# EFFECT OF MEASUREMENT ERROR AND NON-RESPONSE ON ESTIMATION OF POPULATION MEAN

Poonam Singh*[1], Rajesh Singh* and Carlos N. Bouza**
*Department of statistics, Banaras Hindu University (U.P), India
**Facultad MATCOM, Universidad de La Habana, Cuba

**ABSTRACT**
In this paper effect of measurement error and non response error is examined on estimation of unknown population mean of study variable. We have obtained the expression of the MSE (mean square error) of the proposed estimator up to first order of approximation. We have shown theoretically and empirically that the proposed estimator performs better than other estimators considered in this article. For empirically study we have used for different data sets.

**KEYWORDS:** Auxiliary Variable, Measurement Error, Non-Response, Bias, Mean Square Error (MSE).

**MSC:** 62D05

**RESUMEN**
En este paper es examinado el efecto de error de medición de respuesta en la estimación de la media desconocida de la variable de estudio. Hemos obtenido la expresión del MSE (error cuadrático medio) del estimador propuesto hasta el primer orden de aproximación. Hemos mostrado teoréticamente y empíricamente que el propuesto estimador se comparta mejor que los otros considerados en este artículo. Para los estudios empíricos usamos diferentes data.

**PALABRAS CLAVE:** Variable Auxiliar, Error de Medida, Sesgo de No-Respuesta, Error Cuadrático Medio

**MSC:** 62D05

## 1. INTRODUCTION

In a perfect world survey has no non-response, all selected element will participate and provide all of the requested information. However, today reality is very different. Missing data due to non-response is a normal although undesirable feature of any survey. In theory of sample surveys, auxiliary variables play important role. Auxiliary information is used to increase precision of an estimator. Error free measurement of the auxiliary variables on the population frame would thus seems critical for making appropriate finite population inferences. Unfortunately, there has been very little research examining the impact of measurement error in the auxiliary variable on estimation of parameters. Measurement errors occur when answer provided by respondents departs from the true value on the measurement (e.g. failure to report correctly whether he visited doctor in last six months). Measurement errors include observational error, instrument error, respondent error etc. Fuller (1987), Corrol, Ruppert and Stefanski (1995), Meijer (2000), Bound, Brown and Mathiowetz (2001), Hausman (2001), Srivastava and Shalabh(2001), Manisha and Singh (2002), Singh and Karpe (2008,2009), Kumar et al.(2011), Shukla et al.(2012) and Singh and Sharma (2015) are the few references who have studied problem of measurement error.
Besides measurement errors, non-response has always been a matter of concern in sample surveys. Non-response is the failure to get information from some units of the population due to various reasons like unavailability of respondents, lack of information and refusals etc.
Description of non-response error and its effect is described in Cochran (1977). Kalton and Karsprzyk (1986), Merg (1995), Rubin (1996), Kenward and Carpenter (2007) etc. gave several approaches to address non-response in sample surveys. Non-response problem is studied to:-

- Avoid non-response before it has occurred.
- Develop techniques required in estimation when non-response has occurred.

[1] Corresponding Author, ppiyu123@gmail.com

Hansen and Hurwitz (1946) were the first who pioneered technique for estimation of mean when non-response is present in surveys. He simply drawn a simple random sample of size n and mailed questionnaire to sampled units. Then re-contacted some of the non-responding units by drawing a sub-sample from the non-responding units in the initial first attempt.
Cochran (1977) applied Hansen and Hurwitz technique to formulate a ratio estimator of the population mean. Similarly, Rao (1986), Ofkar and Lee (2000) , Tabasum and Khan (2004,2006), Sodipo and Obisesan (2007), Singh and Kumar (2008), Singh et al. (2014), Chaudhary et al. (2014), Singh and Singh (2015) and Sharma and Singh (2015) considered the problem of estimating mean in presence of non-response/ measurement error.
Problem of measurement error and non-response error are studied by many researchers separately. But these problems may creep into survey sampling at the same time. If these errors are small and negligible they can be ignored but if these errors are not negligible, inferences may lead to undesirable consequences. In this paper we will study how both the errors affect efficiency of estimators.

## 2. NOTATIONS

Let us consider a finite population $(U = U_1, U_2, ..U_N)$ of size N such that Y be study variable and X any be auxiliary variable. We draw a sample of size n from a population by using simple random sampling without replacement scheme.
Suppose that $N_1$ units respond for the survey questions and $N_2$ units do not respond. Then by following Hansen Hurwitz (1946) sampling plan, a sub-sample of size $k = \frac{r_2}{h} (h > 1)$ from $N_2$ non-respondents is selected at random and is re-contacted for their direct interview.
Here it is assumed that r units respond to the survey.
Let $(x_i^*, y_i^*)$ be the observed values and $(X_i^*, Y_i^*)$ be the true values of the study variable Y and auxiliary variable X. where (i=1, 2… n) unit in the sample.
Then measurement error is given by-

$$u_i^* = y_i^* - Y_i^* \qquad \text{And} \qquad v_i^* = x_i^* - X_i^* \tag{2.1}$$

Where $(u_i^*, v_i^*)$ are random in nature and both are uncorrelated with mean zero and variance $S_U^2$ and $S_V^2$ are associated with measurement error in study variable Y and auxiliary variable X respectively for the responding part of the population. $S_{U(2)}^2$ and $S_{V(2)}^2$ are the variances associated with measurement error in study variable Y and auxiliary variable X respectively for the non-responding part of the population.
We further assume that mean of study variable Y is unknown and auxiliary variable X is known.
Following symbols have their meaning given below:

