APPLIED DATA MINING IN TTRP WITH FUZZY DEMANDS AND CAPACITIES

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ABSTRACT
Recently, the Truck and Trailer Routing Problem (TTRP) has been tackled with uncertainty in the coefficients of constrains. In order to solve this problem it is necessary to use methods for comparison fuzzy numbers. The problem of ordering fuzzy quantities has been addressed by many authors and there are many indices to perform this task. However, it is impossible to give a final answer to the question on what ranking method is the best in this problem. In this paper we focus our attention on a model to characterize TTRP instances. We use a data mining algorithm to derive a decision tree that determined the best method for comparison based on the characteristics of the TTRP problem to be solved.

KEYWORDS: decision tree, fuzzy coefficients, fuzzy constraints, ranking function, Truck and Trailer Routing Problem.

MSC: 94D05

RESUMEN
Recientemente, el Problema de Planificación de Rutas de Camiones y Remolques (TTRP, por sus siglas en inglés) ha sido tratado considerando incertidumbre en los coeficientes de las restricciones del problema. Cuando esto sucede se utilizan métodos de comparación de cantidades difusas. El problema de ordenar cantidades difusas ha sido abordado por muchos autores y existen disímiles métodos para llevar a cabo esta tarea. Sin embargo, actualmente se hace muy difícil brindar una respuesta absoluta a la pregunta de cuál es el mejor método de ordenación para un problema determinado. El propósito de este trabajo es obtener un modelo que permita caracterizar las instancias del TTRP para determinar que método de ordenación es conveniente utilizar a la hora de resolverlo. Con este fin se aplica un algoritmo de minería de datos que genera un árbol de decisión sobre un conjunto de instancias de la literatura.

1. INTRODUCTION

In optimization problem solution practice the available information can present a high grade of uncertainty, however it is usually assumed to be precise because classical procedures are not suitable for handling linguistic terms or impreciseness in the problem. So generally these procedures simplify the problem forcing these vague values to be exact, thus obliging us to formulate and to solve a problem with a (precise) nature different from the (vague) original one. This simplification of the problem causes the nature of the model to change and may produce very serious errors relating to the obtaining of the solution [1]. The uncertainty in real-world transport operations is particularly recognized by researchers in the field of transportation problems [2]. For example, for the coefficients included in the constraints, we are told that the customer demand is «*about 49 units*» or «*not much more than 72 units*» etc. And the same can happen with the rest of the parameters that define the problem such as travel times, vehicle capacity, etc.
Route planning problems are among the activities that have the highest impact in logistical planning, transport and distribution because of their effects on efficiency in resource management, service levels, and client satisfaction. The Truck and Trailer Routing Problem (TTRP) is an important problem in this field, with a growing interest by its practical relevance in many real-world problems such as milk collection, food distribution and postal delivery. TTRP consist of a heterogeneous fleet composed of trucks and trailers to serve a set of customers. Some customers with accessibility constraints must be served just by truck, while others can be served either by truck or by a truck pulling a trailer [3]. Usually in such problems finding the best possible solution is a complex task. If one adds that the problem or the knowledge about it is imprecise or vague there is a need to have tools with great potential to formulate and solve these problems successfully. Experience shows that the best way of modeling these kinds of problems is using Soft Computing methodologies [4]. In particular, the fuzzy optimization (models and

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methods) is shown as an alternative to model and solve problems with uncertainty. Following, this line in [5, 6] are studied, designed and developed a set of fuzzy models for TTRP. Concretely, in this models the constraints, the coefficients of the objective(s) and the coefficients of the constraints (the right hand value and coefficients in the technological matrix) can be stated as set fuzzy.
In this work, we tackled one of them: a fuzzy model where the demands and capacities are fuzzy numbers. In this situation is usual to use ranking methods. There are several methods for ranking fuzzy numbers that in many cases provide different rankings. In consequence, a long list of different auxiliary models and set of possible different solutions are obtained according to the comparison relation between fuzzy numbers used. In this point the solution to the model is obtained by particularization of the different comparison methods of fuzzy numbers. However, it is important to know which of these methods is more convenient to use or which is the one that gets the most adequate results. These and other questions can arise when solving our model. The purpose of this article is to introduce a data mining model that indicates the best comparison method based on the characteristics of the problem to be solved. This model would be able to offer knowledge on the types of problem instance where each method works better or worse. This knowledge can be of great utility for the final users.
Consequently the rest of the paper is organized as follows. Section 2 presents a brief review about fuzzy optimization models. Section 3 one presents the basic elements of the TTRP. Also, it proposes the model for TTRP with the set of constraints totally fuzzy. Section 4 provides an experimental study to illustrate the usefulness of characterizing the performance of the methods of comparison followed by the conclusions in Section 5.

2. FUZZY OPTIMIZATION

Fuzzy Optimization models and methods has been one of the most and well-studied topics inside the broad area of Soft Computing. Particularly relevant is the field of Fuzzy Linear Programming (FLP) that constitutes the basis for solving fuzzy optimization problems. FLP models are classified according to the way the fuzziness is introduced. In the last past years several kinds of FLP models have been defined in the literature [7], but the four main are:

- FLP models with a fuzzy constraint set, i.e. with a feasible set defined by fuzzy constraints [8, 9, 10, 11].
- FLP models with fuzzy numbers defining the coefficients of the technological matrix, i.e. the coefficients of the constraints and right hand values are defined as fuzzy numbers [11, 12, 13, 14, 15, 16].
- FLP models with fuzzy costs, i.e. with fuzzy numbers defining the costs of the objective function [12, 14, 16, 17, 18].
- FLP models with a fuzzy goal, i.e. with some fuzzy value to be attained in the objective [9, 19].

