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SUPPLY CHAIN INVENTORY MODEL FOR MULTI-ITEMS WITH UP-STREAM PERMISSIBLE DELAY UNDER PRICE SENSITIVE QUADRATIC DEMAND

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ABSTRACT
This paper deals with a two-level supply chain comprising of single manufacturer and single retailer who sells multi-items. The units in inventory of each period are subject to deterioration at a constant rate. The manufacturer has finite production capacity and manufacturing proportional to the demand rate. The manufacturer offers credit period to the retailer. Price sensitive quadratic demand is debated here which is suitable for the items whose demand increase initially and after some time it starts to decrease. Here, comparison between independent and joint decision is studied regarding their decision variables and average profits. The classical optimization is used to optimize the total profit of the supply chain with respect to selling price and cycle time. The model is supported with numerical examples and also established best scenario and best policy of the model. Sensitivity analysis is done to deduce managerial insights. It is observed that the players are encouraged to join hand to hand to run a business by joint decisions.

KEYWORDS: Supply chain, price sensitive quadratic demand, deterioration, up-stream trade credit, independent and joint decision policy.

MSC: 90B05

RESUMEN
En este trabajo se trata de una cadena de suministro de dos niveles que consta de un solo fabricante y minorista que vende solo multi-artículos. Las unidades en el inventario de cada período están sujetos a deterioro a una velocidad constante. El fabricante tiene la capacidad de producción y la fabricación finita es proporcional a la tasa de demanda. El fabricante ofrece un período de crédito al minorista. El precio de la demanda cuadrática sensible se debate aquí, el que es adecuado para los artículos cuyo incremento inicialmente y después de algún tiempo comienza a disminuir la demanda. Aquí, la comparación entre la decisión independiente y conjunta se estudió en cuanto a sus variables de decisión y las ganancias promedio. La optimización clásica se utiliza para optimizar el beneficio total de la cadena de suministro en relación con el precio de venta y el ciclo de tiempo. El modelo es compatible con ejemplos numéricos y también se estableció el mejor escenario y la mejor política del modelo. El análisis de sensibilidad se realiza para deducir ideas de gestión. Se observa que se anima a los jugadores a unirse mano en mano para dirigir un negocio para hacer decisiones conjuntas.

PALABRAS CLAVE: Cadena de suministro, demanda sensitiva cuadrática, deterioro, up-stream trade credit, política conjunta independiente y de decisión conjunta

1. INTRODUCTION

In the today world, coordination policy is very important for the business. A supply chain is a system that is classically comprised of suppliers, manufacturer, distributors and retailers. A main objective of supply chain management research is the supply chain coordination. Quite often, the coordination among the players is a foremost source of benefits which are shared by the players of the chain. In the globalized and modest business environment, the coordination of supply chains is a big challenge. The coordination of supply chain can be done in several ways. Ishii *et al*. (1988) developed joint decision for three players' viz. manufacturer,

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wholesaler and retailer. Haq *et al*. (1991) deliberated coordinated inventory system with one manufacturer, several warehouses and multi-retailers. Goyal and Nebebe (2000) analyzed economic production and shipment policy for a supply chain of a vendor and a buyer. Woo *et al*. (2001) formulated joint policies for a vendor and multi-buyers. They considered vendor to be produce. Rau *et al*. (2003) reduced total cost of the supply chain with three players and deteriorating items. Zhou and Li (2007) determined that the coordination between both players in the ordering strategy increases the expected profit of the retailer and also for the entire supply chain. Shah *et al*. (2009) determined coordinated decision when demand is increasing quadratically. Shah and Shukla (2010) modeled a supply chain inventory model for optimal ordering and pricing policies under the retailer partial trade credit when demand is decreasing. Liu and Cruz (2012) discovered a supply chain inventory model with corporate financial risk and trade credits under economic uncertainty. Soni (2013) formulated optimal policies under two layered and floor constraint stock dependent demand for deteriorating items. Recently, Jiangtao et al. (2014) studied a multi-item inventory model for stock dependent demand of perishable items, adopting two-level trade credit policies and the restriction of inventory capacity. Considering a supply chain with one manufacturer and one retailer environment, Cárdenas-Barrón and Sana (2014) established a production-inventory model that includes variable procurement prices, sales player's initiatives, sensitive demand, backordering, variable production rate and production lot size. Cárdenas-Barrón and Sana (2015) developed multi-item EOQ inventory model in a two-layer supply chain while demand varies with a promotional effort. Shah *et al*. (2015) derived optimal down-stream credit period and replenishment time for deteriorating inventory in a supply chain. Shah and Chaudhari (2015) developed optimal policies for three players with fixed life time and two-level trade credit under time and trade credit dependent demand. Shah *et al*. (2015) established an economic order quantity model under trade credit and consumer returns for selling price-sensitive quadratic demand. Shah and Jani (2016) determined optimal ordering under time dependent and two-level trade credit demand for deteriorating items of fixed-life.

Due to drastic environmental variations, most of the items fatalities its efficiency over time, termed as deterioration. Deterioration of goods likes, volatile fluids, tablets, root vegetable, radioactive substances, medicine, blood etc. Out of several studies on deterioration item only few of them have considered Fixed Life-time issue of deteriorating items. Ghare and Schrader (1963) deliberated effect of deterioration in inventory model. The criticise articles by Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001), Bakker *et al.* (2012), on deteriorating products for inventory system throw light on the part of deterioration. Sarkar (2012) established two-level trade credit policy with time varying deterioration rate and time dependent demand. Chung and Cardenas-Barrón (2013) developed simple algorithm under stock-dependent demand and two-level trade credit in a supply chain comprising of three players for deteriorating items. Some motivating articles are by Ouyang *et al.* (2013), Sarkar *et al*. (2014), Chung *et al*. (2014), Wu *et al*. (2014) and their cited references. Furthermore, Shah and Barrón (2015) deliberated retailer's decision for ordering and credit policies for deteriorating items when a manufacturer offers order-linked trade credit or cash discount. Recently, Shah *et al*. (2016a) studied an integrated production-inventory model for time-varying deteriorating item under time and price sensitive demand with the use of preservation technology investment. Shah *et al*. (2016b) determined optimal policies for time dependent deteriorating item with preservation technology under price and credit limit dependent quadratic demand in a supply chain. Shah *et al*. (2016) analyzed impact of future price increase on ordering policies under quadratic demand for deteriorating items.

