IMPROVED FAMILY OF RATIO TYPE ESTIMATORS FOR ESTIMATING POPULATION MEAN USING CONVENTIONAL AND NON CONVENTIONAL LOCATION PARAMETERS

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ABSTRACT
The present paper deals with estimation of population mean with more precision by incorporating the auxiliary information, as it has been recognised that incorporation of auxiliary information at design or at estimation stage or at both stages, results in more precision in estimating population parameters of interest. We have proposed the improved family of estimators by incorporating the auxiliary information of conventional and non-conventional location parameters and their linear combinations with correlation coefficient and coefficient of variation for estimating the population mean with more precision. The properties associated with the proposed estimators are assessed by mean square error and bias and compared with the classical and existing estimators. By this comparison we found that our proposed estimators are more efficient than the classical and existing estimators. Numerical illustration is also given in support of the theoretical results.

KEYWORDS: Conventional and Non-Conventional location parameters; Correlation Coefficient; Coefficient of Variation; Mean square error; Bias; Efficiency.

MSC: 62D05

RESUMEN
Este paper trata con la estimación de la media de la población con más precisión incorporando una información auxiliar, como ha sido reconocido la incorporación de esta en las etapas de diseño o de estimación, o en ambas etapas, resulta en mayor precisión en la estimación de parámetros poblacionales de interés. Nosotros hemos mejorado la familia de estimadores incorporando información auxiliar de parámetros de posición convencionales y no-convencionales y de combinaciones lineales, de ellos con los coeficientes de correlación y de variación, para estimar la media de la población con mejor precisión. Las propiedades asociadas con los propuestos estimadores son realizados usando el erro cuadrático medio y el sesgo los que se compararon con estimadores clásicos y otros existentes. Con tal comparación hallamos que los propuestos estimadores son más eficientes que los clásicos y existentes estimadores. Ilustraciones numéricas también se dan para soportar los resultados teóricos.

PALABRAS CLAVE: Parámetros de posición Convencionales y No-Convencionales; Coeficiente de Correlación; Coeficiente de Variación; Error cuadrático medio; Sesgo; Eficiencia.

1. INTRODUCTION

The important objective of sampling theory is the estimation part and that estimation part is used to obtain the estimators of the parameter of interest with more precision and to get more precision in estimating the population parameters of interest, we incorporate more information seeking for the estimation procedure, yields better estimators, provided that the information should be valid and proper. Use of that auxiliary information is made through the ratio method of estimation to obtain an improved estimator of the population mean, but the situation is that the auxiliary variable X should be closely related with the main variable Y under study and the correlation between the study and auxiliary variable should be positive and the line should pass through the origin. But when the correlation between the study variable and the auxiliary variable is negative in that case product type estimators are more accurate than the other estimators and regression

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estimators are used when the regression line does not pass through the origin. Cochran (1940) initiated the use of auxiliary information at the estimation stage and proposed a ratio estimator for population mean. It is a well established fact that the ratio type estimator provides better efficiency in comparison to simple mean estimator if the study and auxiliary variable are positively correlated. If the correlation between the study variable and auxiliary variables is negative, product estimator given by Robson (1957) is more efficient than simple mean estimator.

Further improvements are also achieved on the classical ratio estimator by introducing a large number of modified ratio estimators with the use of known parameters like, the coefficient of variation, coefficient of kurtosis, coefficient of skewness and population correlation coefficient. For more detailed discussion one may refer to Cochran (1977), Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Murthy (1967), Prasad (1989), Rao (1991), Singh (2003), Singh and Tailor (2003, 2005), Singh et al (2004), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) and Yan and Tian (2010).

Further, Subramani and Kumarapandiyan (2013) had taken the initiative by proposing a modified ratio estimator for estimating the population mean of the study variable by using the population median of the auxiliary variable. Recently Abid *et al* (2016 a) suggested some modified ratio estimators for the population mean $\bar{Y}$ in simple random sampling using non-conventional location parameters as auxiliary information, which are given in the existing estimators in section 1.2. Abid *et al* (2016 b) suggested the following ratio estimators for the population mean $\bar{Y}$ in simple random sampling using non-conventional measures of dispersion as auxiliary information. Subzar *et al* (2016) had also proposed some modified ratio estimator for estimating the population mean of the study variable by using the population deciles and correlation coefficient of the auxiliary variable, also Subzar *et al* (2017a) had proposed some modified ratio type estimators using the quartile deviation and population deciles of auxiliary variable and Subzar *et al* (2017b) had also proposed an efficient class of estimators by using the auxiliary information of population deciles, median and their linear combination with correlation coefficient and coefficient of variation. Some other authors such as Jeelani *et al* (2017), Sharma and Singh (2014), Sharma *et al* (2016), Verma *et al* (2015), Sharma and Singh (2015a) and Sharma and Singh (2015b) have also done the similar work in different sampling schemes.

The objective of the paper is to a propose new modified ratio type estimators for estimating the population mean by using the auxiliary information provided by population median, Quartile deviation and non-conventional location measures (Tri- mean, Mid-range and Hodges-Lehmann) and their linear combinations with correlation coefficient and coefficient of variation. Let $U = \{U_1, U_2, U_3, ..., U_N\}$ be a finite population of N distinct and identifiable units. Let y and x denotes the study variable and the auxiliary variable taking values y_i and x_i respectively on the i^{th} unit *(i = 1, 2,..., N).* Before discussing about the proposed estimators, we will mention the estimators in Literature using the notations given in the next sub-section.