Of these models (as well as combinations that arise from them), one who has received a lot of attention is the FLP models with the coefficients in the technological matrix and the coefficients of the right hand side are represented by fuzzy numbers, with the costs that define the objective function being real. This type of model represents those circumstances where the data involved in the problem have a vague and imprecise nature. These features present in the data are caused by the ways in which the information is obtained. For example, in transportation problems, it is hard for customers to indicate the exact quantity of goods ordered or the precise time for the schedule of the services. A similar situation occurs with the number of vehicles available or capabilities thereof. These parameters are vague and can be expressed by means of fuzzy numbers. In this paper we focus on this type of model that can be stated as follows:

$$\begin{aligned} & max/\min f(x_j, c_j) \\ s.t: \quad & g(x_j, a_{ij}^f)\,\{\leq_f, \geq_f\}\, b_i^f \quad i = 1, \dots, m, \quad j = 1, \dots, n \qquad (1) \\ & x_j \geq 0 \end{aligned}$$

where x_j are the decision variables, c_j are the coefficients of the objective function, a_{ij}^f are the fuzzy coefficients of the constraints and b_i^f are fuzzy right hand values of the constraints. The functions $f(x_j, c_j)$ and $g(x_j, a_{ij}^f)$ can be linear or nonlinear functions.
The first version of this problem appeared in [13] (although supposing imprecision in the objective as well). In [11] is developed a general solution strategy that manage the imprecision in the comparison by introducing a fuzzy number τ_i for each single constraint, given by the decision maker. This value

represents the allowed maximum violation in the *i-th* constraint. The solution approach transforms the fuzzy model (1) in an equivalent auxiliary traditional model that is expressed as follows.

$$max/\min f(x_j, c_j)$$

$$s.t: \quad g(x_j, a_{ij}^f)\{\leq_g, \geq_g\} b_i^f + \tau_i^f(1-\alpha) \quad i = 1, \dots, m, \quad j = 1, \dots, n \quad (2)$$

$$x_j \geq 0, \alpha \in [0, 1]$$

where the symbols $\leq_g$ and $\geq_g$, stands for a comparison relation between fuzzy numbers, and α is a satisfaction level defined by the decision maker. In order to compare the left hand side to the right hand one the ranking methods for fuzzy numbers are generally used.

2.1. Methods for ranking fuzzy numbers

The problem of ordering fuzzy quantities has been addressed by many authors and there are many indices to perform this task. In problems like as (1) a new model is obtained according to the ranking function that the decision maker wants to use. It becomes patent as fuzzy solutions obtained from these models depend directly on the considered ranking function.
In the study carried out by [20] these methods can be classified into three classes:

- Methods based on the definition of an ordering function: it is constructed a mapping function to transform fuzzy quantities into a positive real number and then simply ranks based on the comparison of the obtained real numbers. In this group are the following methods: Adamo´s approach [21], Yager´s indices [22, 23, 24], Chang´s index [25], Liuo and Wang´s approach [26], Choobineh and Li´s approach [27].
- Methods based on the comparison of alternatives: it is defined some reference set(s) and evaluates each fuzzy quantity by calculating and comparing the closeness of fuzzy quantities to the reference set(s). The definition of the reference set can be done in two forms. Several methods in this class are: Jain´s method [28], Chen´s method [29], Kim and Park´s method [30], Kerre´s method [31].
- Methods based on a relationship of preference: it is constructed a fuzzy binary preference relation to manipulate pairwise comparisons. The result of all pairwise comparisons is used to obtain an order relation among fuzzy quantities. Examples of these methods were proposed by: Delgado et al [11], Dubois and Prade [32], Nakahara [33], Nakamura [34], Yuan [35], Baas and Kwakernaak [36], Baldwin and Guild [37], Kolodziejczyk [38] and Saade y Schwarzlander [39].

Before such variety of methods the following question emerges: which comparison method is more convenient to use in determined problem or which is the one that gets the most adequate results? Studies about this topic where exist comparative analysis are very few. More recent research is presented in [40] that investigate differences/similarities between ranking methods. However, most of the time choosing a method rather than another is a matter of preference or is context dependent.

3. FUZZY TRUCK AND TRAILER ROUTING PROBLEM

TTRP as an extension of the well-known Vehicle Routing Problem (VRP) consists in designing the optimal set of routes to serve a given set of customers, who are serviced by a fleet of vehicles with known capacity. A vehicle in the TTRP is composed by truck pulling a trailer and both are used for transporting goods. A truck plus a trailer is called a complete vehicle, and a vehicle without a trailer is called a pure truck. However, due to practical constraints, including government regulations, limited maneuvering space at customer site, road conditions, etc., some customers may only be serviced by the truck. Therefore, TTRP considers two different kinds of customers: a customer who is accessible with or without a trailer is called a vehicle customer (VC) and one who is only accessible without a trailer is called a truck customer (TC). A solution of the TTRP is generally composed of three types of routes: Pure Vehicle Route performed by a complete vehicle and contains only vehicle customer. Pure Truck Route performed by a truck alone and may visit both customer type and Complete Vehicle Route consisting of a main tour traveled by a complete vehicle, and at least one sub-tour traveled by the truck alone [3]. Also, each route is limited by capacity of vehicle used.
The goal for this NP-hard problem [41] is to find a set of least cost vehicle routes that start and end at the central depot such that each customer is serviced exactly once and the total demand of any vehicle route does not exceed the total capacity of the allocated vehicles used in that route. Also, the number of required trucks and trailers is not greater than available vehicles in the fleet. The solution approaches

published in the literature about this topic can be divided into three groups: exact approaches [42], approximated approaches (including heuristic and metaheuristics) [3], [43, 44, 45, 46, 47, 48], or a combination of these approaches (the so-called matheuristics) [49]. However, to the best of our knowledge, point out that most of models and approaches used for the TTRP in the literature assume that the data available are accurate, still when in many practical problems the available knowledge about some data and parameters of the model involving uncertainty.
In the reality, this problem is very complex and the information is not always available with sufficient precision and completeness as desired for adequate planning and management. In this point, Torres in her PhD research [6] proposes a set of fuzzy model for TTRP. Following, we present one of them.

3.1. TTRP with fuzzy demands and capacities

The model discussed in this paper is an adaptation of the standard TTRP model proposed in [3]. In this model (Fuzzy TTRP, FTTRP) are used the following index, parameters and variables:

Index

- i, j: customer index which represent the localization of the customer.
- k: vehicle index that represents the vehicle.
- l:

Parameters

- n: the number of customers
- c_{ij}: the cost in distance between customer i and customer j
- q_i: the total demand of customer i
- m_c: the number of trucks
- m_r: the number of trailers
- Q_c: the total capacity of a truck
- Q_r: the total capacity of a trailer

Variables

- x_{ij}^{kl}: a binary variable equal to 1 if and only if the vehicle k with ($l = 0$) or without trailer ($l = 1$) is used from customer i to customer j, and 0 otherwise.