In this paper, the demand is dependent on selling price and time. Also, the manufacturer gives trade credit to the retailer to boost his demand. To maximize the total profit of the supply chain, two policy are analysed viz., independent decision and joint decision. The best policy is discussed and analyzed optimality of that policy. Under above assumptions, the objective is to maximize the profit of supply chain with respect to the selling price and cycle time.

The rest of the paper is organized as follow. Section 2 is about the notations and the assumptions that are used. Section 3 is about development of the mathematical model of the proposed inventory problem. Section 4 validates the derived inventory model with numerical examples sensitivity analysis. This section also provides some managerial insights. Finally, Section 5 provides conclusion and future research directions.

2. NOTATION AND ASSUMPTIONS

The proposed inventory problem is based on the following notation and assumptions.

2.1 Notation

Retailer’s parameters:

a_i	Total market potential demand for ith item , $a>0$
b_i	Linear rate of change of demand for ith item , $0\le b<1$
c_i	Quadratic rate of change of demand for ith item, $0\le c<1$
A_r	Ordering cost per order incurred by the retailer ($/order)
p_i	Selling price of the ith item
η_i	Mark-up for selling price of ith item
θ	Constant Deterioration rate; $0\le\theta\le 1$
$I_{ri}(t)$	Inventory level for the retailer of ith item at any time t (units)
T	Cycle time (unit time) of the supply chain (decision variable)
Q_i	Retailer’s order quantity at time t_i
$t_i=\beta_i T$	Production run time of ith item at the manufacturer (unit time)
SR_{ri}	Sales revenue of the retailer for the ith item
w_i	Wholesale price for ith item ($/unit)
h_{ri}	Holding cost for retailer per unit per annum for ith item
$HC_{ri}(t)$	Time dependent holding cost of retailer for ith item ($/unit / unit time)
ie_i	Rate of interest earned for ith item / $ /year
ic_i	Rate of interest charged for ith item / $ /year
$\pi_{ri}(p_i,T)$	Total profit of the retailer for ith item
$\pi_r{}^I(p_i,T)$	Total profit of the retailer for independent decision
$\pi_r{}^J(p_i,T)$	Total profit of the retailer for joint decision

Manufacturer’s Parameters:

P_i	Finite Production rate proportional to the demand rate
M	Credit period offered by manufacturer to retailer (years)
SR_{mi}	Sales revenue of the manufacturer for the ith item
A_{mi}	Ordering cost per order incurred by the manufacturer for ith item ($/order)
C_i	Purchasing cost of raw materials per item i by the manufacturer ($/unit)
h_{mi}	Holding cost for manufacturer per unit per annum for ith item
$HC_{mi}(t)$	Time dependent holding cost of manufacturer for ith item ($/unit / unit time)
e_i	Cost per unit ($/unit time) idle time $(T-t_i)$ of the manufacturer for manufacturing ith item
$\pi_{mi}(p_i,T)$	Total profit of the manufacturer for ith item
$\pi_m{}^I(p_i,T)$	Total profit of the manufacturer for independent decision
$\pi_m{}^J(p_i,T)$	Total profit of the manufacturer for joint decision

Relations between parameters:

- $C_i < w_i < p_i$
- $IE \le IC$
- $0 \le \theta < 1$

Parameters of supply chain:

$R_i\left(p_i,t\right)$	Selling price and time dependent quadratic demand rate for i^th item. $R_i\left(p_i,t\right)=a_i\left(1+b_it-c_it^2\right)p_i^{-\eta}$, where $a_i>0$ is scale demand, $0<b_i,c_i<1$ are rates of change of demand and η_i is mark-up for selling price.
$P_i\left(p_i,t\right)$	Finite production rate proportional to the demand rate for i^th item, $P_i\left(p_i,t\right)=\lambda_i\cdot R_i\left(p_i,t\right),\lambda_i>1$
$\pi_s^{\,I}\left(p_i,T\right)$	Total profit of the supply chain for independent decision
$\pi_s^{\,J}\left(p_i,T\right)$	Total profit of the supply chain for joint decision

2.2 Assumptions

1. The supply chain comprises of single manufacturer and single retailer for Multi- item.
2. The demand rate, $R_i\left(p_i,t\right)=a_i\cdot\left(1+b_it-c_it^2\right)p_i^{-\eta_i}$ (say) is function of time and selling price, $a_i>0$ is total market potential demand, $0\le b_i<1$ denotes the linear rate of change of demand with respect to time, $0\le c_i<1$ denotes the quadratic rate of change of demand and η_i is mark-up for selling price.
3. The stock of inventory at manufacturer and the retailer are different. Both have the common cycle length to avoid shortages or overstocking. The production run-times of the manufacturer are different and less than the common cycle length. The fact that all items have a common cycle length to avoid frequent orders of the products which increase the setup cost and transportation cost of the retailer. Here, the inventory cycle of the manufacturer follows the successor cycle of the retailer, i.e., inventory of $(i+1)^{th}$ cycle of the retailer is produced at i^{th} cycle of the manufacturer.
4. The manufacturer offers a credit period to the retailer only.
5. In a Manufacturer–retailer supply chain system, the retailer acquires a full up-stream credit period of M years from his/her manufacturer (i.e., if the retailer orders items at time 0, and pays off at time M, then there is no interest charges).
6. Due to permissible delay in payments, the retailer can earn interest on the customer's payment with interest rate, interest earn per unit per year. The retailer will have to settle the account at M, a credit period offered by the manufacturer. Retailer will pay interest charges on the unsold stock with rate, interest charged to the manufacturer.
7. The units in inventory system of each player are subject to deterioration at a constant rate. The deteriorated units are not repaired or replaced during the cycle time.
8. The planning horizon is infinite which will facilitate long time agreement.
9. Lead time is zero or negligible.

3 MATHEMATICAL MODEL

In this proposed model, two echelon supply chain model for multi – item under price sensitive quadratic demand. Here, manufacturer is a buyer of raw materials. The manufacturer sales finished goods to retailer by giving trade credit.

At manufacturer end, the lot size R_iT of raw materials is received at the starting of production and the inventory piles up with rate P_i up to time t_i,i.e., t_i is the production run time. Hence, the lot size R_iT is received by the retailer at the end of manufacturer's cycle length t_i, i.e., production run time (Fig. 1).