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i = \text{population mean of Y}$$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i = \text{population mean of X}$$

$\bar{y}$ and $\bar{x}$ are the sample means of y and x respectively. $S_Y^2$ And $S_X^2$ are the population variances of Y and X respectively for the responding part of the population. $S_{Y(2)}^2$ And $S_{X(2)}^2$ are the variances of Y and X respectively for non-responding part of the population.
$\rho$ And $\rho_{(2)}$ are the population correlation coefficient between X and Y for the responding and non-responding part of the population respectively. $C_Y$ And $C_{Y(2)}$ are the coefficients of variation of Y for the responding and non-responding part of the population respectively. Similarly $C_X$ and $C_{X(2)}$ are the coefficients of variation of X for the responding and non-responding part of the population respectively.
In order to obtain MSE of the estimators in presence of non-response and measurement error, following notations are used:
Let

$$w_Y^* = \sum_{i=1}^{n}(Y_i^* - \overline{Y}) \tag{2.2}$$

and

$$w_X^* = \sum_{i=1}^{n}(X_i^* - \overline{X}) \tag{2.3}$$

Then

$$w_U^* = \sum_{i=1}^{n} U_i^* \tag{2.4}$$

and

$$w_V^* = \sum_{i=1}^{n} V_i^* \tag{2.5}$$

Adding (2.2) and (2.4), dividing both sides by n, we have

$$\frac{1}{n}\left(w_Y^* + w_U^*\right) = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i^* - \overline{Y}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(y_i^* - Y_i^*\right) \tag{2.6}$$

Or

$$\frac{1}{n}\left(w_Y^* + w_U^*\right) = \frac{1}{n}\sum_{i=1}^{n} y_i^* - \overline{Y} = \xi_0 \,(\text{say}) \tag{2.7}$$

Similarly adding (2.3) and (2.5), dividing both sides by n we get

$$\frac{1}{n}\left(w_X^* + w_V^*\right) = \frac{1}{n}\sum_{i=1}^{n} x_i^* - \overline{X} = \xi_1 \,(\text{say}) \tag{2.8}$$

On simplification, we get

$$\overline{y}^* = \overline{Y} + \xi_0 \tag{2.9}$$

$$\overline{x}^* = \overline{X} + \xi_1 \tag{2.10}$$

Further,

$$E(\xi_0^2) = \lambda_2\left(S_Y^2 + S_U^2\right) + \theta\left(S_{Y(2)}^2 + S_{U(2)}^2\right) = \nabla_0^2 (say) \tag{2.11}$$

$$E(\xi_1^2) = \lambda_2\left(S_X^2 + S_V^2\right) + \theta\left(S_{X(2)}^2 + S_{V(2)}^2\right) = \nabla_1^2 (say) \tag{2.12}$$

$$E(\xi_0\xi_1) = \lambda_2 \rho_{YX} S_Y S_X + \theta \rho_{YX(2)} S_{Y(2)} S_{X(2)} = \nabla_0 \nabla_1 (say) \tag{2.13}$$

## 3. EXISTING ESTIMATORS

Hansen Hurwitz (1946) estimator for estimating population mean in presence of non-response and measurement error is given by-

$$\overline{y}^* = \left(\frac{n_1}{n}\right)\overline{y}_{n1} + \left(\frac{n_2}{n}\right)\overline{y}_r = t_a (say) \tag{3.1}$$

where $\overline{y}_{n1} = \frac{1}{n_1}\sum_{i=1}^{n_1} y_i$ and $\overline{y}_r = \frac{1}{r}\sum_{i=1}^{r} y_i$.

Expression (2.1) can be written as:

$$t_a = \overline{Y} + \xi_0 \tag{3.2}$$

Subtracting $\overline{Y}$ from both the sides of equation (3.2) and taking expectation, we get bias of estimator $t_a$ given as:

$$\text{Bias}(t_a) = 0 \tag{3.3}$$

Subtracting $\overline{Y}$ from both the sides of equation (3.2) and squaring, we get

$$(t_a - \overline{Y})^2 = \xi_0^{\,2} \tag{3.4}$$

Taking expectations, mean square error of the estimator $t_a$ is obtained up to first order of approximation as:

$$MSE(t_a) = \lambda_2(S_Y^2 + S_U^2) + \theta(S_{Y(2)}^2 + S_{U(2)}^2) \tag{3.5}$$

In case when measurement error is zero, then

$$MSE(t_a) = \lambda_2 S_Y^2 + \theta S_{Y(2)}^2 \tag{3.6}$$

Contribution of measurement error to MSE of estimator $t_a$ is:

$$ME(t_a) = \lambda_2 S_U^2 + \theta S_{U(2)}^2 \tag{3.7}$$

Cochran (1977) ratio estimator in presence of non-response and measurement error is given by-

$$t_r = \frac{\bar{y}^*}{\bar{x}^*}\bar{X} \tag{3.8}$$

Expressing the estimator $t_r$ in terms of $\xi$'s, and then simplifying, we get:

$$t_r = \bar{Y} + \xi_0 - \frac{\bar{Y}}{\bar{X}}\xi_1 - \frac{\xi_0\xi_1}{\bar{X}} + \frac{\bar{Y}^2}{\bar{X}^2}\xi_1^2 \tag{3.9}$$