The formulation for FTTRP is presented below.

$$\min = \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m_c}\sum_{l=0}^{1} c_{ij} x_{ij}^{kl} \tag{5}$$

Subject to:

$$\sum_{i=0}^{n}\sum_{k=1}^{m_c} x_{ij}^{k0} = 1 \qquad j = [1, \ldots n] \tag{6}$$

$$\sum_{i=0}^{n} x_{ij}^{k1} \geq 1 \qquad j = [1, \ldots n] \quad k = [1, \ldots m_c] \tag{7}$$

$$\sum_{i=0}^{n}\sum_{k=1}^{m_c} x_{ji}^{k0} = 1 \qquad j = [1, \ldots n] \tag{8}$$

$$\sum_{i=0}^{n} x_{ji}^{k1} \geq 1 \qquad j = [1, \ldots n] \quad k = [1, \ldots m_c] \tag{9}$$

$$\sum_{i=1}^{n}\sum_{l=0}^{1} x_{0i}^{kl} \leq 1 \qquad k = [1, \ldots m_c] \tag{10}$$

$$\sum_{i=1}^{n}\sum_{l=0}^{1} x_{i0}^{kl} \leq 1 \qquad k = [1, \ldots m_c] \tag{11}$$

$$\sum_{i=0}^{n}\sum_{l=0}^{1} x_{ij}^{kl} - \sum_{i=0}^{n}\sum_{l=0}^{1} x_{ji}^{kl} = 0 \qquad j = [1, \ldots n] \quad k = [1, \ldots m_c] \tag{12}$$

$$\sum_{i=0}^{n}\sum_{j=1}^{n} q_j^f x_{ij}^{k0} + \sum_{i=0}^{n}\sum_{j=1}^{n} q_j^f x_{ij}^{k1} \leq_f \; Q_c^f + Q_r^f \qquad k = [1,\ldots m_c] \tag{13}$$

$$\sum_{i=0}^{n}\sum_{j=1}^{n} q_j^f x_{ij}^{k1} \leq_f \; Q_c^f \qquad k = [1,\ldots m_c] \tag{14}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m_c} x_{0i}^{k0} \leq m_r \tag{15}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{m_c} x_{0i}^{k0} + \sum_{i=1}^{n}\sum_{k=1}^{m_c} x_{0i}^{k1} \leq m_c \tag{16}$$

$$x_{ij}^{kl} \in \{0,1\} \tag{17}$$

$$l \in \{0,1\} \tag{18}$$

$$i \in [0,\ldots n] \tag{19}$$

$$j \in [0,\ldots n] \tag{20}$$

$$k \in [0,\ldots m_c] \tag{21}$$

Equation (5) represents the objective function, which consists in to minimize the total distance traveled by the fleet on all three types of routes. Constraints (6 - 9) guarantee that only one vehicle enters and leaves from one node or that each customer is served exactly once. Furthermore, these constraints can allow multiple visits to VC customers that are root of the sub-tour. Constraints (10) and (11) ensure that each vehicle leaves the depot and returns to it, thereby limiting vehicle use to one trip. Constraint (12) establishes that if a vehicle leaves a customer then it has reached it. Constraints (13) and (14) establish the demand of all customers of any route or sub-tour does not exceed the total capacity of the allocated vehicles used in that route or sub-tour. Constraints (15) and (16) ensure that both the number of vehicle and pure routes are not greater than the number of trailers and trucks available respectively. Lastly, constraints (17 - 21) establish the conditions of the variables:

Observe that constraints (13) and (14) are defined as fuzzy, where also Q_c^f and Q_r^f are fuzzy capacities truck and trailer respectively, and q_j^f is fuzzy demand of customer. Therefore, they can be replaced by the following constraints:

$$\sum_{i=0}^{n}\sum_{j=1}^{n} q_j^f x_{ij}^{k0} + \sum_{i=0}^{n}\sum_{j=1}^{n} q_j^f x_{ij}^{k1} \leq_g \left(Q_c^f + Q_r^f\right) + \tau_1^f (1-\alpha) \quad k = [1,\ldots m_c] \tag{22}$$

$$\sum_{i=0}^{n}\sum_{j=1}^{n} q_j^f x_{ij}^{k1} \leq_g \; Q_c^f + \tau_2^f (1-\alpha) \quad k = [1,\ldots m_c] \tag{23}$$

where the symbols ($\leq_g$) stands for a comparison relation between fuzzy numbers using a ranking function for these constraints. To this end, any of the above comparison methods can be used. Then, the solution to the model is obtained by particularization of the different comparison methods of fuzzy numbers.

4. CASE STUDY

In order to test our approach in the TTRP with fuzzy demands and capacities in this section we present and discuss the computational experiments. This section is organized as follows: Subsection 5.1 describes test instances used in the experiments. Subsection 5.2, presents the configuration of the experiments, and solutions and results of the non-parametric statistical test are analyzed in the Subsection 5.3.

4.1 Instances

We used 21 instances available at the public website http://140.118.201.168/ttrp/. The testbed introduced by Chao [3] were derived from seven classical vehicle routing problem [50] using the following procedure: for each customer *i*, the distance between *i* and its nearest neighbor customer is calculated and denoted by A_i. The generation procedure creates three TTRP instances by defining in the first problem 25% of the customers with the smallest A_i values as TC. For the second and third problem the percentage of the nodes as TC is 50%, and 75% respectively.

Table 1 shows the characteristics of selected problems.

Table 1: Instances TTRP.

Problem ID	Customers		Trucks		Trailers	
	VC	TC	Number	Capacity	Number	Capacity
1	38	12	5	100	3	100
2	25	25				
3	13	37				
4	57	18	9		5	
5	38	37				
6	19	56				
7	75	25	8	150	4	
8	50	50				
9	25	75				
10	113	37	12		6	
11	75	75				
12	38	112				
13	150	49	17		9	
14	100	99				
15	50	149				
16	90	30	7		4	
17	60	60				
18	30	90				
19	75	25	10		5	
20	50	50				
21	25	75				

4.2 Setting experimental

The experiments were performed on a computer Intel Core running at 3.30 GHz under Windows 7 Professional with 6 GB of RAM. We decided to use an algorithm based on local search (Hill Climbing), which is available from the BiCIAM library [51]. The results were obtained with 30 independent runs with 100000 fitness evaluations for each problem. The instances of TTRP were solved for α = {0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0}. The amount demand of each customer and the limited vehicle capacities are considered triangular fuzzy numbers. In each case were obtained in the form of 10% variation in the modal value. Tolerance levels τ_1 and τ_2 are considered fuzzy numbers.
Also, we used six ranking function to obtain a particular order relation between fuzzy numbers. In the following, we will briefly describe each function will be used to test our proposal.