2. 1. Notations used

The following are the notations used in the paper:

N	Population size	n	Sample size
$f = n/N$	Sampling fraction	Y	Study variable
X	Auxiliary variable	$\bar{X}, \bar{Y}$	Population mean
$\bar{x}, \bar{y}$	Sample means	x, y	Sample totals
s_x, s_y	Population standard deviations	s_{xy}	Population covariance between
c_x, c_y	Coefficient of variation	ρ	Correlation coefficient
$B(.)$	Bias of the Estimator	β_2	Kurtosis
$MSE(.)$	Mean square error of the estimator	β_1	Skewness
QD	Quartile deviation	M_d	Median
$D_q \ q = 1, 2, ..., 10.$	Population deciles	TM	Tri- mean

MR	Mid- range	HL	Hodges-Lehmann
$\hat{\hat{Y}}_i$	Existing modified ratio estimator of $\bar{Y}$		
$\hat{\hat{Y}}_{pj}$	Proposed modified ratio estimator of $\bar{Y}$		

Subscript

i	For existing estimators
j	For proposed estimators

Based on the above mentioned notations, the mean ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y is given as

$$\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X},$$

Where $\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X}$.

The bias, constant and the mean square error of the mean ratio estimator are given by

$$B(\hat{\bar{Y}}_r) = \frac{(1-f)}{n}\frac{1}{\bar{X}}(RS_x^2 - \rho S_x S_y), \quad R = \frac{\bar{Y}}{\bar{X}}, \quad MSE(\hat{\bar{Y}}_r) = \frac{(1-f)}{n}(S_y^2 + R^2S_x^2 - 2R\rho S_x S_y).$$

The mean ratio estimator given above is used to improve the precision of the estimate of the population mean in comparison with the sample mean estimator whenever a positive correlation exists between the study variable and the auxiliary variable.

2.2. Estimators in the Literature

Kadilar and Cingi (2004) suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation. They showed that their suggested estimators are more efficient than the traditional ratio estimator in the estimation of the population mean. Their estimators are given by

$$\hat{\hat{Y}}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}}\bar{X}, \quad \hat{\hat{Y}}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)}(\bar{X} + C_x), \quad \hat{\hat{Y}}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_2)}(\bar{X} + \beta_2),$$

$$\hat{\hat{Y}}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + C_x)}(\bar{X}\beta_2 + C_x), \quad \widehat{\bar{Y}}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + b_2)}(\bar{X}C_x + b_2).$$

The biases, related constants, and the MSE for the estimators of Kadilar and Cingi (2004) are, respectively, as follows:

$$B(\hat{\hat{Y}}_1) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_1^2, \quad R_1 = \frac{\bar{Y}}{\bar{X}} \quad MSE(\hat{\hat{Y}}_1) = \frac{(1-f)}{n}(R_1^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{Y}}_2) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_2^2, \quad R_2 = \frac{\bar{Y}}{(\bar{X} + C_x)} \quad MSE(\hat{\hat{Y}}_2) = \frac{(1-f)}{n}(R_2^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{Y}}_3) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_3^2, \quad R_3 = \frac{\bar{Y}}{(\bar{X} + \beta_2)} \quad MSE(\hat{\hat{Y}}_3) = \frac{(1-f)}{n}(R_3^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{Y}}_4) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_4^2, \quad R_4 = \frac{\bar{Y}}{(\bar{X}\beta_2 + C_x)} \quad MSE(\hat{\hat{Y}}_4) = \frac{(1-f)}{n}(R_4^2S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{Y}}_5) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_5^2, \quad R_5 = \frac{\bar{Y}}{(\bar{X}C_x + C_x)} \quad MSE(\hat{\hat{Y}}_5) = \frac{(1-f)}{n}(R_5^2S_x^2 + S_y^2(1-\rho^2)).$$

Kadilar and Cingi (2006) developed some modified ratio estimators using known values of coefficient of correlation, kurtosis, and coefficient of variation as follows:

$$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho), \quad \hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho),$$

$$\hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x), \quad \hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho),$$

$$\hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2).$$

The biases, related constants, and the MSE for the estimators of Kadilar and Cingi (2006) are, respectively, given by

$$B(\hat{\bar{Y}}_6) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_6^2, \quad R_6 = \frac{\bar{Y}}{\bar{X}+\rho} \qquad MSE(\hat{\bar{Y}}_6) = \frac{(1-f)}{n}(R_6^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_7) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_7^2, \quad R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho} \qquad MSE(\hat{\bar{Y}}_7) = \frac{(1-f)}{n}(R_7^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_8) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_8^2, \quad R_8 = \frac{\bar{Y}\rho}{\bar{X}\rho+C_x} \qquad MSE(\hat{\bar{Y}}_8) = \frac{(1-f)}{n}(R_8^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_9) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_9^2, \quad R_9 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho} \qquad MSE(\hat{\bar{Y}}_9) = \frac{(1-f)}{n}(R_9^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{10}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{10}^2, \quad R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2} \qquad MSE(\hat{\bar{Y}}_{10}) = \frac{(1-f)}{n}(R_{10}^2S_x^2+S_y^2(1-\rho^2)).$$

Abid *et al* (2016 a) suggested the some modified ratio estimators for the population mean $\bar{Y}$ in simple random sampling using non-conventional location parameters as auxiliary information. Estimators suggested by Abid *et al* (2016 a) are given as

$$\hat{\bar{Y}}_{11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+TM)}(\bar{X}+TM), \qquad \hat{\bar{Y}}_{12} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+TM)}(\bar{X}C_x+TM),$$

$$\hat{\bar{Y}}_{13} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+TM)}(\bar{X}\rho+TM), \qquad \hat{\bar{Y}}_{14} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+MR)}(\bar{X}+MR),$$

$$\hat{\bar{Y}}_{15} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+MR)}(\bar{X}C_x+MR), \qquad \hat{\bar{Y}}_{16} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+MR)}(\bar{X}\rho+MR),$$

$$\hat{\bar{Y}}_{17} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+HL)}(\bar{X}+HL), \qquad \hat{\bar{Y}}_{18} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+HL)}(\bar{X}C_x+HL),$$

$$\hat{\bar{Y}}_{19} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+HL)}(\bar{X}\rho+HL)$$