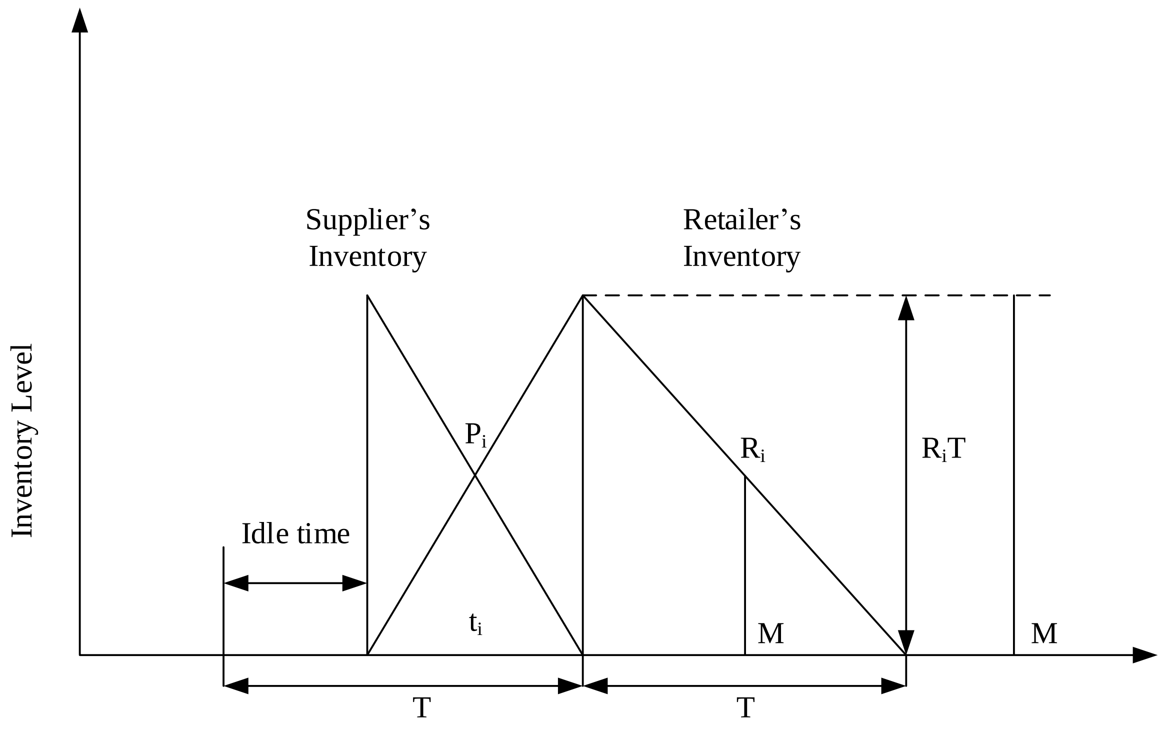

Figure 1. Inventory Level

The manufacturer offers a delay for payments to the retailer. In this period, the retailer may earn interest from selling the products to the customers, whereas the retailer has to pay interest on outstanding amount of purchasing cost to the manufacturer for the delay in payment.

3.1 Retailer's Individual decision Perspective

The retailer's inventory level at time t during a cycle of length T for i^th item is given by

$$\frac{dI_{ri}(t)}{dt} = -R_i(p_i, t) - \theta I_{ri}(t)$$

With boundary condition $I_{ri}(T) = 0$.

Therefore, the order quantity is equal to $I_{ri}(t_i) = Q_i$.

Solving above differential equations we get,

$$I_{ri}(t) = \left(-\frac{\left(-\theta_{ri} b_i + \theta_{ri}{}^2 - 2c_i + \theta_{ri}{}^2 t b_i + 2\theta_{ri} t c_i - c_i t^2 \theta_{ri}{}^2\right) a_i p_i{}^{-\eta_i} e^{\theta_{ri} t}}{\theta_{ri}{}^3} + \frac{\left(-\theta_{ri} b_i + \theta_{ri}{}^2 - 2c_i + \theta_{ri}{}^2 T b_i + 2\theta_{ri} T c_i - c_i T^2 \theta_{ri}{}^2\right) a_i p_i{}^{-\eta_i} e^{\theta_{ri} T}}{\theta_{ri}{}^3} \right) e^{-\theta_{ri} t}$$

Now, the retailer's sales revenue per cycle time T for i^th item is

$$SR_{ri} = \frac{1}{T}\left(p_i \int_0^T R_i\left(p_i, t\right) dt - w_i Q_i \right)$$

Now, the total cost per unit time of retailer for i[th] item is comprised by

- Ordering cost per unit : $OC_r = \frac{A_r}{T}$
- Inventory holding cost per unit : $HC_{ri} = \frac{h_{ri}}{T}\int_0^T I_{ri}\left(t\right) dt$

Scenario 1: $M \le T$

The retailer earns interest; IE_{ri} at the rate ie_i during $\left[0, M\right]$ as $IE_{ri} = \frac{ie_{ri}\, p_i}{T}\int_0^M t\, R_i\left(p_i, t\right) dt$ and

interest charged, IC_{ri} at the rate ic_i per annum during $\left[M, T\right]$ on unsold stock is

$$IC_{ri} = \frac{ic_{ri}\, w_i}{T}\int_M^T I_{ri}\left(t\right) dt .$$

Therefore, the retailer's average profit for n items is

$$\pi_{r1i} = \sum_{i=1}^{n}\left[SR_{ri} - HC_{ri} + IE_{ri} - IC_{ri}\right] - OC_r$$

Scenario 2: $T \le M$

The retailer earns interest; IE_{ri} at the rate ie_i during $\left[0, M\right]$ as

$$IE_{ri} = \frac{ie_{ri}\, p_i}{T}\left[\int_0^T t\, R_i\left(p_i, t\right) dt - Q_i\left(M - T\right)\right]$$

Therefore, the retailer's average profit for n items is

$$\pi_{r2i} = \sum_{i=1}^{n}\left[SR_{ri} - HC_{ri} + IE_{ri}\right] - OC_r$$

Therefore, total profit of the retailer is

$$\pi_r = \begin{cases} \pi_{r1i}\ , M \le T \\ \pi_{r2i}\ , M \ge T \end{cases}$$

3.2 Manufacturer's Individual Perspective

Production run time is always less than or equal to cycle time of retailer's inventory because shortages at any stage are not permitted. The sales revenue earned by manufacturer from i[th] item is

$SR_{mi} = \frac{\left(w_i - C_i\right)}{t_i}\int_0^{t_i} P_i\left(p_i, t\right) dt$. And ordering set-up cost of the manufacturer for i[th] item is

$OC_{mi} = \frac{A_{mi}}{t_i}$. For the raw materials, inventory holding cost per unit for i[th] item is $HC_{mi} = \frac{h_{mi}}{t_i}\int_0^{t_i} t\, P_i\ dt$.