1. In [22, 23, 24], Yager proposed four ranking methods, where he does not assume any hypothesis of normality or convexity. In this study we use the following three methods proposed by Yager:

$$Y_1(\tilde{u}) = \frac{\int_0^1 g(z)\mu_{\tilde{u}}(z)dz}{\int_0^1 \mu_{\tilde{u}}(z)dz} \quad (24)$$

$$Y_2(\tilde{u}) = \int_0^{\alpha-max} M(U_\alpha)d\alpha \quad (25)$$

$$Y_4(\tilde{u}) = \sup_{z\in[0,1]} min\big(z, \mu_{\tilde{u}}(z)\big) \quad (26)$$

2. Chang in [25] proposed a ranking method based on the following index

$$C_I(\tilde{u}) = \int_{z\in\, sup\mu_{\tilde{u}}} z\mu_{\tilde{u}}(z)dz \quad (27)$$

3. Dubois and Prade propose a set of four indices able to completely describe the relative location of two fuzzy numbers [32]. In particular we use:

$$PD(\tilde{u}_i, \tilde{u}_j) = \sup min\left(\mu_{\tilde{u}_i}, \mu_{\tilde{u}_j}\right) \tag{28}$$

$$ND(\tilde{u}_i, \tilde{u}_j) = \inf \sup min\left(1 - \mu_{\tilde{u}_i}, \mu_{\tilde{u}_j}\right) \tag{29}$$

4.3. A characterization of the performance of ordering methods in FTTRP

Table 2 reports the best value of the objective function obtained for each value of the α-cuts in 30 runs. Values contained in brackets are associated with the comparison method obtained the result. Examining

ID	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
1	527.00(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)	526.20(Y4)
2	539.74(Y4)	539.74(Y4)	539.74(Y4)	539.74(Y4)	533.43(Y4)	530.65(Y4)	530.65(Y4)	530.65(Y4)	530.65(Y4)	530.65(Y4)	527.49(Y4)
3	452.16(Y4)	448.96(Y4)	448.96(Y4)	439.84(Y4)	439.84(Y4)	434.56(Y4)	434.56(Y4)	434.56(Y4)	433.20(Y4)	433.20(Y4)	432.99(Y4)
4	750.25(Y4)	724.22(Y4)	733.08(Y4)	730.31(Y4)	721.75(Y4)	694.03(Y4)	694.03(Y4)	721.75(Y4)	707.48(Y4)	707.48(Y4)	707.48(Y4)
5	785.34(Y4)	767.31(Y4)	767.31(Y4)	750.69(Y4)	745.79(Y4)	745.79(Y4)	741.61(Y4)	736.21(Y4)	736.21(Y4)	736.21(Y4)	734.82(Y4)
6	759.22(Y4)	747.64(Y4)	725.8 (Y4)	725.82(Y4)	725.82(Y4)	725.82(Y4)	725.82(Y4)	715.71(Y4)	697.85(Y4)	696.40(Y4)	695.78(Y4)
7	759.94(Y4)	738.22(Y4)	734.60(Y4)	733.49(Y4)	731.55(Y4)	731.55(Y4)	731.16(Y4)	731.16(Y4)	731.16(Y4)	731.16(Y4)	731.16(Y4)
8	785.50(Y4)	763.91(Y4)	763.5 (Y4)	763.55(Y4)	763.55(Y4)	763.55(Y4)	763.55(Y4)	763.55(Y4)	763.55(Y4)	763.55(Y4)	767.93(Y4)
9	672.93(Y4)	655.38(Y4)	655.3 (Y4)	655.38(Y4)	655.38(Y4)	655.38(Y4)	655.38(Y4)	655.38(Y4)	655.38(Y4)	655.38(Y4)	655.38(Y4)
10	1026.63(Y4)	979.30(Y4)	969.66(Y4)	967.03(Y4)	967.03(Y4)	967.03(Y4)	965.23(Y4)	965.23(Y4)	965.23(Y4)	965.23(Y4)	955.79(Y4)
11	1090.85(Y4)	971.14(Y4)	960.83(Y4)	950.96(Y4)	946.23(Y4)	937.00(Y4)	937.00(Y4)	937.00(Y4)	937.00(Y4)	937.00(Y4)	937.00(Y4)
12	774.40(Y4)	747.69(Y4)	745.69(Y4)	744.94(Y4)	744.94(Y4)	744.94(Y4)	744.94(Y4)	744.94(Y4)	744.94(Y4)	744.94(Y4)	744.94(Y4)
13	1473.84(Y4)	1266.14(Y4)	1179.68(Y4)	1134.21(Y4)	1118.08(Y4)	1081.94(Y4)	1060.72(Y4)	1039.32(Y4)	1033.79(Y4)	1028.04(Y4)	1027.23(Y4)
14	1448.05(Y4)	1261.26(Y4)	1174.11(Y4)	1150.7(Y4)	1102.59(Y4)	1090.89(Y4)	1086.90(Y4)	1084.78(Y4)	1078.04(Y4)	1073.38(Y4)	1073.38(Y4)
15	1086.80(Y4)	969.15(Y4)	931.08(Y4)	925.16(Y4)	917.03(Y4)	896.75(Y4)	895.87(Y4)	895.55(Y4)	895.55(Y4)	895.55(Y4)	895.55(Y4)
16	965.23(Y4)	831.71(Y4)	816.49(Y4)	814.68(Y4)	814.68(Y4)	814.27(Y4)	806.79(Y4)	805.28(Y4)	797.74(Y4)	794.63(Y4)	794.63(Y4)
17	805.25(Y4)	796.32(Y4)	792.99(Y4)	789.45(Y4)	789.45(Y4)	789.45(Y4)	789.45(Y4)	789.45(Y4)	789.45(Y4)	789.45(Y4)	789.45(Y4)
18	701.36(Y4)	686.80(Y4)	686.8 (Y4)	686.80(Y4)	686.80(Y4)	686.80(Y4)	686.80(Y4)	686.80(Y4)	696.19(Y4)	696.19(Y4)	696.19(Y4)
19	933.93(GPD)	874.54(GPD)	874.54(GPD)	874.54(GPD)	904.39(GPD)	880.33(GPD)	872.35(GPD)	876.16(GPD)	888.60(GPD)	862.56(GPD)	862.56(GPD)
20	1137.92(GND)	1093.35(GND)	1039.25(GPD)	1037.05(GPD)	1052.69(GPD)	1035.28(GPD)	1035.28(GPD)	1030.43(GPD)	1017.67(GPD)	1008.25(CI)	1007.97(CI)
21	1145.33(GPD)	1113.20(GPD)	1107.13(GPD)	1104.53(GPD)	1067.52(GPD)	1067.52(GPD)	1067.52(GPD)	1042.60(GND)	1021.76(GND)	1020.74(GND)	1017.03(GND)

