
2.75	0.001940131	0.001740131	0.003160336	0.002560336	0.004424558	0.003424558
3.00	0.000996947	0.000796947	0.001597271	0.001197271	0.002277630	0.001677630

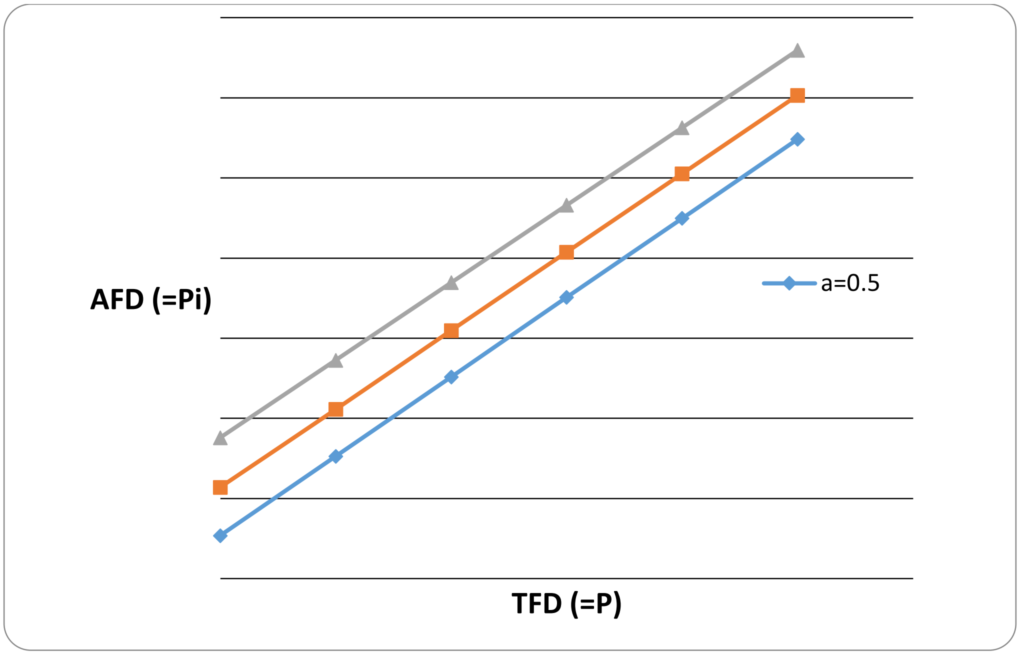

Figure 2 (A): The effect of K on probabilities of misclassification of conforming units (P_1)

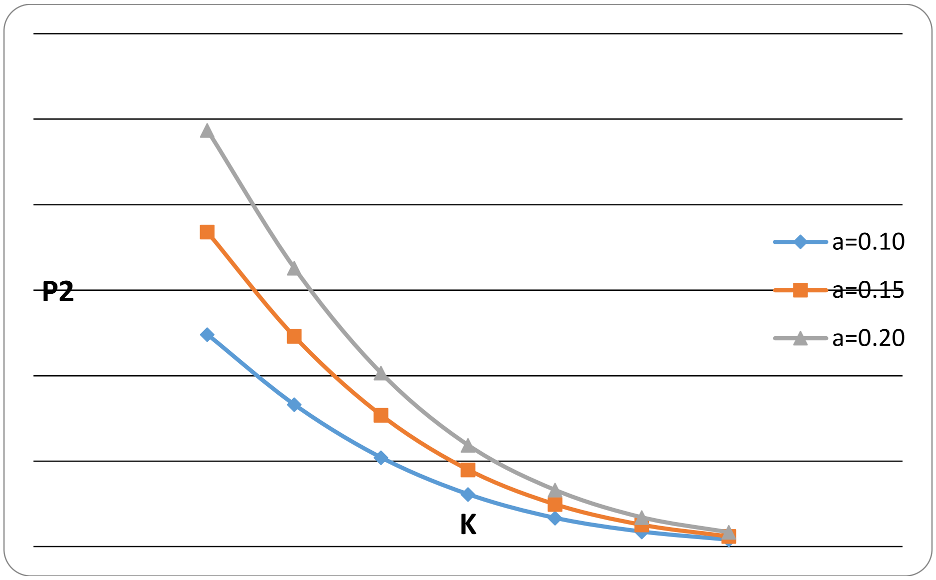

Figure 2 (B): The effect of K on probabilities of misclassification of conforming units (P_2)

Table 4 (A-D) gives us the idea how the values of AFD (π) values influence the control limits for fraction defective charts. It has been observed from the table that for fixed K, the values of both UCL and LCL increase as there is an increase in the values of $a = \sigma_e / \sigma_p$. For fixed $a = \sigma_e / \sigma_p$, the difference between UCL and LCL increases as we go on increasing K when the corresponding values of π decrease (which depends on P_1, P_2 and P). It is observed that the range of UCL and LCL is less when the size of the sample is increased.