The biases, related constants and the mean square error (MSE) for Abid *et al* (2016 a) estimators are respectively given by

$$B(\hat{\bar{Y}}_{11}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{11}^2, \qquad R_{11} = \frac{\bar{Y}}{(\bar{X}+TM)} \qquad MSE(\hat{\bar{Y}}_{11}) = \frac{(1-f)}{n}(R_{11}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{12}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{12}^2, \qquad R_{12} = \frac{\bar{Y}}{(\bar{X}C_X+TM)} \qquad MSE(\hat{\bar{Y}}_{12}) = \frac{(1-f)}{n}(R_{12}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{13}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{13}^2, \qquad R_{13} = \frac{\bar{Y}}{(\bar{X}\rho+TM)} \qquad MSE(\hat{\bar{Y}}_{13}) = \frac{(1-f)}{n}(R_{13}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{14}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{14}^2, \qquad R_{14} = \frac{\bar{Y}}{(\bar{X}+MR)} \qquad MSE(\hat{\bar{Y}}_{14}) = \frac{(1-f)}{n}(R_{14}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{15}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{15}^2, \qquad R_{15} = \frac{\bar{Y}}{(\bar{X}C_X+MR)} \qquad MSE(\hat{\bar{Y}}_{15}) = \frac{(1-f)}{n}(R_{15}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{16}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{16}^2, \qquad R_{16} = \frac{\bar{Y}}{(\bar{X}\rho+MR)} \qquad MSE(\hat{\bar{Y}}_{16}) = \frac{(1-f)}{n}(R_{16}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{17}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{17}^2, \qquad R_{17} = \frac{\bar{Y}}{(\bar{X}+HL)} \qquad MSE(\hat{\bar{Y}}_{17}) = \frac{(1-f)}{n}(R_{17}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{18}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{18}^2, \qquad R_{18} = \frac{\bar{Y}}{(\bar{X}C_X+HL)} \qquad MSE(\hat{\bar{Y}}_{18}) = \frac{(1-f)}{n}(R_{18}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{19}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{19}^2, \qquad R_{19} = \frac{\bar{Y}}{(\bar{X}\rho+HL)} \qquad MSE(\hat{\bar{Y}}_{19}) = \frac{(1-f)}{n}(R_{19}^2 S_x^2 + S_y^2(1-\rho^2)).$$

2. IMPROVED RATIO ESTIMATORS

Motivated by the mentioned estimators in Section 1.2, we propose new modified improved ratio type estimators for estimating the population mean using the population median, quartile deviation and non-conventional location measures and their linear combination with correlation coefficient and coefficient of variation. The aim of proposing these new modified ratio estimators using the above mentioned auxiliary information is that these estimators perform too much better than the existing ratio estimators given in section 1.2. As median, quartile deviation, non conventional location parameters and non conventional measures of dispersion are robust against outliers present the data collected from the population in survey sampling. The proposed estimators are as follows:

$$\hat{\bar{Y}}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\chi_1)}(\bar{X}+\chi_1) \qquad \hat{\bar{Y}}_{p2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\chi_2)}(\bar{X}+\chi_2)$$

$$\hat{\bar{Y}}_{p3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\chi_3)}(\bar{X}+\chi_3) \qquad \hat{\bar{Y}}_{p4} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\chi_4)}(\bar{X}+\chi_4)$$

$$\hat{\bar{Y}}_{p5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\chi_5)}(\bar{X}+\chi_5) \qquad \hat{\bar{Y}}_{p6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\chi_6)}(\bar{X}+\chi_6)$$

$$\hat{\bar{Y}}_{p7} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\chi_1)}(\bar{X}\rho+\chi_1) \qquad \hat{\bar{Y}}_{p8} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\chi_2)}(\bar{X}\rho+\chi_2)$$

$$\hat{\bar{Y}}_{p9} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\chi_3)}(\bar{X}\rho+\chi_3) \qquad \hat{\bar{Y}}_{p10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\chi_4)}(\bar{X}\rho+\chi_4)$$

$$\hat{\hat{\bar{Y}}}_{p11} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \chi_5)}(\bar{X}\rho + \chi_5) \qquad \hat{\hat{\bar{Y}}}_{p12} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \chi_6)}(\bar{X}\rho + \chi_6)$$

$$\hat{\hat{\bar{Y}}}_{p13} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \chi_1)}(\bar{X}C_x + \chi_1) \qquad \hat{\hat{\bar{Y}}}_{p14} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \chi_2)}(\bar{X}C_x + \chi_2)$$

$$\hat{\hat{\bar{Y}}}_{p15} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \chi_3)}(\bar{X}C_x + \chi_3) \qquad \hat{\hat{\bar{Y}}}_{p16} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \chi_4)}(\bar{X}C_x + \chi_4)$$

$$\hat{\hat{\bar{Y}}}_{p17} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \chi_5)}(\bar{X}C_x + \chi_5) \qquad \hat{\hat{\bar{Y}}}_{p18} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \chi_6)}(\bar{X}C_x + \chi_6)$$

Where $\chi_1 = Md \times TM$, $\chi_2 = QD \times TM$, $\chi_2 = Md \times HL$, $\chi_4 = QD \times HL$, $\chi_5 = Md \times MR$ and $\chi_6 = QD \times MR$

The bias, related constant and the MSE for the first proposed estimator can be obtained as follows:
MSE of this estimator can be found using Taylor series method defined as

$$h(\bar{x}, \bar{y}) \cong h(\bar{X}, \bar{Y}) + \frac{\partial h(c,d)}{\partial c}\Big|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{\partial h(c,d)}{\partial d}\Big|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y}) \tag{2.1}$$