Subtracting $\bar{Y}$ from both the sides of equation (3.9), we get

$$(t_r - \bar{Y}) = \xi_0 - \frac{\bar{Y}}{\bar{X}}\xi_1 - \frac{\xi_0\xi_1}{\bar{X}} + \frac{\bar{Y}^2}{\bar{X}^2}\xi_1^2 \tag{3.10}$$

Taking expectation of equation (3.10), we get bias of the estimator $t_r$ estimator as

$$Bias(t_r) = \frac{\bar{Y}^2}{\bar{X}^2}\left\{\lambda_2\left(S_X^2 + S_V^2\right) + \theta\left(S_{X(2)}^2 + S_{V(2)}^2\right)\right\} - \frac{1}{\bar{X}}\left\{\lambda_2\left(\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}\right)\right\} \tag{3.11}$$

Squaring equation (3.10) and then taking expectations, MSE of the estimator $t_r$ is given by

$$MSE(t_r) = \lambda_2\left(S_Y^2 + S_U^2\right) + \theta\left(S_{Y(2)}^2 + S_{U(2)}^2\right) + \frac{\bar{Y}^2}{\bar{X}^2}\left\{\lambda_2\left(S_X^2 + S_V^2\right) + \theta\left(S_{X(2)}^2 + S_{V(2)}^2\right)\right\}$$
$$-\frac{2\bar{Y}}{\bar{X}}\left\{\lambda_2\left(\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}\right)\right\} \tag{3.12}$$

In case when measurement error is zero or negligible,

$$MSE(t_r) = \lambda_2 S_Y^2 + \theta S_{Y(2)}^2 + \frac{\bar{Y}^2}{\bar{X}^2}\left\{\lambda_2 S_X^2 + \theta S_{X(2)}^2\right\}$$
$$-\frac{2\bar{Y}}{\bar{X}}\left\{\lambda_2\left(\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}\right)\right\} \tag{3.13}$$

The contribution of measurement error to the MSE of the estimator $t_r$ is:

$$MSE(t_r) = \lambda_2\bar{Y}^2\left(\frac{S_U^2}{\bar{Y}^2} + \frac{S_V^2}{\bar{X}^2}\right) + \theta\bar{Y}^2\left(\frac{S_{U(2)}^2}{\bar{Y}^2} + \frac{S_{V(2)}^2}{\bar{X}^2}\right) \tag{3.14}$$

Rao's (1991) estimator under non-response and measurement error is given by-

$$t_{ra} = \left[w_1\left(\bar{X} - \bar{x}^*\right) + w_2\bar{y}^*\right] \tag{3.15}$$

Expressing $t_{ra}$ in terms of $\xi$'s, subtracting $\bar{Y}$ and then squaring we get:

$$MSE(t_{ra}) = E\left[-w_1\xi_1 + w_2\xi_0 + w_2\bar{Y} - \bar{Y}\right]^2 \tag{3.16}$$

Or

$$MSE(t_{ra}) = E\left[w_1^2\xi_1^2 + w_2^2\xi_0^2 + w_2^2\bar{Y}^2 - 2w_1w_2\xi_0\xi_1 - 2w_2\bar{Y}^2 + \bar{Y}^2\right] \tag{3.17}$$

Equation (3.17) can be written as

$$MSE(t_{ra}) = \left[w_1^2A_r + w_2^2B_r - 2w_1w_2D_r - 2w_2C_r + \bar{Y}^2\right] \tag{3.18}$$

Where

$$A_r = \nabla_1^2 \tag{3.19}$$

$$B_r = \bar{Y}^2 + \nabla_0^2 \tag{3.20}$$

$$C_r = \bar{Y}^2 \tag{3.21}$$

$$D_r = \nabla_0 \nabla_1 \tag{3.22}$$

Differentiating equation (3.19) with respect to $w_1$ and $w_2$ we get

$$w_1 = \frac{CD}{AB - D^2} \quad \text{and} \quad w_2 = \frac{AC}{AB - D^2} \tag{3.23}$$

Therefore substituting values of equation (3.23) in equation (3.18) MSE becomes

$$MSE(t_{ra}) = \bar{Y}^2 + \frac{AC}{AB - D^2} \tag{3.24}$$

Bahl and Tuteja (1991) estimator in presence of non-response and measurement error is given by-

$$t_{bt} = \bar{y}^* \exp\left[\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right] \tag{3.25}$$

Expressing equation (3.25) in terms of $\xi' s$, we get

$$\bar{y}_{bt} = \bar{Y} + \xi_0 - \frac{\bar{Y}}{2\bar{X}}\xi_1 - \frac{\xi_0\xi_1}{2\bar{X}} + \frac{3}{8}\frac{\bar{Y}}{\bar{X}^2}\xi_1^2 \tag{3.26}$$

Subtracting from both sides of equation and squaring it, MSE of estimator $t_{ra}$ is obtained as:

$$MSE(t_{bt}) = E\left[\xi_0^2 + \frac{\bar{Y}^2}{4\bar{X}^2}\xi_1^2 - \frac{\bar{Y}}{\bar{X}}\xi_0\xi_1\right] \tag{3.27}$$

Or

$$MSE(t_{bt}) = \begin{bmatrix} \left\{\lambda_2\left(S_Y^2 + S_U^2\right) + \theta\left(S_{Y(2)}^2 + S_{U(2)}^2\right)\right\} + \dfrac{\bar{Y}^2}{4\bar{X}^2}\left\{\lambda_2\left(S_X^2 + S_V^2\right) + \theta\left(S_{X(2)}^2 + S_{V(2)}^2\right)\right\} \\ -\dfrac{\bar{Y}}{\bar{X}}\left(\lambda_2\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}\right) \end{bmatrix} \tag{3.28}$$

