MODIFIED RATIO ESTIMATOR UNDER RANK SET SAMPLING

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ABSTRACT
This article deals with the estimation of population mean under rank set sampling using an improved ratio estimator. The expression for bias and mean square error has been obtained up to first degree of approximation. An efficiency comparison has also been considered for the proposed estimator with the classical ratio estimator under rank set sampling. Finally an empirical study is carried out to judge the performance of the proposed estimator.

Keywords: Rank set sampling, Deciles, Mean Square Error.

MSC: 62D05

1. INTRODUCTION:

The idea of ranked set sampling was first proposed by McIntyre (1952) in his effort to find a more efficient method to estimate the yield of pastures. Measuring yield of pasture plots requires mowing and weighing the hay which is time-consuming. But an experienced person may rank by eye inspection fairly accurately the yields of a small number of plots without actual measurement. McIntyre adopted the following sampling scheme:

Each time, a random sample of *k* pasture lots is taken and the lots are ranked by eye inspection with respect to the amount of yield.

From the jth- sample, the lot with rank j is taken for cutting and weighing.

Usually, for obtaining a sample of size n=mk, when each of the ranks from j to *k* has an associated lot being taken for cutting and weighing, the cycle repeats over again and again until a total of m cycles are completed. McIntyre illustrated the gain in efficiency by a computation involving five distributions. He observed that the relative efficiency, defined as the ratio of the variance of the mean of a simple random sample and the variance of the mean of a ranked set sample of the same size, is not much less than *(k +1)/2* for symmetric or moderately asymmetric distributions, and that the relative efficiency diminishes with increasing asymmetry of the underlying distribution but is always greater than 1. In recent past a lot of research has been done in RSS. See for example Bouza (2008), Al-Omari *et al*. (2008, 2009), Jeelani et al. (2013, 2014a, 2014b, 2014c) and Jeelani & Bouza (2015).

2. RATIO ESTIMATION IN RANK SET SAMPLING

Ratio method of estimation is an important type of estimation in sample surveys which utilizes information on an auxiliary variable, with a view on increasing the precision of the estimate of population mean. The famous classical ratio estimator proposed by Samawi and Muttalak (1996) under rank set sampling is given by;

$$\hat{\bar{y}}_{rss} = \frac{\bar{y}_{rss}}{\bar{x}_{rss}} \tag{2.1}$$

where $\bar{y}_{rss} = \frac{1}{mk}\sum_{i=1}^{k} Y_i$ and $\bar{x}_{rss} = \frac{1}{mk}\sum_{i=1}^{k} X_i$, assuming the population mean of the auxiliary variable is known the equation above changes to ;

$$\hat{\bar{y}}_{rss} = \frac{\bar{y}_{rss}}{\bar{x}_{rss}}\bar{X} = \hat{R}_{rss}\bar{X} \tag{2.2}$$

The mean square error of equation (2.2) is given by;

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$$MSE(\hat{\bar{y}}_{rss}) = \bar{Y}\left[\frac{1}{mk}\left\{\left(\frac{S_y}{\bar{Y}}\right)^2 + \left(\frac{S_X}{X}\right)^2 - 2\left(\frac{S_{yx}}{S_yS_x}\right)\langle\frac{S_y}{\bar{Y}}\rangle\langle\frac{S_X}{X}\rangle\right\} - \left\{\omega_{y[i]} - \omega_{x(i)}\right\}^2\right] \quad (2.3)$$

where, $S_y = \sqrt{\frac{\sum_{i=1}^{N}(y_i-\bar{Y})^2}{N-1}}$, $S_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\bar{X})^2}{N-1}}$, $S_{yx} = \sqrt{\frac{\sum(y_i-\bar{Y})(x_i-\bar{X})}{N-1}}$, $\omega_{x(i)}^2 = \frac{1}{k^2m}\frac{1}{\bar{X}^2}\sum_{i=1}^{k}\delta_{x(i)}^2$, $\omega_{y[i]}^2 = \frac{1}{k^2m}\frac{1}{\bar{X}^2}\sum_{i=1}^{k}\delta_{y(i)}^2$, $\omega_{yx(i)} = \frac{1}{k^2m}\frac{1}{\bar{Y}\bar{X}}\sum_{i=1}^{k}\delta_{yx(i)}$, $\delta_{x(i)} = \left(\mu_{x(i)} - \bar{X}\right)$, $\delta_{y[i]} = \left(\mu_{y[i]} - \bar{Y}\right)and\delta_{yx(i)} = \left(\mu_{x(i)} - \bar{X}\right)\left(\mu_{y[i]} - \bar{Y}\right)$

3. THE PROPOSED ESTIMATOR:

In this article we propose a new ratio estimator under RSS based on deciles of an auxiliary variable, which is given below;

$$\hat{\bar{y}}_{\eta rss} = \eta\,\frac{\bar{y}_{rss}}{\bar{x}_{rss}}\bar{X} = \hat{R}_{\tau rss}\bar{X} \quad (3.1)$$

where, $\eta = \left(\frac{\bar{X}}{\bar{x}+1}\right)D_i$, $where\ \ D_i = l + \frac{h}{f}\left(\frac{i\times N}{10} - C\right)$, $i = 1\ to\ 9$

here, $l =$ lower limit of the frequency distribution, $f =$ is the frequency, $h =$ is the magnitude of class containing D_i, $C =$ is the cumulative frequency of the class proceeding the class containing D_i