Where $h(\bar{x}, \bar{y}) = \hat{R}_{p1}$ and $h(\bar{X}, \bar{Y}) = R.$

As shown in Wolter (1985), (2.1) can be applied to the proposed estimator in order to obtain MSE equation as follows:

$$\hat{R}_{p1} - R \cong \frac{\partial((\bar{y} + b(\bar{X} - \bar{x}))/(\bar{x} + \chi_1)}{\partial \bar{x}}\Big|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{\partial((\bar{y} + b(\bar{X} - \bar{x}))/(\bar{x} + \chi_1)}{\partial \bar{y}}\Big|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y})$$

$$\cong -\left(\frac{\bar{y}}{(\bar{x} + \chi_1)^2} + \frac{b(\bar{X} + \chi_1)}{(\bar{x} + \chi_1)^2}\right)\Big|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{1}{(\bar{x} + \chi_1)}\Big|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y})$$

$$E(\hat{R}_{p1} - R)^2 \cong \frac{(\bar{Y} + B(\bar{X} + \chi_1))^2}{(\bar{X} + \chi_1)^4} V(\bar{x}) - \frac{2(\bar{Y} + B(\bar{X} + \chi_1))}{(\bar{X} + \chi_1)^3} Cov(\bar{x}, \bar{y}) + \frac{1}{(\bar{X} + \chi_1)^2} V(\bar{y})$$

$$\cong \frac{1}{(\bar{X} + \chi_1)^2}\left\{\frac{(\bar{Y} + B(\bar{X} + \chi_1))^2}{(\bar{X} + \chi_1)^2} V(\bar{x}) - \frac{2(\bar{Y} + B(\bar{X} + \chi_1)}{(\bar{X} + \chi_1)} Cov(\bar{x}, \bar{y}) + V(\bar{y})\right\}$$

Where $B = \frac{s_{xy}}{s_x^2} = \frac{\rho s_x s_y}{s_x^2} = \frac{\rho s_y}{s_x}.$ Note that we omit the difference of $(E(b) - B)$. Because this assumes a line through the origin, as in the unbiased case for design-based ratio estimator. In Sukhatme (1954), pages 134-144, this condition for bias to ‘vanish’ for SRS makes sense because weighted least square (WLS) regression, and ordinary (homoscedastic) least squares (OLS) regression are both unbiased for b . That is, we see on pages 138-143 of Sukhatme (1954), a derivation of the design based ratio estimator which shows it is unbiased when we have a linear regression through the origin with the regression coefficient being homoscedastic.

$$MSE(\bar{y}_{p1}) = (\bar{X} + \chi_1)^2 E(\hat{R}_{p1} - R)^2 \cong \frac{(\bar{Y} + B(\bar{X} + \chi_1))^2}{(\bar{X} + \chi_1)^2} V(\bar{x}) - \frac{2(\bar{Y} + B(\bar{X} + \chi_1))}{(\bar{X} + \chi_1)} Cov(\bar{x}, \bar{y}) + V(\bar{y})$$

$$\cong \frac{\bar{Y}^2 + 2B(\bar{X} + \chi_1)\bar{Y} + B^2(\bar{X} + \chi_1)^2}{(\bar{X} + \chi_1)^2} V(\bar{x}) - \frac{2\bar{Y} + 2B(\bar{X} + \chi_1)}{(\bar{X} + \chi_1)} Cov(\bar{x}, \bar{y}) + V(\bar{y})$$

$$\cong \frac{(1-f)}{n} \left\{ \left(\frac{\bar{Y}^2}{(\bar{X} + \chi_1)^2} + \frac{2B\bar{Y}}{(\bar{X} + \chi_1)} + B^2 \right) S_x^2 - \left(\frac{2\bar{Y}}{(\bar{X} + \chi_1)} + 2B \right) S_{xy} + S_y^2 \right\}$$

$$\cong \frac{(1-f)}{n} (R^2 S_x^2 + 2BRS_x^2 + B^2 S_x^2 - 2RS_{xy} - 2BS_{xy} + S_y^2)$$

$$MSE(\bar{y}_{p1}) \cong \frac{(1-f)}{n} (R^2 S_x^2 + 2R\rho S_x S_y + \rho^2 S_y^2 - 2R\rho S_x S_y - 2\rho^2 S_y^2 + S_y^2)$$

$$\cong \frac{(1-f)}{n} (R^2 S_x^2 - \rho^2 S_y^2 + S_y^2) \cong \frac{(1-f)}{n} (R^2 S_x^2 + S_y^2(1-\rho^2)$$

Similarly, the bias is obtained as

$$Bias(\bar{y}_{p1}) \cong \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{p1}^2$$

Thus the bias and MSE of the proposed estimator is given below:

$$B(\hat{\bar{Y}}_{p1}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p1}^2, \qquad R_{p1} = \frac{\bar{Y}}{(\bar{X} + \chi_1)} \qquad MSE(\hat{\bar{Y}}_{p1}) = \frac{(1-f)}{n} (R_{p1}^2 S_x^2 + S_y^2(1-\rho^2)),$$

The biases, related constants and the mean square error (MSE) for other improved ratio estimators are respectively given by