this table, we point out that the average of Y_4 function is better than the remaining ranking functions. With the results we performed Friedman test with α = 0.05 as the level of confidence. The results point out that Y4 dominates the other ranking functions and achieved the highest rankings. Also, we can raise the 15 hypotheses of equality among the 6 methods of our study, and apply the post-hoc Shaffer and Holm to contrast them. Nine of these hypotheses confirm the improvement of Y_4 over the rest of the comparison methods. Furthermore, the C_I method was overcome by all methods considered. Finally, only 6 hypotheses can be rejected using these procedures. Each one does not find any significant difference between ND and PD, Y_1 and Y_2. Clearly, this is visible in the graphic of the Figure 1.

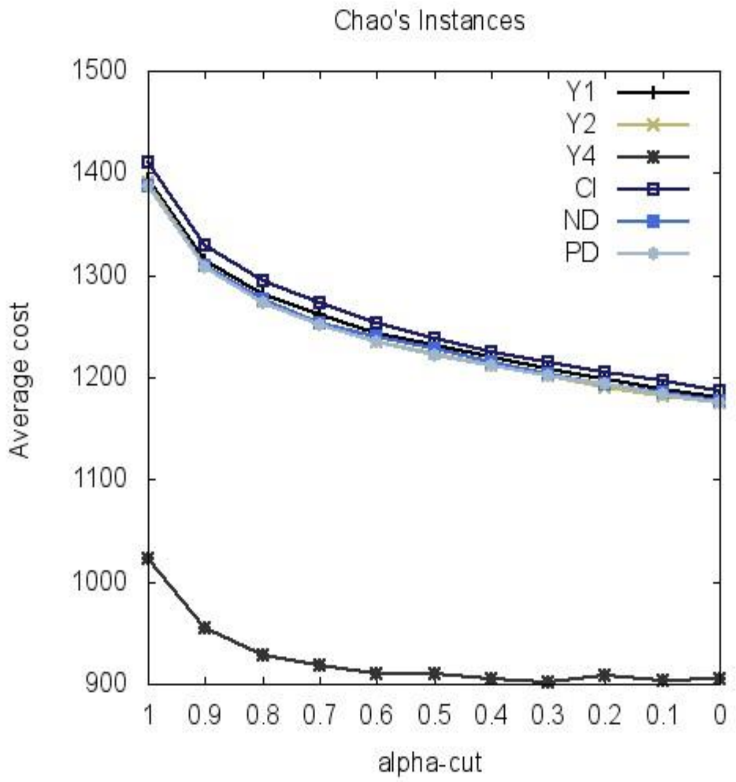

Figure 1: Behavior average of the comparison methods in 21 TTRP instances.

In this point, we decide to generate a decision tree model that indicates the best comparison method based on the characteristics of the problem to be solved. The following figure shows the obtained model using J48 algorithm of tool KNIME on a minable view of 231 tuples (21 instances x 11 α-cuts).
This model comprises a set of rules to determine the best method for comparison based on the characteristics of the problem to be solved. These rules are:

1. Number of Trucks ≤ 9 → $\mathbf{Y_4}$
2. Number of Trucks > 9 **AND** Number of Customers > 120 → $\mathbf{Y_4}$
3. Number of Trucks > 9 **AND** Number of Customers ≤ 120 AND Number of VC > 60 → **PD**
4. Number of Trucks > 9 **AND** Number of Customers ≤ 120 AND Number of VC ≤ 60 AND α > 0.1 → **PD**
5. Number of Trucks > 9 **AND** Number of Customers ≤ 120 **AND** 30 < Number of VC ≤ 60 AND α ≤ 0.1 → $\mathbf{C_I}$
6. Number of Trucks > 9 **AND** Number of Customers ≤ 120 **AND** α ≤ 0.1 **AND** Number of VC ≤ 30 → **ND**