Regression Estimator under measurement error and non-response is given by-

$$t_{lr} = \bar{y}^* + b\left(\bar{X} - \bar{x}^*\right) \tag{3.29}$$

MSE of estimator $t_{lr}$ is obtained as:

$$\begin{aligned} MSE(t_{lr}) &= \lambda_2(S_Y^2 + S_U^2) + \theta(S_{Y(2)}^2 + S_{U(2)}^2) + b^2\left\{\lambda_2\left(S_X^2 + S_V^2\right) + \theta\left(S_{X(2)}^2 + S_{V(2)}^2\right)\right\} \\ &- 2b\left\{\lambda_2\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}\right\} \end{aligned} \tag{3.30}$$

Differentiating MSE of $t_{lr}$ with respect to b and equating it to zero, we get

$$b = \frac{\lambda_2\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}}{\lambda_2(S_X^2 + S_V^2) + \theta(S_{X(2)}^2 + S_{V(2)}^2)} = b_0\,(\text{say}) \tag{3.31}$$

The minimum MSE of the estimator $t_{lr}$ is given by

$$\begin{aligned} \min MSE(t_{lr}) &= \lambda_2(S_Y^2 + S_U^2) + \theta(S_{Y(2)}^2 + S_{V(2)}^2) + b_0^2\left\{\lambda_2\left(S_X^2 + S_V^2\right) + \theta\left(S_{X(2)}^2 + S_{V(2)}^2\right)\right\} \\ &- 2b_0\left\{\lambda_2\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)}\right\} \end{aligned} \tag{3.32}$$

The contribution of measurement error in MSE of Regression estimator is:

$$ME(t_{lr}) = \lambda_2S_U^2 + \theta S_{V(2)}^2 + b_0^2\left\{\lambda_2S_V^2 + \theta S_{V(2)}^2\right\} \tag{3.33}$$

## 4. PROPOSED ESTIMATOR

We propose estimator $t_{sp}$ in presence of non-response and measurement error as

$$t_{sp} = \left[\frac{1}{2}\left\{\bar{y}^* \exp\left(\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}\right) + \bar{y}^* \exp\left(\frac{\bar{x}^*-\bar{X}}{\bar{x}^*+\bar{X}}\right)\right\} + \alpha_1\left(\bar{X}-\bar{x}^*\right) + \alpha_2 \bar{y}^*\right] \exp\left[\frac{\bar{X}^{"}-\bar{x}^{*"}}{\bar{X}^{"}+\bar{x}^{*"}}\right] \tag{4.1}$$

where $\bar{X}^{"} = \bar{X}p$ and $\bar{x}^{*"} = \bar{x}^* + \bar{X}(p-1)$

Further $p = \dfrac{\rho_{xy}+1}{4}$ is a suitably chosen constant.

Expressing equation (3.1) in terms of $\xi$'s we have

$$t_{sp} = \left[(\bar{Y} + \xi_0 - \frac{\bar{Y}}{8\bar{X}^2}\xi_1^2) - \alpha_1\xi_1 + \alpha_2\bar{Y} + \alpha_2\xi_0\right] \exp\left[\frac{\xi_1}{2\bar{X}\,p} - \frac{\xi_1^2}{4\bar{X}^2 p^2}\right] \tag{4.2}$$

Simplifying equation (4.2) we get:

$$\begin{aligned} t_{sp} = \Bigg[&\left(\bar{Y} + \xi_0 + \frac{\bar{Y}}{8\bar{X}^2}\xi_1^2 - \frac{\bar{Y}}{2\bar{X}}\frac{\xi_1}{p} - \frac{\xi_0\xi_1}{2\bar{X}p} + \frac{3}{8}\frac{\bar{Y}}{\bar{X}^2}\frac{\xi_1^2}{p^2}\right) - \alpha_1\left(\xi_1 - \frac{\xi_1^2}{2\bar{X}p}\right) \\ &+ \alpha_2\left(\bar{Y} + \xi_0 - \frac{\bar{Y}}{2\bar{X}p}\xi_1 - \frac{\xi_0\xi_1}{2\bar{X}p} + \frac{3}{8}\frac{\bar{Y}^2}{\bar{X}^2p^2}\xi_1^2\right)\Bigg] \end{aligned} \tag{4.3}$$

Or

$$\begin{aligned} t_{sp} - \bar{Y} = \Bigg[&\left(\xi_0 + \frac{\bar{Y}}{8\bar{X}^2}\xi_1^2 - \frac{\bar{Y}}{2\bar{X}}\frac{\xi_1}{p} - \frac{\xi_0\xi_1}{2\bar{X}p} + \frac{3}{8}\frac{\bar{Y}}{\bar{X}^2}\frac{\xi_1^2}{p^2}\right) - \alpha_1\left(\xi_1 - \frac{\xi_1^2}{2\bar{X}p}\right) \\ &+ \alpha_2\left(\bar{Y} + \xi_0 - \frac{\bar{Y}}{2\bar{X}p}\xi_1 - \frac{\xi_0\xi_1}{2\bar{X}p} + \frac{3}{8}\frac{\bar{Y}^2}{\bar{X}^2p^2}\xi_1^2\right)\Bigg] \end{aligned} \tag{4.4}$$