$$B(\hat{\bar{Y}}_{p2}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p2}^2, \qquad R_{p2} = \frac{\bar{Y}}{(\bar{X} + \chi_2)} \qquad MSE(\hat{\bar{Y}}_{p2}) = \frac{(1-f)}{n} (R_{p2}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p3}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p3}^2, \qquad R_{p3} = \frac{\bar{Y}}{(\bar{X} + \chi_3)} \qquad MSE(\hat{\bar{Y}}_{p3}) = \frac{(1-f)}{n} (R_{p3}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p4}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p4}^2, \qquad R_{p4} = \frac{\bar{Y}}{(\bar{X} + \chi_4)} \qquad MSE(\hat{\bar{Y}}_{p4}) = \frac{(1-f)}{n} (R_{p4}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p5}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p5}^2, \qquad R_{p5} = \frac{\bar{Y}}{(\bar{X} + \chi_5)} \qquad MSE(\hat{\bar{Y}}_{p5}) = \frac{(1-f)}{n} (R_{p5}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p6}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p6}^2, \qquad R_{p6} = \frac{\bar{Y}}{(\bar{X} + \chi_6)} \qquad MSE(\hat{\bar{Y}}_{p6}) = \frac{(1-f)}{n} (R_{p6}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p7}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p7}^2, \qquad R_{p7} = \frac{\bar{Y}\rho}{(\bar{X}\rho + \chi_1)} \qquad MSE(\hat{\bar{Y}}_{p7}) = \frac{(1-f)}{n} (R_{p7}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p8}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p8}^2, \qquad R_{p8} = \frac{\bar{Y}\rho}{(\bar{X}\rho + \chi_2)} \qquad MSE(\hat{\bar{Y}}_{p8}) = \frac{(1-f)}{n} (R_{p8}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p9}) = \frac{(1-f)}{n} \frac{s_x^2}{\bar{Y}} R_{p9}^2, \qquad R_{p9} = \frac{\bar{Y}\rho}{(\bar{X}\rho + \chi_3)} \qquad MSE(\hat{\bar{Y}}_{p9}) = \frac{(1-f)}{n} (R_{p9}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p10}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p10}^2, \qquad R_{p10} = \frac{\bar{Y}\rho}{(\bar{X}\rho+\chi_4)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p10}) = \frac{(1-f)}{n}(R_{p10}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p11}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p11}^2, \qquad R_{p11} = \frac{\bar{Y}\rho}{(\bar{X}\rho+\chi_5)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p11}) = \frac{(1-f)}{n}(R_{p11}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p12}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p12}^2, \qquad R_{p12} = \frac{\bar{Y}\rho}{(\bar{X}\rho+\chi_6)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p12}) = \frac{(1-f)}{n}(R_{p12}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p13}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p13}^2, \qquad R_{p13} = \frac{\bar{Y}C_x}{(\bar{X}C_x+\chi_1)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p13}) = \frac{(1-f)}{n}(R_{p13}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p14}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p14}^2, \qquad R_{p14} = \frac{\bar{Y}C_x}{(\bar{X}C_x+\chi_2)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p14}) = \frac{(1-f)}{n}(R_{p14}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p15}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p15}^2, \qquad R_{p15} = \frac{\bar{Y}C_x}{(\bar{X}C_x+\chi_3)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p15}) = \frac{(1-f)}{n}(R_{p15}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p16}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p16}^2, \qquad R_{p16} = \frac{\bar{Y}C_x}{(\bar{X}C_x+\chi_4)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p16}) = \frac{(1-f)}{n}(R_{p16}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p17}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p17}^2, \qquad R_{p17} = \frac{\bar{Y}C_x}{(\bar{X}C_x+\chi_5)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p17}) = \frac{(1-f)}{n}(R_{p17}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{\bar{Y}}}_{p18}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{p18}^2, \qquad R_{p18} = \frac{\bar{Y}C_x}{(\bar{X}C_x+\chi_6)} \qquad MSE(\hat{\hat{\bar{Y}}}_{p18}) = \frac{(1-f)}{n}(R_{p18}^2 S_x^2 + S_y^2(1-\rho^2)),$$

3. EFFICIENCY COMPARISONS

In this section, the efficiency conditions for the proposed ratio estimators have been derived algebraically according to traditional ratio estimator and existing ratio estimators in literature.

3.1. Comparison with the Traditional Ratio Estimator

The proposed ratio estimators in Section 2 are more efficient than the traditional ratio estimator when the below given inequality hold as follow

$$MSE(\hat{\hat{\bar{Y}}}_{pj}) \le MSE(\hat{\hat{\bar{Y}}}_r),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2) \le \frac{(1-f)}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y),$$

$$R_{pj}^2 S_x^2 - \rho^2 S_y^2 - R^2 S_x^2 + 2R\rho S_x S_y \le 0,$$

$$(\rho S_y - RS_x)^2 - R_{pj}^2 S_x^2 \ge 0,$$

$$(\rho S_y - RS_x + R_{pj}^2)(\rho S_y - RS_x - R_{pj} S_x) \ge 0.$$

Condition I: $(\rho S_y - RS_x + R_{pj} S_x) \le 0$ and $(\rho S_y - RS_x - R_{pj} S_x) \le 0$

After solving the condition I, we get

$$\left(\frac{RS_y - RS_x}{S_x}\right) \le R_{pj} \le \left(\frac{RS_x - \rho S_y}{S_x}\right).$$

Hence,

$MSE(\hat{\bar{Y}}_{pj}) \leq MSE(\hat{\bar{Y}}_r)$, When the above given inequality hold

$$\left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pj} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right), \quad \text{Or} \quad \left(\frac{RS_x - \rho S_y}{S_x}\right) \leq R_{pj} \leq \left(\frac{\rho S_y - RS_x}{S_x}\right).$$

Where $j = 1, 2, \ldots, 18.$

3.3. Comparisons with Estimators in the Literature

From the expressions of the MSE of the proposed estimators and the estimators in literature, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as follows:

$$MSE(\hat{\bar{Y}}_{pj}) \leq MSE(\hat{\bar{Y}}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2) \leq \frac{(1-f)}{n}(R_i{}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$R_{pj}^2 S_x^2 \leq R_i^2 S_x^2,$$

$$R_{pj} \leq R_i,$$

Where $j = 1, 2, \ldots, 18.$ and $i = 1, 2, \ldots., 19.$

4. APPLICATIONS

The performances of the proposed ratio estimators are evaluated and compared with the mentioned ratio estimators in Section 1.2 by using the data of the three natural populations.
For the population I and II we use the data of Singh and Chaudhary (1986) page 177 and for the population III we use the data of Murthy (1967) page 228 in which fixed capital is denoted by X (auxiliary variable) and output of 80 factories are denoted by Y (study variable). We apply the proposed, classical and the existing estimators to these data sets and the data statistics of these populations are given in Table 1.
From Table 2 and Table 3, we observe that the proposed estimators are more efficient than the classical ratio and all the existing ratio estimators in literature for the Population I, II and III respectively.
The percentage relative efficiency (PRE) of the proposed estimators (p), with respective to the existing estimators, is computed by

$$PRE = \frac{MSE\,of\;\;existing\;estimator}{MSE\,of\;\;proposed\;estimator} \times 100$$

The PRE values are given in Table 4 for the Population I. Similarly we can calculate the percent relative efficiency of the proposed estimators with the classical and the existing estimators by using the above formulae for population II and III respectively. From the table 4, it is clearly seen that the proposed estimators are quiet efficient with respect to the classical and the existing ratio estimators in literature.