As a manufacturer received the raw material at the starting of the production rate with full payment to the third party, the average cost of idle time $\left(T - t_i\right)$ is $ITC_{mi} = e_i\left(\frac{P_i\left(p_i, t\right)}{R_i\left(p_i, t\right)} - 1\right)$. This idle time is considered at the starting of the cycle of manufacturer to avoid extra inventory cost of the whole lot $R_i T$ during the idle time $(T - t_i)$ if production starts at very beginning of the cycle T .

Scenario 1: $M \le T$

The manufacturer earns interest from the retailer; IE_{mi} at the rate ie_i during $[M, T]$ for i[th] item is

$IE_{mi} = \frac{ie_i w_i}{T} \int_0^{T-M} t \, P_i \, dt$ and interest charged, IC_{mi} at the rate ic_i per annum during $[0, M]$ on raw material is $IC_{mi} = \frac{ic_i C_i}{t_i} \int_0^{M} P_i \, dt$.

Therefore, the manufacturer's average profit for n items is

$$\pi_{m1i} = \sum_{i=1}^{n} \left[SR_{mi} - OC_{mi} - HC_{mi} + IE_{mi} - IC_{mi} - ITC_{mi} \right]$$

Scenario 2: $M \ge T$

The manufacturer interest charged, IC_{mi} at the rate ic_i per annum during $[0, M]$ on raw material is

$$IC_{mi} = \frac{ic_i C_i}{t_i} \int_0^{M} P_i \, dt .$$

Therefore, the manufacturer's average profit for n items is

$$\pi_{m2i} = \sum_{i=1}^{n} \left[SR_{mi} - OC_{mi} - HC_{mi} - IC_{mi} - ITC_{mi} \right]$$

Therefore, total profit of the manufacturer is

$$\pi_m = \begin{cases} \pi_{m1i} , \ M \le T \\ \pi_{m2i} \ , M \ge T \end{cases} .$$

3.3 Independent decision policy

In independent decision policy, retailer is decision maker of the whole supply chain. The retailer will set selling prices of two items and time to order and policy is followed by the manufacturer. With these decisions, manufacturer will deduce his profit.

3.4 Joint decision policy:

In the joint policy, the joint total profit of the supply chain for scenario 1 and scenario 2 are

$$\pi_S{}^J = \begin{cases} \pi_{m1i} + \pi_{r1i} , & M \le T \\ \pi_{m2i} + \pi_{r2i} , & M \ge T \end{cases}$$

$$\pi_S{}^J = \pi_m{}^J + \pi_r{}^J$$

Here, the decision variables will be obtained by setting partial derivatives of the objective function to be zero. For obtained values of selling prices of two items and cycle time, joint profit of the supply chain will be computed.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1 Numerical Example

Example 1 (Scenario 1 : $M \le T$ **):** We have considered the values of parameters of two types of items in appropriate units as follows: $a_1 = 15000$ units, $b_1 = 0.2$, $c_1 = 0.08$, $\eta_1 = 1.1$, $\lambda_1 = 1.1$,

$w_1 = \$5$ per unit , $A_r = \$\ 50$ per lot, $M = 0.4$ week, $h_{r1} = \$0.19$ per unit per cycle, $ic_1 = 0.12$, $ie_1 = 0.09$, $\beta_1 = 0.4$, $a_2 = 17000$ units, $b_2 = 0.1$, $c_2 = 0.07$, $\eta_2 = 1.3$, $\lambda_2 = 1.1$, $w_2 = \$4$ per unit, $h_{r2} = \$0.18$ per unit per cycle, $ic_2 = 0.11$, $ie_2 = 0.08$, $\beta_2 = 0.3$, $\theta = 0.15$, $C_1 = \$2$ per unit, $C_2 = \$2.5$ per unit, $A_{m1} = \$\ 30$ per lot, $A_{m2} = \$\ 40$ per lot, $h_{m1} = \$0.8$ per unit per cycle, $h_{m2} = \$0.9$ per unit per cycle, $e_1 = \$80$ per lot per week, $e_2 = \$60$ per lot per week then the optimal solution for individual decision of retailer for this scenario is $T = 0.93$ week , $p_1 = \$\ 37$ per unit , $p_2 = \$\ 13.40$ per unit, $\pi_r{}^I = \$\ 16392.33$ per week, $\pi_m{}^I = \$\ 1353.23$ per week, $\pi_s{}^I = \$\ 17745.57$ per year and the optimal solution for joint decision of both retailer and manufacturer is $T = 1.26$ week , $p_1 = \$\ 7.31$ per unit , $p_2 = \$\ 7.7$ per unit, $\pi_r{}^J = \$\ 12340.75$ per week, $\pi_m{}^J = \$\ 6926.73$ per week, $\pi_s{}^J = \$\ 19267.48$ per year. The above results show that individual decision is more profitable for retailer than joint decision. But, if we want to focus for whole supply chain then it is cleared from table1 and fig. 1 that joint decision is more profitable for whole supply chain as compare to independent decision. Profit of the whole supply chain is increased by 8.57% if players follow joint decision.