Taking expectation on both the sides of equation (4.4), we get the bias of the estimator $t_{sp}$ as:

$$Bias(t_{sp}) = \alpha_2\bar{Y} + \left(\frac{\bar{Y}}{8\bar{X}^2} + \frac{\alpha_1}{2\bar{X}p} + \frac{3}{8}\frac{\bar{Y}}{\bar{X}^2p^2} + \frac{3}{8}\frac{\alpha_2\bar{Y}}{\bar{X}^2p^2}\right)\nabla_1^2 - \left(\frac{1}{2\bar{X}p} + \frac{\alpha_2}{2\bar{X}p}\right)\nabla_0\nabla_1 \tag{4.5}$$

Squaring equation (4.4) and taking expectations on both the sides, MSE of the estimator $t_{sp}$ is given by

$$\begin{aligned} MSE(t_{sp}) = {}& \alpha_1^2\nabla_1^2 + \alpha_2^2\left(\bar{Y}^2 + \nabla_0^2 + \frac{\bar{Y}^2}{\bar{X}^2p^2}\nabla_1^2 - \frac{2\bar{Y}}{\bar{X}p}\nabla_0\nabla_1\right) - 2\alpha_1\alpha_2\left(\nabla_0\nabla_1 - \frac{\bar{Y}}{\bar{X}p}\nabla_1^2\right) \\ & - 2\alpha_1\left(\nabla_0\nabla_1 - \frac{\bar{Y}}{2\bar{X}p}\right) + 2\alpha_2\left(\frac{\bar{Y}}{8\bar{X}^2}\nabla_1^2 - \frac{3}{2}\frac{\bar{Y}}{\bar{X}p}\nabla_0\nabla_1 + \nabla_0^2 + \frac{5}{8}\frac{\bar{Y}^2}{\bar{X}^2}\frac{\nabla_1^2}{p^2}\right) \\ & + \left(\nabla_0^2 + \frac{\bar{Y}^2}{4\bar{X}^2}\frac{\nabla_1^2}{p^2} - \frac{\bar{Y}}{\bar{X}}\frac{\nabla_0\nabla_1}{p}\right) \end{aligned} \tag{4.6}$$

Where

$$\nabla_0^2 = E\left(\xi_0^2\right) = \lambda_2\left(S_Y^2 + S_U^2\right) + \theta\left(S_{Y(2)}^2 + S_{U(2)}^2\right) \tag{4.7}$$

$$\nabla_1^2 = E\left(\xi_1^2\right) = \lambda_2\left(S_X^2 + S_V^2\right) + \theta\left(S_{X(2)}^2 + S_{V(2)}^2\right) \tag{4.8}$$

$$\nabla_0\nabla_1 = E\left(\xi_0\,\xi_1\right) = \lambda_2\rho_{YX}S_YS_X + \theta\rho_{YX(2)}S_{Y(2)}S_{X(2)} \tag{4.9}$$

Equation (4.6) can be written as:

$$MSE(t_{sp}) = \alpha_1^2A_s + \alpha_2^2B_s - 2\alpha_1\alpha_2C_s - 2\alpha_1D_s + 2\alpha_2E_s + F_s \tag{4.10}$$

Where

$$A_s = \nabla_1^2 \tag{4.11}$$

$$B_s = \left(\bar{Y}^2 + \nabla_0^2 + \frac{\bar{Y}^2}{\bar{X}^2p^2}\nabla_1^2 - 2\frac{\bar{Y}}{\bar{X}p}\nabla_0\nabla_1\right) \tag{4.12}$$

$$C_s = \left( \nabla_0 \nabla_1 - \frac{\bar{Y}}{\bar{X}p} \nabla_1^2 \right) \tag{4.13}$$

$$D_s = \left( \nabla_0 \nabla_1 - \frac{\bar{Y}}{2\bar{X}p} \nabla_1^2 \right) \tag{4.14}$$

$$E_s = \left( \frac{\bar{Y}}{8\bar{X}} \nabla_1^2 - \frac{3}{2} \frac{\bar{Y}}{\bar{X}p} \nabla_0 \nabla_1 + \nabla_0^2 + \frac{5}{8} \frac{\bar{Y}^2}{\bar{X}^2} \frac{\nabla_1^2}{p^2} \right) \tag{4.15}$$

$$F_s = \left( \nabla_0^2 - \frac{\bar{Y}}{\bar{X}p} \nabla_0 \nabla_1 + \frac{\bar{Y}^2}{4\bar{X}^2} \frac{\nabla_1^2}{p^2} \right) \tag{4.16}$$

Differentiating equation (4.10) with respect to $\alpha_1$, $\alpha_2$ and equating it to zero we get:

$$\alpha_1 A_s - \alpha_2 C_s = D_s \tag{4.17}$$

$$-\alpha_1 C_s + \alpha_2 B_s = -E_s \tag{4.18}$$

Solving equation (4.17) and (4.18) we get optimum values of $\alpha_1$ and $\alpha_2$ :

$$\alpha_1 = \frac{B_s D_s - C_s E_s}{A_s B_s - C_s{}^2} = \alpha_{10} \tag{4.19}$$

$$\alpha_2 = \frac{A_s E_s - C_s D_s}{A_s B_s - C_s{}^2} = \alpha_{20} \tag{4.20}$$