5. CONCLUSION

Thus from the above study we conclude that by proposing the modified improved ratio type estimators using the known values of population median, quartile deviation and non-conventional location measures (Tri-mean, Mid-range and Hodges-Lehmann) and their linear combinations with correlation coefficient and coefficient of variation of the auxiliary variable have lower mean square errors as well as bias than the classical and the existing ratio estimators and also according to the percent relative efficiency criteria, they are more efficient than the classical and the existing ratio estimators. Hence we strongly recommend that our proposed improved ratio type estimators preferred over the classical ratio and existing ratio estimators for the use in practical applications.

Table 1: Characteristics of these populations

Parameters	Population 1	Population 2	Population 3
N	34	34	80
n	20	20	20
$\bar{Y}$	856.4117	856.4117	5182.637
$\bar{X}$	208.8823	199.4412	1126.463
ρ	0.4491	0.4453	0.941
S_y	733.1407	733.1407	1835.659
C_y	0.8561	0.8561	0.354193
S_x	150.5059	150.2150	845.610
C_x	0.7205	0.7531	0.7506772
β_2	0.0978	1.0445	-0.063386
β_1	0.9782	1.1823	1.050002
M_d	150	142.5	757.5
TM	162.25	165.562	931.562
MR	284.5	320	1795.5
HL	190	184	1040.5
QD	80.25	89.375	588.125
DM	234.82	206.944	1150.7

Table 2: The Statistical Analysis of the Estimators for 4these Populations

Estimators	Population I		Population II		Population III	
	Constant	Bias	Constant	Bias	Constant	Bias
$\hat{\bar{Y}}_r$	4.100	4.270	4.294	4.940	4.601	60.877
$\hat{\bar{Y}}_1$	4.100	9.1539	4.294	10.0023	4.601	36.5063
$\hat{\bar{Y}}_2$	4.086	9.0911	4.278	9.9272	4.598	36.4577
$\hat{\bar{Y}}_3$	4.098	9.1454	4.272	9.8983	4.601	36.5104
$\hat{\bar{Y}}_4$	3.960	8.5387	4.279	9.9303	4.650	37.2861
$\hat{\bar{Y}}_5$	4.097	9.1420	4.264	9.8646	4.601	36.5117
$\hat{\bar{Y}}_6$	4.091	9.1147	4.284	9.9578	4.597	36.4453
$\hat{\bar{Y}}_7$	4.088	9.0995	4.281	9..9432	4.596	36.4251
$\hat{\bar{Y}}_8$	4.069	9.0149	4.258	9.8348	4.598	36.4546
$\hat{\bar{Y}}_9$	4.011	8.7630	4.285	9.9597	4.662	37.4882
$\hat{\bar{Y}}_{10}$	4.096	9.1349	4.244	9.7711	4.601	36.5106

$\hat{\bar{Y}}_{11}$	2.3076	2.900	2.3463	2.986	2.518	32.81
$\hat{\bar{Y}}_{12}$	1.9730	2.120	2.0427	2.263	2.189	24.79
$\hat{\bar{Y}}_{13}$	1.5021	1.229	1.4993	1.219	2.449	31.03
$\hat{\bar{Y}}_{14}$	1.7358	1.641	1.6487	1.475	1.774	16.23
$\hat{\bar{Y}}_{15}$	1.4185	1.096	1.3718	1.021	1.473	11.23
$\hat{\bar{Y}}_{16}$	1.0167	0.563	0.9329	0.472	1.708	15.10
$\hat{\bar{Y}}_{17}$	2.147	2.510	2.233	2.706	2.392	29.59
$\hat{\bar{Y}}_{18}$	1.812	1.788	1.930	2.021	2.063	22.01
$\hat{\bar{Y}}_{19}$	1.355	1.000	1.398	1.060	2.322	27.90
$\hat{\bar{Y}}_{20}$	1.108	0.6686	1.028	0.5727	1.124	2.1783
$\hat{\bar{Y}}_{21}$	2.3076	2.900	2.3463	2.986	2.518	32.81
$\hat{\bar{Y}}_{22}$	1.9730	2.120	2.0427	2.263	2.189	24.79
$\hat{\bar{Y}}_{23}$	1.5021	1.229	1.4993	1.219	2.449	31.03
$\hat{\bar{Y}}_{24}$	1.7358	1.641	1.6487	1.475	1.774	16.23
$\hat{\bar{Y}}_{25}$	1.4185	1.096	1.3718	1.021	1.473	11.23
$\hat{\bar{Y}}_{26}$	1.0167	0.563	0.9329	0.472	1.708	15.10
$\hat{\bar{Y}}_{27}$	2.147	2.510	2.233	2.706	2.392	29.59
$\hat{\bar{Y}}_{28}$	1.812	1.788	1.930	2.021	2.063	22.01
$\hat{\bar{Y}}_{29}$	1.355	1.000	1.398	1.060	2.322	27.90
$\hat{\bar{Y}}_{p1}$	0.03488	0.000663	0.03599	0.000703	0.00733	0.000278
$\hat{\bar{Y}}_{p2}$	0.06474	0.002282	0.05711	0.001769	0.00944	0.000461
$\hat{\bar{Y}}_{p3}$	0.02983	0.000485	0.03242	0.000570	0.00656	0.000223
$\hat{\bar{Y}}_{p4}$	0.05541	0.001672	0.05145	0.001436	0.00845	0.000369
$\hat{\bar{Y}}_{p5}$	0.01997	0.000217	0.01869	0.000190	0.00381	0.000075
$\hat{\bar{Y}}_{p6}$	0.03717	0.000752	0.02974	0.000480	0.00490	0.000124
$\hat{\bar{Y}}_{p7}$	0.01574	0.000135	0.01610	0.000141	0.00690	0.000246
$\hat{\bar{Y}}_{p8}$	0.02933	0.000468	0.02562	0.000356	0.00888	0.000408
$\hat{\bar{Y}}_{p9}$	0.01345	0.000099	0.01449	0.000114	0.00618	0.000197
$\hat{\bar{Y}}_{p10}$	0.02507	0.000342	0.02307	0.000289	0.00796	0.000328