The concavity of the profit function is obtained by the well-known Hessian matrix. Now, Hessian matrix is for the above supply chain for joint decision of the players is

$$H(p_1, p_2, T) = \begin{pmatrix} \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial p_1{}^2} & \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial p_1 \partial p_2} & \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial p_1 \partial T} \\ \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial p_2 \partial p_1} & \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial p_2{}^2} & \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial p_2 \partial T} \\ \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial T \partial p_1} & \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial T \partial p_2} & \dfrac{\partial^2 \pi_s(p_1, p_2, T)}{\partial T^2} \end{pmatrix}$$

Using the above example 1, we get the hessian matrix $H(p_1, p_2, T)$ at the point (p_1, p_2, T)

$$H(p_1, p_2, T) = \begin{pmatrix} -25.0303 & 96.3578 & 0 \\ 96.3578 & -1564.8575 & 0 \\ 0 & 0 & -48.0609 \end{pmatrix}$$

As in Barrón and Sana (2015), if the eigenvalues of the Hessian matrix at the solution (p_1, p_2, T) are all negative, then the profit function $\pi_s{}^J(p_1, p_2, T)$ is maximum at that solution. Here, Eigenvalues of above Hessian matrix are $\lambda_1 = -19.02$ $\lambda_2 = -1570.86$ $\lambda_3 = -48.06$. So, the profit function $\pi_s{}^J(p_1, p_2, T)$ is maximum. Also, the concavity of the profit obtained in figs. 2-4 with respect to two variables at a time.

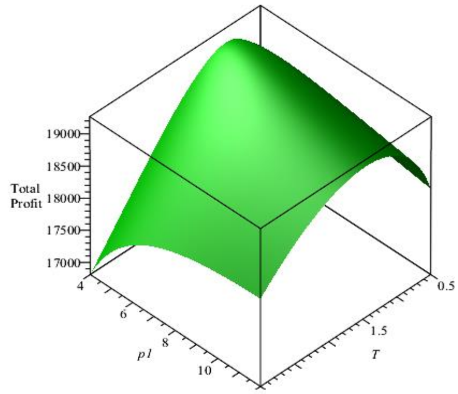

Figure. 2: Concavity of total profit V_s selling price $\left(p_1\right)$ and cycle time $\left(T\right)$.

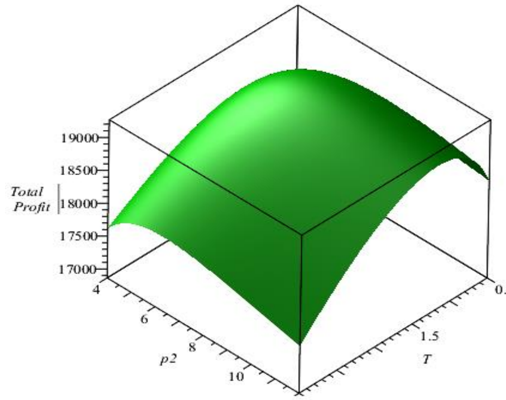

Figure. 3: Concavity of total profit V_s selling price $\left(p_2\right)$ and cycle time $\left(T\right)$.

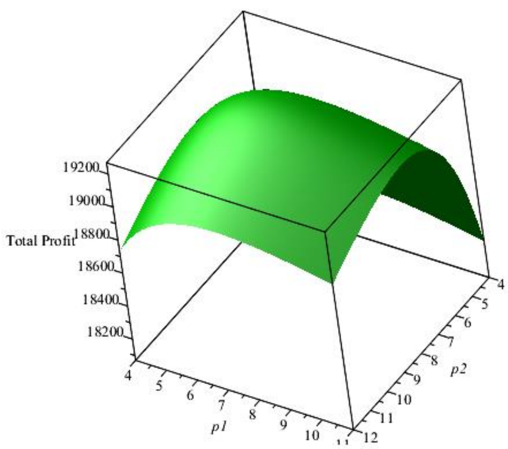

Figure. 4: Concavity of total profit V_s selling price $\left(p_1\right)$ and selling price $\left(p_2\right)$.

Example 2 (For Scenario 1 $M \geq T$): We have considered the values of parameters of two types of items in appropriate units as follows: $a_1 = 25000$ units, $b_1 = 0.3$, $c_1 = 0.25$, $\eta_1 = 1.18$, $\lambda_1 = 1.5$, $w_1 = \$13$ per unit , $A_r = \$10$ per lot, $M = 2$ week, $h_{r1} = \$3$ per unit per cycle, $ic_1 = 0.12$, $ie_1 = 0.09$, $\beta_1 = 0.4$, $a_2 = 2700$ units, $b_2 = 0.1$, $c_2 = 0.09$, $\eta_2 = 1.2$, $\lambda_2 = 1.2$, $w_2 = \$18$ per unit, $h_{r2} = \$2$ per unit per cycle, $ic_2 = 0.11$, $ie_2 = 0.08$, $\beta_2 = 0.3$, $\theta = 0.15$, $C_1 = \$7$ per unit, $C_2 = \$11$ per unit, $A_{m1} = \$10$ per lot, $A_{m2} = \$30$ per lot, $h_{m1} = \$0.8$ per unit per cycle, $h_{m2} = \$0.9$ per unit per cycle, $e_1 = \$4$ per lot per week, $e_2 = \$5$ per lot per week then the optimal solution for individual decision of retailer for this scenario is $T = 1.49$ week , $p_1 = \$66.90$ per unit , $p_2 = \$88.51$ per unit, $\pi_r^{\ I} = \$11650.33$ per week, $\pi_m^{\ I} = \$828.40$ per week, $\pi_s^{\ I} = \$12478.73$ per year and the optimal solution for joint decision of both retailer and manufacturer is $T = 1.98$ week , $p_1 = \$28.74$ per unit, $p_2 = \$68.75$ per unit, $\pi_r^{\ J} = \$10234.55$ per week, $\pi_m{}^J = \$ \; 2854.35$ per week, $\pi_s{}^J = \$ \; 13088.90$ per year. The above results show that individual decision is more profitable for retailer than joint decision. But, if we want to focus for whole supply chain then it is cleared from table1 and fig. 4 that the joint decision is more profitable for whole supply chain as compare to independent decision. Profit of the whole supply chain is increased by 4.88% if players follow joint decision.

Table 1. Joint decision Vs Independent decision

Scenario	**Decision**	**Decision maker**	π_r	π_m	π_s	**% change**
$M \le T$	Individual	Retailer	16392.33	1353.23	17745.57	-
	Joint	-	12340.75	6926.73	19267.49	8.57
$M \ge T$	Individual	Retailer	11650.33	828.40	12478.73	-
	Joint	-	10234.55	2854.35	13088.90	4.88

Total profit for the above two different scenarios can be described by following bar graph Fig.5. The optimum solution is exhibited in Table 2.