Substituting these optimum values of $\alpha_1$ and $\alpha_2$ in equation (4.10), min MSE of estimator $t_{sp}$ is:

$$\min MSE(t_{sp}) = \bar{Y}^2 + \left[ \frac{2C_s D_s E_s - B_s D_s{}^2 - A_s E_s{}^2}{A_s B_s - C_s{}^2} \right] \tag{4.21}$$

## 5. EMPIRICAL STUDY

We use following data sets for empirical study: Source: Muhammad Azeem & Muhammad Hanif (2016)

**Population I:**

N=5000, $\bar{Y} = 4.9271$ , $\bar{X} = 4.9243$, $S_Y^2 = 102.007$, $S_X^2 = 101.411$, $S_U^2 = 8.8621$, $S_V^2 = 9.0013$ , $\rho_{XY} = 0.9950$

| $N_1$ | $N_2$ | $S^2_{Y(2)}$ | $S^2_{X(2)}$ | $S^2_{U(2)}$ | $S^2_{V(2)}$ | $\rho_{YX(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 99.99174 | 99.8747 | 9.1505 | 8.756 | 0.9949 |
| 4250 | 750 | 100.8224 | 100.822 | 9.05382 | 8.766 | 0.9955 |
| 4000 | 1000 | 103.2349 | 103.234 | 8.8212 | 8.339 | 0.9954 |

**Population II:**

N=5000, $\bar{Y} = 4.9966$ , $\bar{X} = 5.0135$, $S_Y^2 = 97.1206$, $S_X^2 = 95.9580$, $S_U^2 = 23.96055$, $S_V^2 = 24.1928$, $\rho_{XY} = 0.9948$

| $N_1$ | $N_2$ | $S^2_{Y(2)}$ | $S^2_{X(2)}$ | $S^2_{U(2)}$ | $S^2_{V(2)}$ | $\rho_{YX(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 97.0278 | 94.5457 | 22.8055 | 25.4326 | 0.9945 |
| 4250 | 750 | 98.2761 | 97.4267 | 23.2783 | 24.1382 | 0.9949 |
| 4000 | 1000 | 96.0935 | 94.7192 | 24.4297 | 23.0307 | 0.9946 |

**Population III:**

N=5000, $\overline{Y} = 4.7309$ , $\overline{X} = 4.7419$, $S_Y^2 = 101.2633$, $S_X^2 = 100.2288$, $S_U^2 = 9.1025$, $S_V^2 = 9.0520$, $\rho_{XY} = 0.9951$

| $N_1$ | $N_2$ | $S^2_{Y(2)}$ | $S^2_{X(2)}$ | $S^2_{U(2)}$ | $S^2_{V(2)}$ | $\rho_{YX(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 102.75 | 101.2097 | 9.0951 | 8.8123 | 0.9950 |
| 4250 | 750 | 99.559 | 99.4976 | 9.2336 | 8.8058 | 0.9953 |
| 4000 | 1000 | 105.433 | 103.89 | 9.2777 | 9.0721 | 0.9951 |

**Population IV:**

N=5000, $\overline{Y} = 1.9600$ , $\overline{X} = 1.9433$, $S_Y^2 = 25.441$, $S_X^2 = 100.228$, $S_U^2 = 6.0404$, $S_V^2 = 6.2244$, $\rho_{XY} = 0.9808$

| $N_1$ | $N_2$ | $S^2_{Y(2)}$ | $S^2_{X(2)}$ | $S^2_{U(2)}$ | $S^2_{V(2)}$ | $\rho_{YX(2)}$ |
|---|---|---|---|---|---|---|
| 4500 | 500 | 24.527 | 23.6120 | 6.3354 | 5.5894 | 0.97911 |
| 4250 | 750 | 28.596 | 27.553 | 6.1242 | 6.2996 | 0.9821 |
| 4000 | 1000 | 25.877 | 25.213 | 5.9383 | 6.2722 | 0.9825 |

Since real data set was not available for this problem so using above four populations from Muhammad Azeem & Muhammad Hanif (2016) mean square errors of the estimators in presence of non-response and measurement error were computed. We have also computed the percent relative efficiencies (PREs) of the various estimators with respect to usual unbiased estimator $\overline{y}^*$ by using the formula:

$$\mathrm{PRE}(t, \overline{y}^*) = \frac{\mathrm{MSE}(\overline{y}^*)}{\mathrm{MSE}(t)} \times 100\,, \tag{5.1}$$

where $t = t_r, t_{ra}, t_{bt}, t_{lr}, t_{sp}$

The findings are presented in the following tables :