$\hat{\bar{Y}}_{p11}$	0.00899	0.000044	0.00835	0.000038	0.00358	0.000066
$\hat{\bar{Y}}_{p12}$	0.01677	0.000153	0.01329	0.000096	0.00462	0.000110
$\hat{\bar{Y}}_{p13}$	0.02519	0.000346	0.02716	0.000401	0.00551	0.000157
$\hat{\bar{Y}}_{p14}$	0.04685	0.001195	0.04315	0.001010	0.00709	0.000260
$\hat{\bar{Y}}_{p15}$	0.02154	0.000253	0.02446	0.000324	0.00493	0.000126
$\hat{\bar{Y}}_{p16}$	0.04007	0.000874	0.03886	0.000819	0.00635	0.000209
$\hat{\bar{Y}}_{p17}$	0.01044	0.000113	0.01409	0.000109	0.00286	0.000042
$\hat{\bar{Y}}_{p18}$	0.02685	0.000393	0.02243	0.000273	0.00368	0.000070

Table 3: The Statistical Analysis of the Estimators for these Populations

Estimators	**Pop I**	**Pop II**	**Pop III**	**Estimators**	**Pop I**	**Pop II**	**Pop III**
	MSE	**MSE**	**MSE**		**MSE**	**MSE**	**MSE**
$\hat{\bar{Y}}_r$	10539.27	10960.76	189775.10	$\hat{\bar{Y}}_{19}$	9690.50	9779.43	158990.70
$\hat{\bar{Y}}_1$	16673.45	17437.65	581994.20	$\hat{\bar{Y}}_{p1}$	8834.72	8872.37	14472.25
$\hat{\bar{Y}}_2$	16619.64	17373.31	581238.50	$\hat{\bar{Y}}_{p2}$	8836.11	8873.28	14473.20
$\hat{\bar{Y}}_3$	16666.14	17348.62	582058.10	$\hat{\bar{Y}}_{p3}$	8834.57	8872.25	14471.97
$\hat{\bar{Y}}_4$	16146.61	17376.04	594119.80	$\hat{\bar{Y}}_{p4}$	8835.58	8872.99	14472.73
$\hat{\bar{Y}}_5$	16663.31	17319.75	582079.30	$\hat{\bar{Y}}_{p5}$	8834.34	8871.93	14471.20
$\hat{\bar{Y}}_6$	16639.85	17399.52	581046.80	$\hat{\bar{Y}}_{p6}$	8834.80	8872.17	14471.46
$\hat{\bar{Y}}_7$	16626.87	17387.08	580732.70	$\hat{\bar{Y}}_{p7}$	8834.27	8871.88	14472.09
$\hat{\bar{Y}}_8$	16554.40	17294.19	581191.40	$\hat{\bar{Y}}_{p8}$	8834.55	8872.07	14472.93
$\hat{\bar{Y}}_9$	16338.65	17401.14	597260.90	$\hat{\bar{Y}}_{p9}$	8834.24	8871.86	14471.83
$\hat{\bar{Y}}_{10}$	16654.19	17239.66	582062.10	$\hat{\bar{Y}}_{p10}$	8834.45	8872.01	14472.51
$\hat{\bar{Y}}_{11}$	11317.28	11429.08	184446.2	$\hat{\bar{Y}}_{p11}$	8834.19	8871.79	14471.15
$\hat{\bar{Y}}_{12}$	10649.40	10809.99	142903.2	$\hat{\bar{Y}}_{p12}$	8834.28	8871.85	14471.38
$\hat{\bar{Y}}_{13}$	9886.21	9915.81	175238.7	$\hat{\bar{Y}}_{p13}$	8834.45	8872.11	14471.62
$\hat{\bar{Y}}_{14}$	10239.11	10134.39	98755.61	$\hat{\bar{Y}}_{p14}$	8835.18	8872.63	14472.16

$\hat{\bar{Y}}_{15}$	9772.39	9745.79	72582.52	$\hat{\bar{Y}}_{p15}$	8834.37	8872.04	14471.46
$\hat{\bar{Y}}_{16}$	9316.02	9275.87	92644.60	$\hat{\bar{Y}}_{p16}$	8834.90	8872.46	14471.89
$\hat{\bar{Y}}_{17}$	10983.77	11189.04	167778.6	$\hat{\bar{Y}}_{p17}$	8834.25	8871.86	14471.03
$\hat{\bar{Y}}_{18}$	10365.55	10602.02	128487.6	$\hat{\bar{Y}}_{p18}$	8834.49	8871.99	14471.17

Table 4: PRE of the Proposed Estimators with the Estimators in Literature for population I.