The Bias and Mean Square Error (MSE) of the proposed estimator can be obtained by using the following Taylor series development

$$\bar{y}_{rss} = \bar{Y}(1+\varepsilon_0)\ ;\ \bar{x}_{rss} = \bar{X}(1+\varepsilon_1)\ ;\ E(\varepsilon_0) = E(\varepsilon_1) = 0 \quad (3.3)$$

Then

$$V(\varepsilon_0) = E({\varepsilon_0}^2) = \frac{V(\bar{y}_{rss})}{\bar{Y}^2} = \frac{1}{km}\frac{1}{\bar{Y}^2}\left[\frac{1}{k}\sum_{i=1}^{k}\delta_{y[i]}^2\right] = \left|\frac{1}{km}\left(\frac{S_y}{\bar{Y}}\right)^2 - \omega_{y[i]}^2\right| \quad (3.4)$$

Similarly we obtain that

$$V(\varepsilon_1) = E({\varepsilon_1}^2) = \left|\frac{1}{km}\left(\frac{S_x}{\bar{X}}\right)^2 - \omega_{x(i)}^2\right| \quad (3.5)$$

$$Cov(\varepsilon_0,\varepsilon_1) = E(\varepsilon_0,\varepsilon_1) = \frac{Cov(\bar{y}^*\bar{x}^*)}{\bar{Y}\bar{X}} = \frac{1}{\bar{Y}\bar{X}}\frac{1}{km}\left[\sigma_{yx} - \frac{1}{k}\sum_{i=1}^{k}\delta_{yx(i)}\right] = \left(\frac{1}{km}\left(\frac{S_{yx}}{S_yS_x}\right)\left(\frac{S_y}{\bar{Y}}\right)\left(\frac{S_x}{\bar{X}}\right) - \omega_{yx(i)}\right) \quad (3.6)$$

where,

$S_y = \sqrt{\frac{\sum_{i=1}^{N}(y_i-\bar{Y})^2}{N-1}}$, $S_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\bar{X})^2}{N-1}}$,$S_{yx} = \sqrt{\frac{\sum(y_i-\bar{Y})(x_i-\bar{X})}{N-1}}$,$\omega_{x(i)}^2 = \frac{1}{k^2m}\frac{1}{\bar{X}^2}\sum_{i=1}^{k}\delta_{x(i)}^2$, $\omega_{y[i]}^2 = \frac{1}{k^2m}\frac{1}{\bar{X}^2}\sum_{i=1}^{k}\delta_{y(i)}^2$, $\omega_{yx(i)} = \frac{1}{k^2m}\frac{1}{\bar{Y}\bar{X}}\sum_{i=1}^{k}\delta_{yx(i)}$, $\delta_{x(i)} = \left(\mu_{x(i)} - \bar{X}\right)$, $\delta_{y[i]} = \left(\mu_{y[i]} - \bar{Y}\right)and\delta_{yx(i)} = \left(\mu_{x(i)} - \bar{X}\right)\left(\mu_{y[i]} - \bar{Y}\right)$

Now we assume that the sample size is large enough to get the errors $\varepsilon_0, \varepsilon_1$ sufficiently as small for supporting that the terms involving $\varepsilon_0, \varepsilon_1$ in a degree greater than two aree negligible, then; also,

$$\hat{\bar{y}}_{nrss} = \eta\,\frac{\bar{y}_{rss}}{\bar{x}_{rss}}\bar{X} = \eta\frac{\bar{Y}(1+\iota_0)}{\bar{X}(1+\iota_1)}\bar{X} = \eta\bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{-1} \quad (3.7)$$

$$Bais\ (\hat{\bar{y}}_{nrss}) \cong (\eta-1)\bar{Y} - \eta\bar{Y}\left(\frac{1}{km}\left(\frac{S_x}{\bar{X}}\right)^2 - \omega_{x(i)}^2 - \frac{1}{km}\left(\frac{S_{yx}}{S_yS_x}\right)\langle\frac{S_y}{\bar{Y}}\rangle\langle\frac{S_X}{X}\rangle + \omega_{yx(i)}\right) \quad (3.8)$$