Table 2. Optimal Solution

Scenario	Decision	Profit of the Players ($)	Total Profit of the Supply Chain π_s ($)	Decision (in weeks & in $)
$M \le T$ & $M = 0.4$	Individual	$\pi_r{}^I = 16392.33$ $\pi_m{}^I = 1353.23$	17745.57	$T = 0.92$ $p_1 = 37.00$ $p_2 = 13.40$
	Joint	$\pi_r{}^J = 12340.75$ $\pi_m{}^J = 6926.73$	**19267.49**	$T = 1.26$ $p_1 = 7.32$ $p_2 = 7.70$
$M \ge T$ & $M = 2$	Individual	$\pi_r{}^I = 11650.33$ $\pi_m{}^I = 828.40$	12478.73	$T = 1.49$ $p_1 = 66.90$ $p_2 = 88.51$
	Joint	$\pi_r{}^J = 10234.55$ $\pi_m{}^J = 2854.35$	13088.90	$T = 1.98$ $p_1 = 28.74$ $p_2 = 68.75$

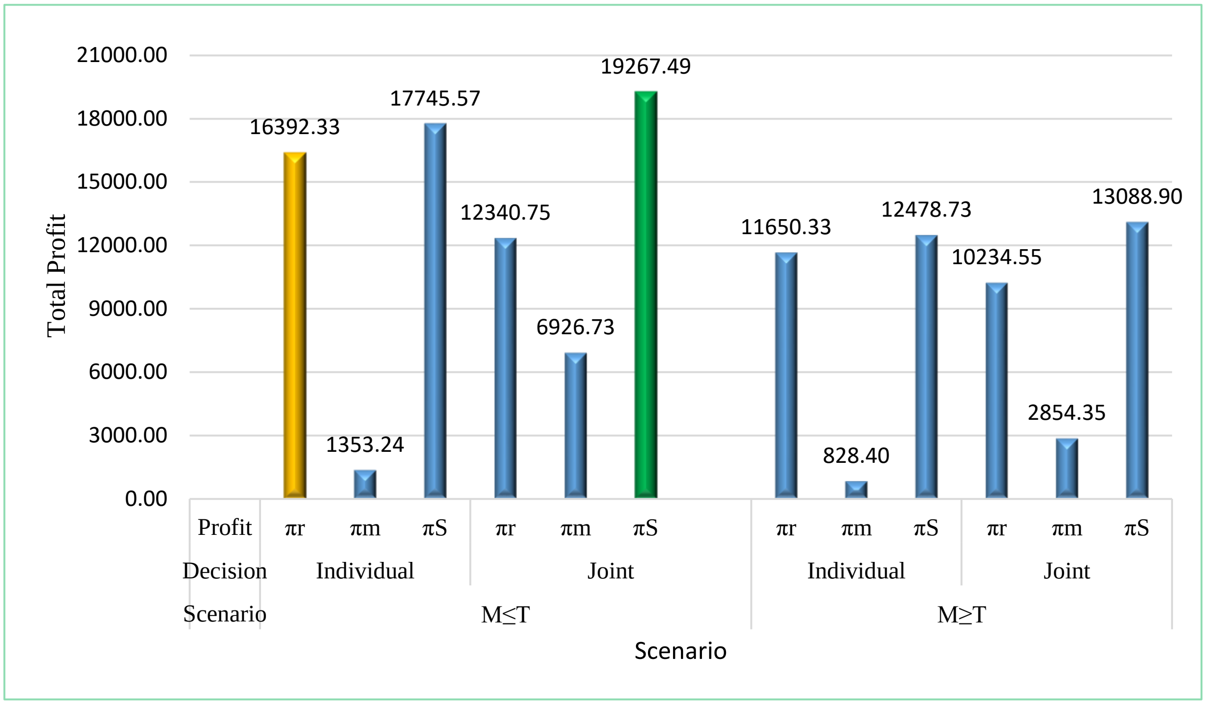

Figure 5. Optimal Solution

From fig.. 5 and table 2 it is cleared that in scenario 1 ($M \leq T$), if retailer is a decision maker then his profit is maximum when he takes autonomous decision. But, for the whole supply chain joint decision is the best policy to earn maximum profit. Moreover, in the scenario 2 ($M \geq T$), if the retailer is decision maker then he earns maximum profit when the retailer takes independent decision. But for the supply chain joint decision is the best policy. Finally from the whole analysis scenario 1($M \leq T$) and joint decision policy is best for this model. So, terms and conditions should be fabricated that both the players willingly take joint decision (for scenario 1) to maximize total profit.

4.2 Sensitivity Analysis for the Inventory Parameters

Therefor for the different inventory parameters, the sensitivity analysis of example 1 for joint decision is carried out by changing one variable at a time as $-20\%, -10\%$, 10% and 20%.

Table 3: Sensitivity analysis

Parameter	Values	p_1	p_2	T	Total Profit πs^J
a_1	12000	7.56	7.65	1.32	16834.03
	13500	7.43	7.67	1.29	18050.22
	15000	7.32	7.70	1.26	19267.49
	16500	7.21	7.73	1.23	20485.68
	18000	7.12	7.75	1.21	21704.68
b_1	0.16	6.15	7.82	1.14	19131.31
	0.18	6.69	7.76	1.19	19194.85
	0.20	7.32	7.70	1.26	19267.49
	0.22	8.04	7.63	1.34	19350.26