**Table 5.1**: PRE's of estimators with respect to $\overline{y}^*$ for population I.

| $N_1$ | | $N_2$ | Estimators | PRE with measurement error | | | PRE without measurement error | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h=2 | h=4 | h=8 | h=2 | h=4 | h=8 |
| 4500 | | 500 | $\overline{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | $t_r$ | 586.0808 | 584.4539 | 582.4221 | 10,087.16 | 10,039.54 | 9,980.247 |
| | | | $t_{ra}$ | 612.2445 | 610.7793 | 609.0847 | 10,096.39 | 10,051.12 | 9,995.222 |
| | | | $t_{bt}$ | 297.5699 | 297.3415 | 297.055 | 390.6204 | 390.8468 | 391.132 |
| | | | $t_{lr}$ | 611.3325 | 609.6874 | 607.6332 | 10,095.55 | 10,050.11 | 9,993.889 |
| | | | $t_{sp}$ | **636.6662** | **639.9666** | **647.8538** | **13,360.24** | **14,161.74** | **16,129.83** |
| | | | $\overline{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4250 | | 750 | $t_r$ | 587.1388 | 587.1021 | 587.0624 | 10,248.54 | 10,427.13 | 10,627.41 |
| | | | $t_{ra}$ | 613.3782 | 613.637 | 614.1666 | 10,258.23 | 10,439.78 | 10,644.02 |
| | | | $t_{bt}$ | 297.7318 | 297.7451 | 297.7594 | 390.8179 | 391.2838 | 391.7889 |
| | | | $t_{lr}$ | 612.4202 | 612.4071 | 612.3929 | 10,257.35 | 10,438.65 | 10,642.39 |
| | | | $t_{sp}$ | **639.1152** | **646.7875** | **662.2565** | **13,868.23** | **15,764.65** | **20,800.16** |
| 4000 | | 1000 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | $t_r$ | 593.61 | 601.4833 | 609.1461 | 10,254.12 | 10,419.62 | 10,581.45 |
| | | | $t_{ra}$ | 619.36 | 626.9757 | 634.7779 | 10,260.05 | 10,423.84 | 10,584.87 |
| | | | $t_{bt}$ | 298.1597 | 298.6841 | 299.1832 | 390.2786 | 390.0507 | 389.835 |
| | | | $t_{lr}$ | 618.3516 | 625.5946 | 632.6514 | 10259.12 | 10422.57 | 10,582.91 |
| | | | $t_{sp}$ | **646.6072** | **664.6857** | **693.5958** | **14,084.42** | **16,555.63** | **24,029.92** |

**Table 5.2**: PRE's of estimators with respect to $\bar{y}^*$ for population II.

| $N_1$ | $N_2$ | Estimators | PRE with measurement error | | | PRE with no measurement error | | |
|---|---|---|---|---|---|---|---|---|
| | | | h=2 | h=4 | h=8 | h=2 | h=4 | h=8 |
| 4500 | 500 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 246.8139 | 246.3256 | 245.7158 | 9,605.463 | 9,516.803 | 9,408.218 |
| | | $t_{ra}$ | 273.9812 | 273.851 | 273.8332 | 9,627.647 | 9,548.192 | 9,453.234 |
| | | $t_{bt}$ | 219.3226 | 219.2884 | 219.2455 | 384.3351 | 383.5184 | 382.5021 |
| | | $t_{lr}$ | 273.0122 | 272.69 | 272.2883 | 9,626.869 | 9,547.259 | 9,451.989 |
| | | $t_{sp}$ | **284.9563** | **287.0743** | **291.5858** | **12,010.92** | **12,431.58** | **13,462.85** |
| 4250 | 750 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 247.8297 | 248.7546 | 249.7567 | 9,712.634 | 9,775.975 | 9,844.793 |
| | | $t_{ra}$ | 275.0537 | 276.4282 | 278.1861 | 9,728.151 | 9,789.833 | 9,857.214 |
| | | $t_{bt}$ | 219.7493 | 220.2895 | 220.8733 | 385.095 | 385.3791 | 385.6841 |
| | | $t_{lr}$ | 274.0347 | 275.1171 | 276.2908 | 9,727.333 | 9,788.778 | 9,855.687 |
| | | $t_{sp}$ | **286.643** | **291.5057** | **300.313** | **12,355.68** | **13,484.17** | **16,238.12** |
| 4000 | 1000 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 244.4634 | 245.8319 | 247.1578 | 9,720.503 | 9,787.652 | 9,853.052 |
| | | $t_{ra}$ | 271.9547 | 273.1309 | 274.6739 | 9,739.698 | 9,809.953 | 9,878.888 |
| | | $t_{bt}$ | 218.5734 | 218.5575 | 218.5423 | 384.7701 | 384.6362 | 384.5076 |
| | | $t_{lr}$ | 270.8887 | 271.6787 | 272.4493 | 9,738.844 | 9,808.791 | 9,877.11 |
| | | $t_{sp}$ | **284.0312** | **289.4657** | **299.6114** | **12,498.39** | **13,965.25** | **17,801.28** |

**Table 5.3**: PRE's of estimators with respect to $\bar{y}^*$ for population III.

| $N_1$ | $N_2$ | Estimators | PRE with measurement error | | | PRE without measurement error | | |
|---|---|---|---|---|---|---|---|---|
| | | | h=2 | h=4 | h=8 | h=2 | h=4 | h=8 |
| 4500 | 500 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 579.6391 | 581.718 | 584.3259 | 10372.41 | 10322.25 | 10260.54 |
| | | $t_{ra}$ | 602.5743 | 604.575 | 607.2361 | 10381.95 | 10334.08 | 10275.63 |
| | | $t_{bt}$ | 294.0979 | 294.1538 | 294.2234 | 386.6437 | 386.3398 | 385.9626 |
| | | $t_{lr}$ | 601.5868 | 603.3876 | 605.649 | 10381.04 | 10332.99 | 10274.18 |
| | | $t_{sp}$ | **627.3482** | **634.4098** | **647.2312** | **13920.59** | **14780.38** | **16897.79** |
| 4250 | 750 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 577.7208 | 576.9875 | 576.1954 | 10441.01 | 10488 | 10539.43 |
| | | $t_{ra}$ | 601.0964 | 600.9336 | 601.0279 | 10446.07 | 10490.43 | 10541.19 |
| | | $t_{bt}$ | 294.1564 | 294.2815 | 294.4171 | 387.3762 | 388.0877 | 388.8621 |
| | | $t_{lr}$ | 600.063 | 599.6086 | 599.1197 | 10445.12 | 10489.22 | 10539.44 |
| | | $t_{sp}$ | **627.0971** | **634.4653** | **649.7234** | **14305.99** | **16064.17** | **20923.44** |
| 4000 | 1000 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 580.9977 | 584.2545 | 587.352 | 10368.01 | 10323.23 | 10281.49 |
| | | $t_{ra}$ | 603.9336 | 607.2549 | 610.8464 | 10378.7 | 10337.44 | 10299.79 |
| | | $t_{bt}$ | 294.1975 | 294.3565 | 294.5062 | 386.515 | 386.1235 | 385.7562 |
| | | $t_{lr}$ | 602.841 | 605.7524 | 608.5238 | 10377.69 | 10336.06 | 10297.66 |
| | | $t_{sp}$ | **631.4019** | **645.205** | **669.8931** | **14405.81** | **16541.35** | **23228.66** |