Table 4: Values of π and control limits (LCL and UCL) for different values of K, P_1, P_2, $a = \sigma_e / \sigma_p$ and n for fixed $p = \bar{p} = 0.2$

Table 4 A: $a = \sigma_e / \sigma_p = 0.05$

				$n=15$		$n=50$	
K	P_1	P_2	π	LCL	UCL	LCL	UCL
1.50	0.012929020	0.012929020	0.2078	0.0506	0.3649	0.1217	0.2939
1.75	0.008609618	0.008609618	0.2052	0.0227	0.3877	0.1053	0.3051
2.00	0.005385544	0.005385544	0.2032	0	0.4110	0.0894	0.3170
2.25	0.003164848	0.003164848	0.2019	0	0.4351	0.0742	0.3296
2.50	0.001746808	0.001746808	0.2010	0	0.4597	0.0593	0.3427
2.75	0.000905740	0.000905740	0.2006	0	0.4849	0.0449	0.3563
3.00	0.000441156	0.000441156	0.2003	0	0.5102	0.0305	0.3701

Table 4 B: $a = \sigma_e / \sigma_p = 0.15$

				$n=15$		$n=50$	
K	P_1	P_2	π	LCL	UCL	LCL	UCL
1.50	0.042012700	0.036812700	0.2262	0.0642	0.3882	0.1375	0.3149
1.75	0.028007440	0.024607440	0.2175	0.0311	0.4039	0.1154	0.3195
2.00	0.017588894	0.015388894	0.2110	0.0003	0.4217	0.0956	0.3264
2.25	0.010404278	0.009004278	0.2065	0	0.4417	0.0777	0.3353
2.50	0.005762864	0.004962864	0.2036	0	0.4635	0.0612	0.3460
2.75	0.003160336	0.002560336	0.2020	0	0.4871	0.0459	0.3581
3.00	0.001597271	0.001197271	0.2010	0	0.5114	0.0310	0.3710

Table 4 C: $a = \sigma_e / \sigma_p = 0.25$

				$n=15$		$n=50$	

K	P_1	P_2	π	LCL	UCL	LCL	UCL
1.50	0.071186380	0.060586380	0.2448	0.0783	0.4113	0.1536	0.3360
1.75	0.049251220	0.040251220	0.2314	0.0408	0.4220	0.1270	0.3358
2.00	0.032063260	0.025263260	0.2210	0.0067	0.4353	0.1036	0.3384
2.25	0.019712582	0.014912582	0.2128	0	0.4506	0.0826	0.3430
2.50	0.010920462	0.008320462	0.2071	0	0.4687	0.0638	0.3504
2.75	0.005955246	0.004355246	0.2039	0	0.4900	0.0472	0.3606
3.00	0.003105350	0.002105350	0.2021	0	0.5132	0.0317	0.3725

Table 4 D: $a = \sigma_e / \sigma_p = 0.50$

				$n=15$		$n=50$	
K	P_1	P_2	π	LCL	UCL	LCL	UCL
1.50	0.164087200	0.117487200	0.3078	0.1290	0.4866	0.2099	0.4057
1.75	0.116417220	0.080217220	0.2771	0.0822	0.4793	0.1663	0.3879
2.00	0.080074420	0.052274420	0.2536	0.0289	0.4783	0.1305	0.3767
2.25	0.050922120	0.031922120	0.2344	0	0.4805	0.0996	0.3692
2.50	0.031422820	0.018822820	0.2214	0	0.4894	0.0746	0.3682
2.75	0.017912106	0.010312106	0.2123	0	0.5027	0.0533	0.3713
3.00	0.010021712	0.005421712	0.2069	0	0.5207	0.0350	0.3788

Table 5 (A-K) shows the different values of power of control chart (P_d) for the corresponding values of π. Here we observe how power curve (P_d) changes for different values of n, K, $a = \sigma_e / \sigma_p$, UCL and LCL. From the Table 5 (A, B, C) it is observed that values of P_d go on decreasing as we increase K (K =1.5 to K =3) for fixed $a = \sigma_e / \sigma_p$ and $P_1 = P_2$. Also no change in the values of P_d being observed if there is marginal increase in the values of $a = \sigma_e / \sigma_p$ for fixed n and fixed K. But if we increase the size of the sample (Table 5 D) for fixed K and $P_1 = P_2$ there is a change in the values of P_d. The values of the power (P_d) is less if the size of the sample is larger for fixed $a = \sigma_e / \sigma_p$. It is also understood from the Table 5 (E), that the values of the power (P_d) is more, if n increased along with the value of $a = \sigma_e / \sigma_p$.