ID	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
1	527.00$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$	526.20$_{(Y4)}$
2	539.74$_{(Y4)}$	539.74$_{(Y4)}$	539.74$_{(Y4)}$	539.74$_{(Y4)}$	533.43$_{(Y4)}$	530.65$_{(Y4)}$	530.65$_{(Y4)}$	530.65$_{(Y4)}$	530.65$_{(Y4)}$	530.65$_{(Y4)}$	527.49$_{(Y4)}$
3	452.16$_{(Y4)}$	448.96$_{(Y4)}$	448.96$_{(Y4)}$	439.84$_{(Y4)}$	439.84$_{(Y4)}$	434.56$_{(Y4)}$	434.56$_{(Y4)}$	434.56$_{(Y4)}$	433.20$_{(Y4)}$	433.20$_{(Y4)}$	432.99$_{(Y4)}$
4	750.25$_{(Y4)}$	724.22$_{(Y4)}$	733.08$_{(Y4)}$	730.31$_{(Y4)}$	721.75$_{(Y4)}$	694.03$_{(Y4)}$	694.03$_{(Y4)}$	721.75$_{(Y4)}$	707.48$_{(Y4)}$	707.48$_{(Y4)}$	707.48$_{(Y4)}$
5	785.34$_{(Y4)}$	767.31$_{(Y4)}$	767.31$_{(Y4)}$	750.69$_{(Y4)}$	745.79$_{(Y4)}$	745.79$_{(Y4)}$	741.61$_{(Y4)}$	736.21$_{(Y4)}$	736.21$_{(Y4)}$	736.21$_{(Y4)}$	734.82$_{(Y4)}$
6	759.22$_{(Y4)}$	747.64$_{(Y4)}$	725.8 $_{(Y4)}$	725.82$_{(Y4)}$	725.82$_{(Y4)}$	725.82$_{(Y4)}$	725.82$_{(Y4)}$	715.71$_{(Y4)}$	697.85$_{(Y4)}$	696.40$_{(Y4)}$	695.78$_{(Y4)}$
7	759.94$_{(Y4)}$	738.22$_{(Y4)}$	734.60$_{(Y4)}$	733.49$_{(Y4)}$	731.55$_{(Y4)}$	731.55$_{(Y4)}$	731.16$_{(Y4)}$	731.16$_{(Y4)}$	731.16$_{(Y4)}$	731.16$_{(Y4)}$	731.16$_{(Y4)}$
8	785.50$_{(Y4)}$	763.91$_{(Y4)}$	763.5 $_{(Y4)}$	763.55$_{(Y4)}$	763.55$_{(Y4)}$	763.55$_{(Y4)}$	763.55$_{(Y4)}$	763.55$_{(Y4)}$	763.55$_{(Y4)}$	763.55$_{(Y4)}$	767.93$_{(Y4)}$
9	672.93$_{(Y4)}$	655.38$_{(Y4)}$	655.3 $_{(Y4)}$	655.38$_{(Y4)}$	655.38$_{(Y4)}$	655.38$_{(Y4)}$	655.38$_{(Y4)}$	655.38$_{(Y4)}$	655.38$_{(Y4)}$	655.38$_{(Y4)}$	655.38$_{(Y4)}$

10	1026.63$_{(Y4)}$	979.30$_{(Y4)}$	969.66$_{(Y4)}$	967.03$_{(Y4)}$	967.03$_{(Y4)}$	967.03$_{(Y4)}$	965.23$_{(Y4)}$	965.23$_{(Y4)}$	965.23$_{(Y4)}$	965.23$_{(Y4)}$	955.79$_{(Y4)}$
11	1090.85$_{(Y4)}$	971.14$_{(Y4)}$	960.83$_{(Y4)}$	950.96$_{(Y4)}$	946.23$_{(Y4)}$	937.00$_{(Y4)}$	937.00$_{(Y4)}$	937.00$_{(Y4)}$	937.00$_{(Y4)}$	937.00$_{(Y4)}$	937.00$_{(Y4)}$
12	774.40$_{(Y4)}$	747.69$_{(Y4)}$	745.69$_{(Y4)}$	744.94$_{(Y4)}$	744.94$_{(Y4)}$	744.94$_{(Y4)}$	744.94$_{(Y4)}$	744.94$_{(Y4)}$	744.94$_{(Y4)}$	744.94$_{(Y4)}$	744.94$_{(Y4)}$
13	1473.84$_{(Y4)}$	1266.14$_{(Y4)}$	1179.68$_{(Y4)}$	1134.21$_{(Y4)}$	1118.08$_{(Y4)}$	1081.94$_{(Y4)}$	1060.72$_{(Y4)}$	1039.32$_{(Y4)}$	1033.79$_{(Y4)}$	1028.04$_{(Y4)}$	1027.23$_{(Y4)}$
14	1448.05$_{(Y4)}$	1261.26$_{(Y4)}$	1174.11$_{(Y4)}$	1150.7$_{(Y4)}$	1102.59$_{(Y4)}$	1090.89$_{(Y4)}$	1086.90$_{(Y4)}$	1084.78$_{(Y4)}$	1078.04$_{(Y4)}$	1073.38$_{(Y4)}$	1073.38$_{(Y4)}$
15	1086.80$_{(Y4)}$	969.15$_{(Y4)}$	931.08$_{(Y4)}$	925.16$_{(Y4)}$	917.03$_{(Y4)}$	896.75$_{(Y4)}$	895.87$_{(Y4)}$	895.55$_{(Y4)}$	895.55$_{(Y4)}$	895.55$_{(Y4)}$	895.55$_{(Y4)}$
16	965.23$_{(Y4)}$	831.71$_{(Y4)}$	816.49$_{(Y4)}$	814.68$_{(Y4)}$	814.68$_{(Y4)}$	814.27$_{(Y4)}$	806.79$_{(Y4)}$	805.28$_{(Y4)}$	797.74$_{(Y4)}$	794.63$_{(Y4)}$	794.63$_{(Y4)}$
17	805.25$_{(Y4)}$	796.32$_{(Y4)}$	792.99$_{(Y4)}$	789.45$_{(Y4)}$	789.45$_{(Y4)}$	789.45$_{(Y4)}$	789.45$_{(Y4)}$	789.45$_{(Y4)}$	789.45$_{(Y4)}$	789.45$_{(Y4)}$	789.45$_{(Y4)}$
18	701.36$_{(Y4)}$	686.80$_{(Y4)}$	686.8 $_{(Y4)}$	686.80$_{(Y4)}$	686.80$_{(Y4)}$	686.80$_{(Y4)}$	686.80$_{(Y4)}$	686.80$_{(Y4)}$	696.19$_{(Y4)}$	696.19$_{(Y4)}$	696.19$_{(Y4)}$
19	933.93$_{(GPD)}$	874.54$_{(GPD)}$	874.54$_{(GPD)}$	874.54$_{(GPD)}$	904.39$_{(GPD)}$	880.33$_{(GPD)}$	872.35$_{(GPD)}$	876.16$_{(GPD)}$	888.60$_{(GPD)}$	862.56$_{(GPD)}$	862.56$_{(GPD)}$
20	1137.92$_{(GND)}$	1093.35$_{(GND)}$	1039.25$_{(GPD)}$	1037.05$_{(GPD)}$	1052.69$_{(GPD)}$	1035.28$_{(GPD)}$	1035.28$_{(GPD)}$	1030.43$_{(GPD)}$	1017.67$_{(GPD)}$	1008.25$_{(CI)}$	1007.97$_{(CI)}$
21	1145.33$_{(GPD)}$	1113.20$_{(GPD)}$	1107.13$_{(GPD)}$	1104.53$_{(GPD)}$	1067.52$_{(GPD)}$	1067.52$_{(GPD)}$	1067.52$_{(GPD)}$	1042.60$_{(GND)}$	1021.76$_{(GND)}$	1020.74$_{(GND)}$	1017.03$_{(GND)}$

Table 2: Results for 21 instances TTRP with fuzzy demands and capacities.