**Table 5.4**: PRE's of estimators with respect to $\bar{y}^*$ for population IV.

| $N_1$ | $N_2$ | Estimators | PRE with measurement error | | | PRE without measurement error | | |
|---|---|---|---|---|---|---|---|---|
| | | | h=2 | h=4 | h=8 | h=2 | h=4 | h=8 |
| 4500 | 500 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 236.0505 | 236.1577 | 236.2926 | 2604.001 | 2570.671 | 2529.699 |
| | | $t_{ra}$ | 264.4871 | 264.4013 | 264.5374 | 2611.026 | 2578.072 | 2537.761 |
| | | $t_{bt}$ | 215.3613 | 214.8848 | 214.2881 | 364.6081 | 364.2541 | 363.8071 |
| | | $t_{lr}$ | 262.8514 | 262.4443 | 261.9378 | 2609.706 | 2576.497 | 2535.675 |
| | | $t_{sp}$ | **285.011** | **288.7275** | **296.554** | **3010.02** | **3051.212** | **3167.222** |
| 4250 | 750 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 238.9165 | 242.8124 | 246.9824 | 2654.337 | 2690.853 | 2729.431 |
| | | $t_{ra}$ | 267.7365 | 272.083 | 277.2414 | 2661.282 | 2698.123 | 2737.467 |
| | | $t_{bt}$ | 216.8776 | 218.4431 | 220.088 | 365.1299 | 365.4987 | 365.8784 |
| | | $t_{lr}$ | 265.9904 | 269.7947 | 273.8686 | 2659.867 | 2696.261 | 2734.712 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $t_{sp}$ | **290.0738** | **301.9229** | **322.4102** | **3108.209** | **3322.39** | **3756.298** |
| 4000 | 1000 | $\bar{y}^*$ | 100 | 100 | 100 | 100 | 100 | 100 |
| | | $t_r$ | 236.8203 | 237.8343 | 238.8079 | 2667.974 | 2719.149 | 2769.596 |
| | | $t_{ra}$ | 266.0461 | 268.056 | 270.6769 | 2676.232 | 2729.543 | 2782.804 |
| | | $t_{bt}$ | 216.3226 | 217.0976 | 217.8408 | 365.9304 | 367.2356 | 368.4837 |
| | | $t_{lr}$ | 264.2399 | 265.5873 | 266.8832 | 2674.771 | 2727.543 | 2779.726 |
| | | $t_{sp}$ | **289.2459** | **300.5426** | **322.2754** | **3149.877** | **3442.595** | **4052.483** |

From the Tables (5.1), (5.2), (5.3) and (5.4), it is clear that the proposed estimator $t_{sp}$ has largest PRE (percentage relative efficiency) i.e. $t_{sp}$ is the most efficient estimator among all other estimators considered in this paper. We have also computed PRE for estimators in presence of measurement error and in absence of measurement error. From our findings we conclude that estimators show unexpected increase in efficiency when measurement error is not considered. The PRE's (in case of without measurement error) becomes approximately double than the PRE's (in case of with measurement error).

## 6. CONCLUSIONS

In this paper, we have proposed an estimator $t_{sp}$ in presence of non-response and measurement error and derived its MSE (mean square error) up to first order of approximation. We have also compared efficiency of estimators (in case of without measurement error) with efficiency of estimators (in case of with measurement error). From our empirical study we conclude that PRE (percentage relative efficiency) of our proposed estimator $t_{sp}$ is maximum among all the estimators that we have considered here. We have also found that measurement error and non-response error effects PRE of estimators at high rate. As we can observe in Table (5.1), (5.2), (5.3) and (5.4) that PRE of estimators $t_r, t_{ra}, t_{lr}, t_{sp}$ has unexpectedly declined when measurement error is taken into account and it has approximately fallen down to half of the case when measurement error is not considered in estimation. However estimator $t_{bt}$ is not as much affected as other estimators and it shows small decline in PRE in presence of measurement error. Hence we conclude that estimation of population parameters under the assumption that all the data is observable without any measurement error and non-response error is highly incorrect. As it is very clear in the tables (5.1), (5.2), (5.3) and (5.4) that the measurement error and non-response errors are heavily affecting estimators considered here. Our proposed estimator $t_{sp}$ is efficient estimator as compared to usual estimator $\bar{y}^*$ as well as other estimators considered here.

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