	0.24	8.88	7.55	1.44	19444.40
c_1	0.064	8.06	7.63	1.34	19305.29
	0.072	7.66	7.67	1.30	19285.05
	0.080	7.32	7.70	1.26	19267.49
	0.088	7.02	7.73	1.23	19252.10
	0.096	6.76	7.75	1.20	19238.56
η_1	1.10	7.32	7.70	1.26	19267.49
	1.21	3.26	8.05	0.95	17377.22
	1.32	2.24	8.21	0.86	16311.64
λ_1	0.88	14.73	7.51	1.50	18576.13
	0.99	11.14	7.57	1.42	18857.42
	1.10	7.32	7.70	1.26	19267.49
	1.21	3.17	8.03	0.96	20058.95
w_1	4.00	12.56	7.50	1.51	18745.33
	4.50	10.08	7.57	1.42	18967.30
	5.00	7.32	7.70	1.26	19267.49
	5.50	4.05	8.01	0.98	19766.81
	6.00	0.96	8.58	0.69	21307.05
A_r	40.00	7.29	7.71	1.25	19275.44
	45.00	7.30	7.70	1.26	19271.46
	50.00	7.32	7.70	1.26	19267.49
	55.00	7.33	7.70	1.26	19263.52
	60.00	7.34	7.70	1.27	19259.57
M	0.32	6.55	7.34	1.16	19423.09
	0.36	6.96	7.52	1.22	19340.58
	0.40	7.32	7.70	1.26	19267.49
	0.44	7.62	7.88	1.30	19202.26
	0.48	7.90	8.05	1.33	19143.83
h_{r1}	0.152	7.08	7.69	1.27	19316.02
	0.171	7.20	7.70	1.27	19291.46
	0.190	7.32	7.70	1.26	19267.49
	0.209	7.43	7.71	1.25	19244.06
	0.228	7.55	7.71	1.25	19221.16
ic_1	0.096	6.35	7.73	1.23	19419.85
	0.108	6.84	7.71	1.25	19340.92
	0.120	7.32	7.70	1.26	19267.49
	0.132	7.79	7.69	1.27	19198.80
	0.144	8.26	7.68	1.28	19134.27
ie_1	0.072	7.50	7.66	1.31	19266.18
	0.081	7.41	7.68	1.28	19267.17
	0.090	7.32	7.70	1.26	19267.49
	0.099	7.23	7.72	1.24	19267.18

	0.108	7.14	7.74	1.22	19266.30
β_1	0.32	13.52	7.50	1.51	18649.34
	0.36	10.51	7.57	1.42	18913.15
	0.40	7.32	7.70	1.26	19267.49
	0.44	3.71	8.03	0.96	19866.59
	0.48	0.48	8.70	0.65	22289.69
a_2	13600	7.20	7.73	1.23	17806.20
	15300	7.26	7.71	1.25	18536.71
	17000	7.32	7.70	1.26	19267.49
	18700	7.37	7.69	1.27	19998.49
	20400	7.42	7.68	1.29	20729.71
b_2	0.08	7.18	7.64	1.22	19207.61
	0.09	7.24	7.67	1.24	19237.29
	0.10	7.32	7.70	1.26	19267.49
	0.11	7.39	7.73	1.28	19298.21
	0.12	7.47	7.77	1.30	19329.50
c_2	0.056	7.47	7.75	1.30	19298.15
	0.063	7.39	7.73	1.28	19282.57
	0.070	7.32	7.70	1.26	19267.49
	0.077	7.25	7.68	1.24	19252.86
	0.084	7.19	7.66	1.23	19238.64
η_2	1.04	6.61	47.38	1.07	26445.80
	1.17	7.04	12.34	1.19	21750.35
	1.30	7.32	7.70	1.26	19267.49
	1.43	7.50	5.88	1.31	17669.77
	1.56	7.62	4.91	1.34	16549.51
λ_2	0.88	6.79	9.03	1.12	18965.92
	0.99	7.01	8.39	1.18	19106.00
	1.10	7.32	7.70	1.26	19267.49
	1.21	7.75	6.94	1.37	19458.98
	1.32	8.43	6.04	1.55	19696.93
w_2	3.20	7.02	9.10	1.18	18926.79
	3.60	7.14	8.41	1.22	19087.38
	4.00	7.32	7.70	1.26	19267.49
	4.40	7.56	6.97	1.32	19472.78
	4.80	7.95	6.18	1.42	19712.74
h_{r2}	0.144	7.42	7.57	1.29	19297.89
	0.162	7.37	7.64	1.27	19282.52
	0.180	7.32	7.70	1.26	19267.49
	0.198	7.27	7.76	1.25	19252.76
	0.216	7.22	7.83	1.24	19238.34

ic_2	0.088	7.34	7.31	1.27	19382.34
	0.099	7.33	7.50	1.26	19323.94
	0.110	7.32	7.70	1.26	19267.49
	0.121	7.31	7.90	1.26	19212.87
	0.132	7.30	8.10	1.25	19159.99
ie_2	0.064	6.95	8.06	1.16	19181.61
	0.072	7.11	7.89	1.21	19222.56
	0.080	7.32	7.70	1.26	19267.49
	0.088	7.57	7.48	1.33	19317.62
	0.096	7.91	7.20	1.41	19374.92
β_2	0.24	9.86	12.67	1.92	18185.14
	0.27	9.01	10.17	1.69	18650.64
	0.30	7.32	7.70	1.26	19267.49
	0.33	5.71	5.34	0.70	19748.93
	0.36	4.04	3.82	0.32	19134.27
θ	0.120	7.16	7.43	1.34	19401.85
	0.135	7.24	7.57	1.30	19333.09
	0.150	7.32	7.70	1.26	19267.49
	0.165	7.40	7.83	1.22	19204.71
	0.180	7.48	7.94	1.19	19144.50
C_1	1.60	0.13	8.89	0.59	24320.53
	1.80	3.56	8.08	0.93	19881.69
	2.00	7.32	7.70	1.26	19267.49
	2.20	10.47	7.57	1.42	18925.31
	2.40	13.33	7.50	1.52	18678.76
C_2	2.00	7.65	5.02	1.35	20242.85
	2.25	7.44	6.37	1.29	19688.66
	2.50	7.32	7.70	1.26	19267.49
	2.75	7.24	9.03	1.24	18931.20
	3.00	7.18	10.36	1.22	18653.43
A_{m1}	24.00	7.28	7.71	1.25	19279.43
	27.00	7.30	7.70	1.26	19273.45
	30.00	7.32	7.70	1.26	19267.49
	33.00	7.33	7.70	1.26	19261.54
	36.00	7.35	7.69	1.27	19255.62
A_{m2}	32.00	7.26	7.71	1.25	19288.77
	36.00	7.29	7.71	1.25	19278.10
	40.00	7.32	7.70	1.26	19267.49
	44.00	7.34	7.69	1.27	19256.94
	48.00	7.37	7.69	1.27	19246.44
h_{m1}	0.64	6.91	7.69	1.28	19349.40
	0.72	7.11	7.69	1.27	19307.64