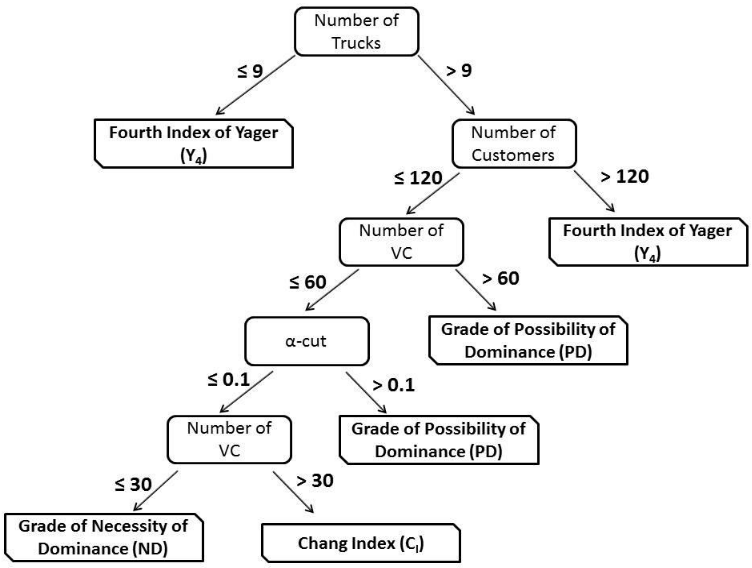

Figure 2: Decision tree model

For example, this model suggests using the method Grade of Necessity of Dominance (ND) when the parameter number of trucks is strictly greater than (>) 9, the number of customers is less than or equal to (≤) 120 with no more than 30 customers of type VC and α-cut equal to 0.0 or 0.1 (rule 6). Another conclusion is that the methods First Index of Yager (Y_1) and Second Index of Yager (Y_2) are not adequate to solve any of the instances. Also, it is important to note that the most important parameters to decide the best method are number of trucks, number of customers, number of VC and α-cuts.

5. CONCLUSIONS

In this paper we introduce a decision model useful for users as it allows defining strategies for selecting comparison methods in solving the TTRP with fuzzy demands and capacities. This knowledge can be generalized into a learning mechanism to determine which methods to use depending on the characteristics of a problem. Furthermore, the model can be improved if new TTRP problems are incorporated or comparison methods.

RECEIVED: FEBRUARY 2016
REVISED: JULY, 2016.

REFERENCES

[1] CADENAS, J. and VERDEGAY, J. L. (1997): Using Fuzzy Numbers in Linear Programming. **IEEE Transactions on Systems, Man. and Cybernetics**, 27, 1017-1022.
[2] SANCHEZ-RODRIGUES, V., STANTCHEV, D., POTTER, A., NAIM, M. and WHITEING, A. (2008): Establishing a transport operation focussed uncertainty model for the supply chain. **International Journal of Physical a Distribution and Logistics Management**, 38, 388-411.
[3] CHAO, I-M. (2002): A tabu search method for the truck and trailer routing problem. **Computers & Operation Research**, 29, 33-51.
[4] VERDEGAY, J., YAGER, R. and BONISSONE, P. (2008): On heuristic as a fundamental constituent of soft computing. **Fuzzy Sets and Systems**, 159, 846-855.
[5] TORRES, I., CRUZ, C. and VERDEGAY, J. (2015): Solving the Truck and Trailer Routing Problem with Fuzzy Constraints. **International Journal of Computational Intelligence Systems**, 8, 713-724.
[6] TORRES, I. (2016): **Modelos basados en Soft Computing para el diseño de rutas de vehículos: soluciones en diferentes** entornos. Universidad de Granada, España.
[7] BAYKASOGLU, A. and TOLUNAY, G. (2008): A review and classification of fuzzy mathematical programs. **Journal of Intelligent & Fuzzy Systems**, 19, 205-229.
[8] TANAKA, H., OKUDA, T. and ASAI, K. (1974): On fuzzy-mathematical programming. **Journal of Cybernetics**, 3, 37-46.
[9] ZIMMERMANN, H.-J. (1976): Description and optimization of fuzzy systems. **International Journal of General Systems**, 2, 209-215.
[10] VERDEGAY, J. (1982): Fuzzy mathematical programming. In: Gupta, M. and Sanchez, E. **Fuzzy Information and Decision Processes**, 231-237. North-Holland Publishing Company, North-Holland.
[11] DELGADO, M., VERDEGAY, J. and VILA, M. (1989): A general model for fuzzy linear programming problem. **Fuzzy Sets and Systems**, 29, 21-29.
[12] TANAKA, H. and ASAI, K. (1984): Fuzzy linear programming problems with fuzzy numbers. **Fuzzy Sets and Systems**, 13, 1-10.
[13] TANAKA, H., ICHIHASHI, H. and ASAI, K. (1984): A formulation of fuzzy linear programming problems based on comparison of fuzzy numbers. **Control and Cybernet**, 13, 185-194.
[14] DELGADO, M., VERDEGAY, J. and VILA, M. (1987): Imprecise costs in mathematical programming problems. **Control and Cybernet**, 16, 113-121.
[15] CAMPOS, L. and GONZÁLEZ, A. (1989): A subjective approach for ranking fuzzy numbers. **Fuzzy Sets and Systems**, 29, 145-153.
[16] ROMMELFANGER, H., HANUSCHECK, R. and Wolf, J. (1989): Linear programming with fuzzy objectives. **Fuzzy Sets and Systems**, 29, 31-48.
[17] ISHIBUCHI, H., and TANAKA, H. (1990): Multiobjective programming in optimization of the interval objective function. **European Journal of Operational Research**, 48, 219-225.
[18] DELGADO, M., VERDEGAY, J. and VILA, M. (1990): Relating different approaches to solve linear programming problems with imprecise costs. **Fuzzy Sets and Systems**, 37, 33-42.
[19] VERDEGAY, J. (1984): A dual to approach to solve the fuzzy linear programming problem. Fuzzy Sets and Systems, 14, 131-141.
[20] WANG, X. and KERRE, E. (1996): On the Classification and the Dependencies of the Ordering Methods Advances in Intelligent. In: Ruan, D. **Systems Research Fuzzy Logic Foundations and Industrial Applications (Part I)**, 73-90. Springer, US.
[21] ADAMO, J. (1980): Fuzzy decision trees. **Fuzzy Sets and Systems**, 4, 207-219.
[22] YAGER, R. (1978): Ranking fuzzy subsets over the unit interval. **Proceeding 1978 CDC**.
[23] YAGER, R. (1980): On choosing between fuzzy subsets. **Kybernetes**, 9, 151-154.
[24] YAGER, R. (1981): A procedure for ordering fuzzy subsets of the unit interval. **Information Sciences**, 24, 143-161.
[25] CHANG, W. (1981): Ranking of fuzzy utilities with triangular membership functions. **Proceeding of International Conference on Policy Analysis and Systems**.
[26] LIUO, T and WANG, J. (1992): Ranking fuzzy numbers with integral value. **Fuzzy Sets and Systems**, 50, 247-255.
[27] CHOOBINEH, F. and LI, H. (1993): An index for ordering fuzzy numbers. **Fuzzy Sets and Systems**, 54, 287-294.
[28] JAIN, R. (1977): A procedure for multiple-aspect decision making using fuzzy set. **International Journal of Systems Sciences**, 8, 1-7.
[29] CHEN, S. (1985): Ranking fuzzy numbers with maximizing set and minimizing set. **Fuzzy Sets and Systems**, 17, 113-129.
[30] KIM, K. and PARK, K. (1990): Ranking fuzzy numbers with index of optimism. **Fuzzy Sets and Systems**, 35, 143-150.