	$\hat{\bar{Y}}_r$	$\hat{\bar{Y}}_1$	$\bar{Y}_2$	$\hat{\bar{Y}}_3$	$\hat{\bar{Y}}_4$	$\hat{\bar{Y}}_5$	$\hat{\bar{Y}}_6$	$\hat{\bar{Y}}_7$	$\hat{\bar{Y}}_8$	$\hat{\bar{Y}}_9$
$\hat{\bar{Y}}_{p1}$	119.293	188.726	188.117	188.643	182.763	188.611	188.346	188.199	187.378	184.936
$\hat{\bar{Y}}_{p2}$	119.275	188.696	188.087	188.614	182.734	188.582	188.316	188.169	187.349	184.907
$\hat{\bar{Y}}_{p3}$	119.295	188.729	188.120	188.646	182.766	188.614	188.349	188.202	187.382	184.940
$\hat{\bar{Y}}_{p4}$	119.282	188.708	188.099	188.625	182.745	188.593	188.327	188.180	187.360	184.918
$\hat{\bar{Y}}_{p5}$	119.298	188.734	188.125	188.651	182.771	188.619	188.354	188.207	187.386	184.944
$\hat{\bar{Y}}_{p6}$	119.292	188.724	188.115	188.642	182.761	188.609	188.344	188.197	187.377	184.935
$\hat{\bar{Y}}_{p7}$	119.299	188.736	188.126	188.653	182.772	188.621	188.355	188.208	187.388	184.946
$\hat{\bar{Y}}_{p8}$	119.296	188.73	188.121	188.647	182.766	188.615	188.349	188.202	187.382	184.940
$\hat{\bar{Y}}_{p9}$	119.300	188.736	188.127	188.653	182.773	188.621	188.356	188.209	187.389	184.946
$\hat{\bar{Y}}_{p10}$	119.297	188.732	188.123	188.649	182.768	188.617	188.351	188.204	187.384	184.942
$\hat{\bar{Y}}_{p11}$	119.300	188.737	188.128	188.655	182.774	188.623	188.357	188.210	187.390	184.947
$\hat{\bar{Y}}_{p12}$	119.299	188.735	188.126	188.653	182.772	188.621	188.355	188.208	187.388	184.946
$\hat{\bar{Y}}_{p13}$	119.297	188.732	188.123	188.649	182.768	188.617	188.351	188.204	187.384	184.942
$\hat{\bar{Y}}_{p14}$	119.287	188.716	188.107	188.633	182.753	188.601	188.336	188.189	187.369	184.927
$\hat{\bar{Y}}_{p15}$	119.298	188.733	188.124	188.6511	182.770	188.619	188.353	188.206	187.386	184.944
$\hat{\bar{Y}}_{p16}$	119.291	188.722	188.113	188.639	182.759	188.607	188.342	188.195	187.375	184.933
$\hat{\bar{Y}}_{p17}$	119.300	188.736	188.127	188.653	182.772	188.621	188.356	188.209	187.388	184.946
$\hat{\bar{Y}}_{p18}$	119.296	188.731	188.122	188.648	182.767	188.616	188.351	188.204	187.383	184.941
	$\hat{\bar{Y}}_{10}$	$\hat{\bar{Y}}_{11}$	$\hat{\bar{Y}}_{12}$	$\hat{\bar{Y}}_{13}$	$\hat{\bar{Y}}_{14}$	$\hat{\bar{Y}}_{15}$	$\hat{\bar{Y}}_{16}$	$\hat{\bar{Y}}_{17}$	$\hat{\bar{Y}}_{18}$	$\hat{\bar{Y}}_{19}$

$\widehat{\overline{\overline{Y}}}_{p1}$	188.508	128.100	120.540	111.901	115.896	110.613	105.447	124.325	117.327	109.686
$\widehat{\overline{\overline{Y}}}_{p2}$	188.478	128.079	120.521	111.884	115.878	110.596	105.431	124.305	117.309	109.669
$\widehat{\overline{\overline{Y}}}_{p3}$	188.511	128.102	120.542	111.903	115.898	110.615	105.449	124.327	117.329	109.688
$\widehat{\overline{\overline{Y}}}_{p4}$	188.490	128.087	120.528	111.890	115.885	110.602	105.437	124.312	117.316	109.675
$\widehat{\overline{\overline{Y}}}_{p5}$	188.516	128.105	120.545	111.906	115.901	110.618	105.452	124.330	117.332	109.691
$\widehat{\overline{\overline{Y}}}_{p6}$	188.506	128.098	120.539	111.900	115.895	110.612	105.446	124.323	117.326	109.685
$\widehat{\overline{\overline{Y}}}_{p7}$	188.518	128.106	120.546	111.907	115.902	110.619	105.453	124.331	117.333	109.692
$\widehat{\overline{\overline{Y}}}_{p8}$	188.512	128.102	120.542	111.903	115.898	110.615	105.449	124.327	117.329	109.688
$\widehat{\overline{\overline{Y}}}_{p9}$	188.518	128.107	120.546	111.907	115.902	110.619	105.453	124.331	117.333	109.692
$\widehat{\overline{\overline{Y}}}_{p10}$	188.514	128.104	120.544	111.905	115.899	110.616	105.451	124.328	117.331	109.689
$\widehat{\overline{\overline{Y}}}_{p11}$	188.519	128.107	120.547	111.908	115.903	110.620	105.454	124.332	117.334	109.693
$\widehat{\overline{\overline{Y}}}_{p12}$	188.517	128.106	120.546	111.907	115.902	110.619	105.453	124.331	117.333	109.692
$\widehat{\overline{\overline{Y}}}_{p13}$	188.514	128.104	120.544	111.905	115.899	110.616	105.451	124.328	117.331	109.689
$\widehat{\overline{\overline{Y}}}_{p14}$	188.498	128.093	120.534	111.896	115.890	110.607	105.442	124.318	117.321	109.680
$\widehat{\overline{\overline{Y}}}_{p15}$	188.515	128.105	120.545	111.906	115.900	110.617	105.452	124.33	117.332	109.690
$\widehat{\overline{\overline{Y}}}_{p16}$	188.504	128.097	120.537	111.899	115.893	110.611	105.445	124.322	117.325	109.684
$\widehat{\overline{\overline{Y}}}_{p17}$	188.518	128.106	120.546	111.907	115.902	110.619	105.453	124.331	117.333	109.692
$\widehat{\overline{\overline{Y}}}_{p18}$	188.513	128.103	120.543	111.904	115.899	110.616	105.450	124.328	117.330	109.689

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