It has been observed from the Table 5 that as we go on increasing the shift of the process parameter, there is an increase in the power of the control chart for fixed $a = \sigma_e / \sigma_p$, n, K, UCL and LCL. Thus, smaller the deviation, smaller the power of the test. As we increase $a = \sigma_e / \sigma_p$, keeping other parameters fixed, it has been observed from the table that P_2 (type II error) value increases for fixed deviation, and P_2 values tend to decrease as there is an increase in deviation. Higher values of P_2 may involve cost. Thus, where it is necessary to have a sample of small size, P_d should be set at a relatively high level so that the resultant P_2 value does not become a matter of excessive concern. Increase in the sample size n also shifts the power curve upward. Graphical representation for some values of P_d for the Table 5 (A, B, C) is shown in Figure 3 for different values of K.

Table 5: Power of control chart for proportions under misclassification (due to measurement error)

	Table: 5 A $a = \sigma_e / \sigma_p = 0.05$ $p = \bar{p} = 0.2$ $n = 15$, $K = 1.5$	**Table: 5 B** $a = \sigma_e / \sigma_p = 0.05$ $p = \bar{p} = 0.2$ $n = 15$, $K = 2$	**Table: 5 C** $a = \sigma_e / \sigma_p = 0.05$ $p = \bar{p} = 0.2$ $n = 15$, $K = 3$

	$P_1 = P_2 = 0.01292902$ $CL = 3.1,\ UCL = 5,$ $LCL = 1$	$P_1 = P_2 = 0.005385544$ $CL = 3,\ UCL = 5,\ LCL = 0$	$P_1 = P_2 = 0.0004411564$ $CL = 3.005,\ UCL = 7,$ $LCL = 0$
π	P_d	P_d	P_d
0.01	0.9904	0.8601	0.8601
0.02	0.9674	0.7386	0.7386
0.04	0.8811	0.5421	0.5421
0.05	0.8296	0.4634	0.4633
0.07	0.7196	0.3370	0.3367
0.09	0.6117	0.2443	0.2432
0.10	0.5617	0.2081	0.2062
0.15	0.3354	0.1042	0.0910
0.20	0.3313	0.0963	0.0533
0.25	0.3937	0.1618	0.0700
0.35	0.6623	0.4373	0.2468
0.45	0.8813	0.7393	0.5479
0.50	0.9413	0.8491	0.6964
0.65	0.9972	0.9876	0.9578
0.75	0.9999	0.9992	0.9958

	Table: 5 D $a = \sigma_e / \sigma_p = 0.05$ $p = \bar{p} = 0.2$ $n = 20$, $K = 1.5$ $P_1 = P_2 = 0.01292902$ $CL = 4.156,$ $UCL = 7,\ LCL = 1$	**Table: 5 E** $a = \sigma_e / \sigma_p = 0.50$ $p = \bar{p} = 0.2$ $n = 20$, $K = 1.5$ $P_1 = 0.1640872$, $P_2 = 0.1174872$ $CL = 6.16,$ $UCL = 9,\ LCL = 3$	**Table: 5 F** $a = \sigma_e / \sigma_p = 0.50$ $p = \bar{p} = 0.2$ $n = 20$, $K = 3$ $P_1 = 0.010021712$, $P_2 = 0.005421712$ $CL = 4,\ UCL = 9,$ $LCL = 0$
π	P_d	P_d	P_d
0.01	0.9831	1.0000	0.8179
0.02	0.9401	0.9994	0.6676
0.04	0.8103	0.9926	0.4420
0.05	0.7358	0.9841	0.3585
0.10	0.3941	0.8671	0.1217
0.20	0.1559	0.4214	0.0215
0.25	0.2385	0.2661	0.0441
0.35	0.5855	0.2820	0.2378
0.40	0.7505	0.4204	0.4044
0.45	0.8702	0.5906	0.5857
0.50	0.9432	0.7496	0.7483
0.65	0.9887	0.9804	0.9804