	0.80	7.32	7.70	1.26	19267.49
	0.88	7.52	7.71	1.25	19228.82
	0.96	7.72	7.72	1.24	19191.53
h_{m2}	0.72	7.47	7.50	1.30	19314.70
	0.81	7.39	7.60	1.28	19290.70
	0.90	7.32	7.70	1.26	19267.49
	0.99	7.25	7.80	1.24	19245.00
	1.08	7.18	7.89	1.22	19223.18
e_1	64	7.32	7.70	1.26	19269.09
	72	7.32	7.70	1.26	19268.29
	80	7.32	7.70	1.26	19267.49
	88	7.32	7.70	1.26	19266.69
	96	7.32	7.70	1.26	19265.89
e_2	48	7.32	7.70	1.26	19268.69
	54	7.32	7.70	1.26	19268.09
	60	7.32	7.70	1.26	19267.49
	66	7.32	7.70	1.26	19266.89
	72	7.32	7.70	1.26	19266.29

In order to examine the sensitivity of the model parameters on the optimal solution, we consider the data as given in numerical Example 1 of the scenario 1 ($M \le T$) for the joint decision policy. Optimal solutions for different values of $a_1, b_1, c_1, \eta_1, \lambda_1, w_1, A_r, M$, $h_{r1}, ic_1, ie_1, \beta_1, a_2, b_2, c_2, \eta_2, \lambda_2, w_2, h_{r2}, ic_2, ie_2, \beta_2, \theta$, $C_1, C_2, A_{m1}, A_{m2}, h_{m1}, h_{m2}, e_1$ and e_2 are presented in Table 3. The following observation could be made from Table 3.

1. In Table 3, linear rate of change of demand and quadratic rate of change of demand of item 1 increases selling price of item 1 $\left(p_1\right)$ rapidly whereas purchasing cost of the manufacturer of item 1, mark-up for selling price of the item 1, production rate of item 1 and wholesale price of item 1 decreases selling price of item 1 rapidly. Moreover, total market potential demand of item 1, quadratic rate of change of demand for item 2, rate of interest earned for item 1, holding cost rate for retailer for item 2, rate of interest charged for item 2, purchasing cost of item 2 by the manufacturer and holding cost rate for manufacturer for item 2 decreases selling price of item 1 slowly. However, ordering cost per order incurred by the retailer, credit period offered by manufacturer to retailer , holding cost rate for retailer for item 1, rate of interest charged for item 1, total market potential demand for item 2, linear rate of change of demand for item 2, mark-up for selling price for item 2, production rate of item 2, wholesale price of item 2, rate of interest earned for item 2, deterioration rate, ordering cost per order incurred by the manufacturer for both the items and holding cost rate for manufacturer for item 1 increases selling price of item1 slowly. In addition, change in cost for idle time $\left(T - t_i\right)$ of the manufacturer for manufacturing both the items selling price for item 1 remain constant.
2. In Table 3, mark-up for selling price of the item 1 and purchasing cost of item 2 by the manufacturer increases selling price of item 2 $\left(p_2\right)$ rapidly whereas mark-up for selling price for item 2, production rate of item 2 and wholesale price of item 2 decreases selling price of item 2 rapidly. Moreover, quadratic rate of change of demand of item 1, total market potential demand of item 1, production rate of item 1, wholesale price of item 1, holding cost rate for retailer for item 1, credit period offered by manufacturer to retailer, rate of interest earned for both the items, linear rate of change of demand for item 2, holding cost rate for manufacturer for item 2, rate of interest charged for item 2, deterioration rate and holding cost rate for manufacturer for both the items increases selling price of item 2 slowly. However, linear rate of change of

demand of item 1, rate of interest charged for item 1, total market potential demand for item 2, quadratic rate of change of demand for item 2, purchasing cost of the manufacturer for item 2 and ordering cost per order incurred by the manufacturer for both the items decreases selling price of item 2 slowly. In addition, change in cost for idle time $\left(T - t_i\right)$ of the manufacturer for manufacturing both the items and ordering cost per order incurred by the retailer, selling price for item 2 remain constant.

3. In Table 3, production rate of item 2 and purchasing cost of item 1 increases cycle time rapidly whereas mark-up for selling price of the item 1 decreases cycle time rapidly. Moreover, linear rate of change of demand for item 1, ordering cost per order incurred by the retailer, credit period offered by manufacturer to retailer, rate of interest charged for item 1, rate of interest earned for the item 1, total market potential demand for item 2, linear rate of change of demand for item 2, mark-up for selling price of the item 2, wholesale price of item 2, rate of interest earned for the item 2, purchasing cost of item 2 and ordering cost per order incurred by the manufacturer for both the items increases cycle time slowly however linear rate of change of demand for item 1, quadratic rate of change of demand for item 1, production rate of item 1, wholesale price of item 1, holding cost rate for retailer for item 2, rate of interest charged for item 2, deterioration rate and holding cost rate for manufacturer for both the items decreases cycle time slowly. In addition, change in cost for idle time $\left(T - t_i\right)$ of the manufacturer for manufacturing both the items cycle time remain constant.

4. In Table 3, total market potential demand for both items, production rate of item 1 and wholesale price of item 1 increases total profit rapidly whereas mark-up for selling price of both the items and purchasing cost of both the items decreases total profit rapidly. Moreover, quadratic rate of change of demand for both the items, production rate of item 2, wholesale price of item 2 and rate of interest earned for the item 2 increase total profit slowly. However, quadratic rate of change of demand for both the items, deterioration rate, holding cost rate for manufacturer for both the items, holding cost rate for retailer for both the items, ordering cost per order incurred by the manufacturer for both the items, ordering cost of the retailer, rate of interest charged for both the items and credit period offered by manufacturer to retailer decreases total profit slowly. In addition, change in cost for idle time $\left(T - t_i\right)$ of the manufacturer for manufacturing both the items total profit remain constant.

5. CONCLUSION

In this paper, we consider supply chain inventory model for constant deterioration under selling price and cycle time dependent demand. Also, manufacturer gives trade credit to retailer to boost his demand. To maximize the total profit of the supply chain, two policies are analyzed, independent decision and joint decision with respect to selling price of the i^{th} item and cycle time. The best policy is analyzed for optimality. For numerical examples, retailer reaches the maximum profit and carry-out sensitivity analysis with respect to inventory parameters. Current research have several possible extension like, model can be further generalized by allowing shortages and taken more items at a time. One can also analyzed Multi layered supply chain.

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