[31] KERRE, E. (1982): The use of fuzzy set theory in eletrocardiological diagnostics. In: Gupta, M. and Sanchez, E. **Approximate reasoning in decision-analysis**, 277-282. North-Holland Publishing Company, Amsterdam.
[32] DUBOIS, D. and PRADE, H. (1983): Ranking fuzzy numbers in the setting of possibility theory. **Information Sciences**, 30, 183-224.
[33] NAKAHARA, Y. (1998): User oriented ranking criteria and its application to fuzzy mathematical programming problems. **Fuzzy Sets and Systems**, 94, 275-286.
[34] NAKAMURA, K. (1986): Preference relations on a set of fuzzy utilities as a basis for decision making. **Fuzzy Sets and Systems**, 20, 147-162.
[35] YUAN, Y. (1991): Criteria for evaluating fuzzy ranking methods. **Fuzzy Sets and Systems**, 43, 139-157.
[36] BAAS, S. and KWAKERNAAK, H. (1977): Rating and ranking of multiple-aspect alternatives using fuzzy sets. **Automatic**, 13, 47-58.
[37] BALDWIN, J. and GUILD, N. (1979): Comparison of fuzzy sets on the same decision space. **Fuzzy Sets and Systems**, 2, 213-231.
[38] KOLODZIEJCZYK, W. (1990): Orlovsky´s concept of decision-making with fuzzy preference relation: further results. **Fuzzy Sets and Systems**, 19, 197-212.
[39] SAADE, J. and SCHWARZLANDER, H. (1992): Ordering fuzzy sets over the real line: an approach based on decision making under uncertainty. **Fuzzy Sets and Systems**, 50, 237-246.
[40] BRUNELLI, M. and MEZEIB, J. (2013): How different are ranking methods for fuzzy numbers? A numerical study. **International Journal of Approximate Reasoning**, 54, 627-639.
[41] GAREY, M. and JOHNSON, D. (1979): **Computers and intractability: a guide to the theory of NP-completeness**. W. H. Freeman. San Francisco. USA.
[42] DREXL, M. (2011): Branch and price and heuristic colum generation for the generalized truck and trailer routing problem. **Journal of Quantitative Methods for Economics and Bussines Administration**, 12, 5-38.
[43] SCHEUERER, S. (2006): A tabu search heuristic for the truck and trailer routing problem. **Computers & Operation Research**, 33, 894-909.
[44] LIN, S-W., YU, V. F. and CHOU, S.Y. (2009): Solving the truck and trailer routing problem based on a simulated annealing heuristic. **Computers & Operation Research**, 36, 1683-1692.
[45] VILLEGAS, J.G., PRINS, C., PRODHON, C., MEDAGLIA, A. L. and VELASCO, N. (2011): A GRASP with evolutionary path relinking for the truck and trailer routing problem. **Computers & Operation Research**, 38, 1319-1334.
[46] DERIGS, U., PULLMANN, M. and VOGEL, U. (2013): Truck and trailer routing – problems, heuristics and computational experience. **Computers & Operations Research**, 40, 536-546.
[47] MIRMOHAMMADSADEGHI, S., AHMED, S. and NADIRAH, E. (2014): Application of memetic algorithm to solve truck and trailer routing problems. **Proceedings of the 2014 International Conference on Industrial Engineering and Operations Management**, Bali.
[48] CARAMIA, M. and GUERRIERO, F. (2010): A heuristic approach for the truck and trailer routing problem. **Journal of the Operational Research Society**, 61, 1168-1180.
[49] VILLEGAS, J.G., PRINS, C., PRODHON, C., MEDAGLIA, A. L. and VELASCO, N. (2013): A matheuristic for the truck and trailer routing problem. **European Journal of Operational Research**, 230, 231-244.
[50] CHRISTOFIDES, N., MINGOZZI, A. and TOTH, P. (1979): Combinatorial Optimization. In: Christofides, N., Mingozzi, A., Toth, P. and Sandi, C. **The vehicle routing problem**, 315-338. Wiley, Chichester, UK.
[51] FAJARDO, J., MASEGOSA, A. and PELTA, D. (2015): Algorithm portfolio based scheme for dynamic optimization problems. **International Journal of Computational Intelligence Systems**, 8, 667-689.