	Table: 5 G $a = \sigma_e / \sigma_p = 0.15$ $p = \bar{p} = 0.2$ $n = 50$, $K = 1.5$ $P_1 = 0.0420127$, $P_2 = 0.0368127$ $CL = 11.31,\ UCL = 19,\ LCL = 3$	**Table: 5 H** $a = \sigma_e / \sigma_p = 0.15$ $p = \bar{p} = 0.2$ $n = 50$, $K = 3$ $P_1 = 0.0015972714$, $P_2 = 0.0011972714$ $CL = 10,\ UCL = 25,\ LCL = 0$

π	P_d	P_d
0.01	0.9984	0.6050
0.02	0.9822	0.3642
0.05	0.7604	0.0769
0.07	0.5327	0.0266
0.10	0.2503	0.0052
0.20	0.0082	0
0.25	0.0293	0.0001
0.35	0.3784	0.0207
0.40	0.6644	0.0978
0.50	0.9675	0.5561
0.60	0.99948	0.942656
0.65	0.99997	0.989956
0.75	0.9999	0.999962

	Table: 5 I (when $AFD = TFD$) $\pi = p = \overline{p} = 0.2$ $n = 15$, $K = 1.5$ $CL = 3, UCL = 5,$ $LCL = 1$	**Table: 5 J** (when $AFD = TFD$) $\pi = p = \overline{p} = 0.2$ $n = 20$, $K = 1.5$ $CL = 3,$ $UCL = 6,\ LCL = 1$	**Table: 5 K** (when $AFD = TFD$) $\pi = p = \overline{p} = 0.2$ $n = 50$, $K = 3$ $CL = 3, UCL = 18,$ $LCL = 2$
$\pi = p$	P_d	P_d	P_d
0.01	0.9904	0.9904	0.9862
0.02	0.9647	0.9647	0.9216
0.05	0.8296	0.7361	0.5404
0.10	0.5617	0.4030	0.1117
0.20	0.3313	0.2650	0.0004
0.25	0.3937	0.4071	0.0552
0.35	0.6623	0.7567	0.4940
0.40	0.7879	0.8749	0.7631
0.45	0.8813	0.9948	0.9235
0.50	0.9413	0.9793	0.9836
0.65	0.9973	0.9997	0.9999

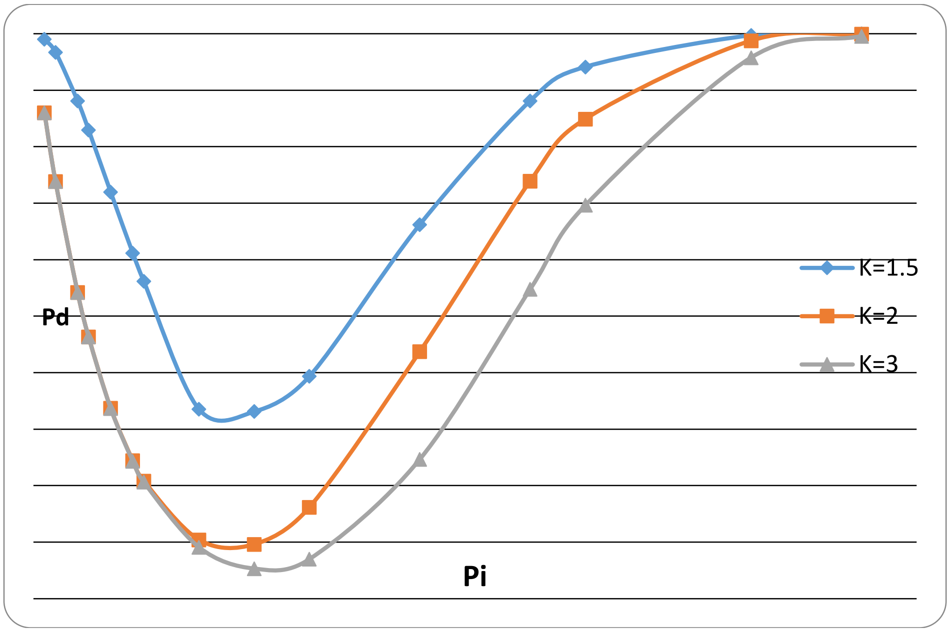

Figure 3: Relationship between π and P_d for the Table 5 (A, B, C) for different values of

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