Then the mean square of$\hat{\bar{y}}_{nrss}$ is given by

$$MSE(\hat{\bar{y}}_{nrss}) = E(\hat{\bar{y}}_{nrss} - R_{rss})^2 \quad (3.9)$$

$\because MSE(X) = E(\bar{x}-\mu)^2$ $\quad E(\hat{\bar{y}}_{nrss} - R_{rss})^2 = \bar{Y}^2E(\varepsilon_0 - \eta\varepsilon_1 + \eta^2{\varepsilon_1}^2 - 2\eta\varepsilon_0\varepsilon_1)^2 = \bar{Y}^2E({\varepsilon_0}^2 + \eta^2{\varepsilon_1}^2 - \eta\varepsilon_0\varepsilon_1)$=

$$= \bar{Y}^2\left(\frac{1}{km}\left(\frac{S_y}{\bar{Y}}\right)^2 - \omega_{y[i]}^2 + \eta^2\left(\frac{1}{km}\left(\frac{S_x}{\bar{X}}\right)^2 - \omega_{x(i)}^2\right) - 2\eta\left(\frac{1}{km}\left(\frac{S_{yx}}{S_yS_x}\right)\langle\frac{S_y}{\bar{Y}}\rangle\langle\frac{S_X}{\bar{X}}\rangle - \omega_{yx(i)}\right)\right)$$

$$= \bar{Y}^2\left[\frac{1}{km}\left\{\left(\frac{S_y}{\bar{Y}}\right)^2 + \eta^2\left(\frac{S_x}{\bar{X}}\right)^2 - 2\left(\frac{S_{yx}}{S_yS_x}\right)\langle\frac{S_y}{\bar{Y}}\rangle\langle\frac{S_X}{\bar{X}}\rangle\right\} - \left\{\omega_{y[i]}^2 + \eta^2\omega_{x(i)}^2 - 2\eta\omega_{yx(i)}\right\}\right]=$$

$$= \left[\frac{1}{km}\left\{\left(\frac{S_y}{\bar{Y}}\right)^2 + \eta^2\left(\frac{S_x}{\bar{X}}\right)^2 - 2\left(\frac{S_{yx}}{S_yS_x}\right)\langle\frac{S_y}{\bar{Y}}\rangle\langle\frac{S_X}{\bar{X}}\rangle\right\} - \left\{\omega_{y[i]} - \eta\omega_{x(i)}\right\}^2\right] \quad (3.10)$$

3. EFFICIENCY COMPARISON:

Let

$$ב = \bar{Y}^2\left[\frac{1}{km}\left\{\left(\frac{S_y}{\bar{Y}}\right)^2 + \eta^2\left(\frac{S_x}{\bar{X}}\right)^2 - 2\left(\frac{S_{yx}}{S_yS_x}\right)\langle\frac{S_y}{\bar{Y}}\rangle\langle\frac{S_X}{\bar{X}}\rangle\right\} - \left\{\omega_{y[i]} - \eta\omega_{x(i)}\right\}^2\right],$$

If ב is non-negative, the mean square error of (3.1) is less than (2.2). Provided that ב ≥ 0.
$MSE(\hat{\hat{y}}_{rss}) - MSE(\hat{\hat{y}}_{nrss}) =$ ב ≥ 0.
Hence approximately, $\{\omega_{y[i]} - \eta\omega_{x(i)}\}^2$ב ≥ 0 and $\cong \omega_{y[i]} \geq \eta\omega_{x(i)}$.

4. AN EMPIRICAL STUDY :

For obtaining an empirical illustration 800 replicates from a population taken from Singh and Chaudhary (1986), page number 177, with $\mu_x = 170.32, \mu_y = 654.63, \sigma_x = 125.21, \sigma_y = 658.14$with ρ $(0.99, 0.70, 0.40)$. Based on 800 replications, the results for *m= 2, 6, 14, 18* using deciles (60.60, 83.00, 102.70, 111.20, 142.50, 210.20, 264.50, 304.40, 373.20, 643.00. The results are given in Table 1.

Table 1: Efficiency comparison

ρ	**D_1**				**D_2**				**D_3**			
	2	**6**	**14**	**18**	**2**	**6**	**14**	**18**	**2**	**6**	**14**	**18**
0.85	1.49	1.64	1.82	1.94	1.23	1.25	1.28	1.31	1.32	1.39	1.42	1.49
0.65	1.20	1.22	1.26	1.29	1.21	1.30	1.34	1.38	1.30	1.40	1.52	1.59
0.35	1.20	1.20	1.25	1.28	1.21	1.31	1.39	1.43	1.27	1.46	1.55	1.64
	D_4				**D_5**				**D_6**			
0.85	1.52	1.67	1.85	1.97	1.26	1.28	1.31	1.34	1.35	1.42	1.45	1.52
0.65	1.23	1.25	1.29	1.32	1.24	1.33	1.37	1.41	1.33	1.43	1.55	1.62
0.35	1.20	1.23	1.28	1.31	1.24	1.34	1.42	1.46	1.30	1.49	1.58	1.67
	D_7				**D_8**				**D_9**			
0.85	1.65	2.15	2.64	3.09	1.56	1.84	2.15	2.40	1.39	1.54	1.73	1.80
0.65	1.61	2.05	2.52	2.87	1.48	1.70	1.92	2.06	1.35	1.50	1.62	0.70
0.35	1.56	1.93	2.27	2.63	1.42	1.61	1.80	1.90	1.33	1.43	1.56	1.60

5. CONCLUSION :

It is concluded from the Table 1 that the proposed estimator performs better than the classic ratio estimator based on RSS. Utilizing the knowledge of the deciles of the auxiliary variable *X*, a gain in efficiency is obtained for estimating the population mean of the variable of interest *Y*.Finally the efficiency in case of the modified ratio estimator RSS estimators increases as the set size *k* increases.

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