SUPPLY CHAIN MODEL FOR IMPERFECT QUALITY ITEMS WITH TRADE CREDIT FINANCING: A GAME THEORETICAL APPROACH

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ABSTRACT
The concepts of advanced technology and equipment are adopted widely in the industry to determine the production of high quality items, regardless, it is observed that a few items produced are of imperfect quality. These defective items might be the result of common operations or static maintenance. Thereby, to sort the imperfect quality items, an inspection process is performed with each lot of items delivered from the seller to the buyer, which are then sold at a discount. Generally, shortages appear due to defective items, which can be restrained by scaling an order when the inventory level meets the demand concurrent to screening process. Further, it is generally assumed that the buyer would process payments on an immediate basis to the seller after receiving the consignment of the gross items purchased. While in practice, the seller does offer a certain fixed duration for the buyer to trigger his supply. The interest is only levied to the buyer by the seller beyond the fixed period as per agreed terms and conditions. With the outline, a supply chain model has been developed with imperfect quality items with allowable late payments wherein end demand of the product depends upon the retail price. Optimal polices of the seller and buyer are obtained under co-operative and non-cooperative analogue, which will enhance the supply chain profit. Co-operative relationship is established by a Pareto efficient solution method, and non-cooperative is obtained by Seller-Stackelberg approach. Finally, numerical illustrations with sensibility analysis are stated to exemplify the theory of the paper.

KEYWORDS: Imperfect quality items; Trade credit Game theory; Non-cooperative game; Co-operative games; Supply chain.

MSC: 90B05

RESUMEN
La industria está dotada de los conceptos de tecnología avanzada y equipamiento para determinar la producción de productos de alta calidad, sin embargo se observa que algunos ítems producidos son de calidad imperfecta. Estos ítems defectuosos pueden ser resultado de operaciones comunes de mantenimiento estático. De ahí que para tratar con ellos un proceso de inspección se lleva a cabo con cada lote de ítems, enviado por el vendedor al comprador, los que se venden con un descuento. Generalmente aparecen carencias debido a los ítems defectuosos, la orden es re-escalada cuando el nivel de inventario satisface la demanda concurrente para el proceso del ´screening´. Además, se asume generalmente que el comprador procesará los pagos al vendedor inmediatamente después de servirse la consigna del grueso de los ítems a recibir. Mientras, en la práctica, el vendedor ofrece una cierta duración fija para proceder a enviar su suplemento. El interés del comprador solo es recabado por el vendedor más allá del periodo fijo a partir de términos acordados. Con la descripción, una cadena de suministro ha sido desarrollado con ítems de calidad imperfecta con pagos permisiblemente demorados, donde la demanda final del producto depende del precio de venta. Políticas optimales del vendedor y el comprador se obtienen bajo los análogos co-operativo y no-cooperativo, los que permitirían incrementar la ganancia de la cadena de suministro. La relación co-operativa es establecidas para un método de solución Pareto-eficiente y de uno no-cooperativo usando el enfoque de Seller-Stackelberg. Finalmente se presentan ilustraciones numéricas con un análisis de sensibilidad para ilustrar la teoría desarrolla en el paper.

PALABRAS CLAVE: Calidad imperfecta de ítems; Teoría de Juegos de comercio-crédito, Juegos no-cooperativos, juego co-operativos; cadena de suministro.

1. INTRODUCTION

It is observed that products with imperfect quality have the direct implication on supply chain system, which is well acknowledged by industry and receive the attention of the researchers. During the last few decades, many researchers and academicians put a lot of amount of efforts to developed supply chain models for imperfect quality items. Initially, (Schwaller 1988; Porteus 1986; Rosenblatt and Lee 1986) explored *EOQ* models on defective items as a result of the imperfect quality process of production. Salameh and Jaber

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(2000) expanded the *EOQ/EPQ* model for imperfect quality items and these items are then sold at a discount at the completion of the inspection process in a single lot. (Cárdenas-Barrón 2000) improved the optimum order quantity formula obtained by Salameh and Jaber (2000). Goyal and Cárdenas-Barrón (2002) presented a simple approach for *EOQ* model for imperfect quality items and compared results with Salameh and Jaber (2000). Wee et al. (2007) expanded the model of Salameh and Jaber (2000) where shortages were ordered back in each cycle. Maddah and Jaber (2008) improved Salameh and Jaber's (2000) work by changing the technique of calculating the expected total profit per unit time by applying Renewal-reward theorem (Ross 1996).

Sarkar et al. (2014) developed EPQ model for single-stage manufacturing system under the concept of random defective rate, rework and backorder. Inventory models are developed for three different distribution (triangular, uniform and beta) to compare the optimal results. Taleizadeh et al. (2014) investigated an economic production quantity inventory model with scrap, rework and interruption in process, in which shortage is permitted and these are fully backordered. Cycle length and backordered quantities of each product are considered as decisión variables which minimize the expected total cost. Taleizadeh et al. (2015) developed EPQ model with rework in which selling price, replenishment lot size and the number of shipments are jointly determined. A practical algorithm is developed to find the optimal results and average long-run benefit function is maximized. Wang et al. (2015) developed an EOQ model for imperfect quality items with partial backorders and screening constraint. Expected total profit per unit time is obtained by using Renewal-reward theory. Pasandideh et al. (2015) developed an economic production quantity (EPQ) inventory model for a multi-product single-machine lot sizing problem with scrap and rework, where reworks are classified into several groups based on failure severity, the shortage is allowed and is backordered. The aim is to determine the optimal period length, the lot size and the allowable shortages of each product, thus, the total cost is minimized. Nobil et al. (2016), developed multi-machine multi-product (EPQ) for an imperfect manufacturing system as a mixed integer nonlinear programming (MINLP) where the convexity property of multi-product single machine EPQ model is used to convert the problem into a bi-level decision-making problem. Hybrid genetic algorithm (HGA) is proposed to find the optimal results.

Most of the supply chain models developed with the presumption that payment of quantity will be made by the sellerto the buyer immediately after receiving it. Although with routine procedure, this assumption is not appreciated much, as mostly seller offers a certain fixed duration of credit to trigger their supply. The interest is only levied to the buyer by the seller beyond the fixed period as per agreed terms and conditions. Thereby, most of the supply chain industries are using credit period policy to enhance the profit of the partners in the supply chain. During the last few years, depending on this fact, a great amount of research work has been conducted on seller-buyer models with allowable delay in payment. Haley and Higgins (1973) analyzed the buyer's lot size problem with a contract of trade credit by taking a fixed demand and showed that lot size is not affected by the length of credit period. Hwang and Shinn (1997) showed in his study that the buyer's order quantity varies with the trade credit period's length by taking demand varies with price.

Kim et al. (1995) constructed a model to derive the optimal trade credit period with the presumption that the selling price of the seller is fixed. Jaber and Osman (2006) explored a supply chain model in which the credit period policy is used by the seller with a motive to enhance the buyer's order quantity and he will charge whenever payment is delayed from the buyer's side. Chung and Huang (2006) merged the idea of an inspection of imperfect quality items with trade credit. Further, under allowable delay in payments, Jaggi et al. (2013) investigated an inventory model for imperfect quality items in which shortages are permitted and fully backlogged, which are cancelled out during the screening process under the presumption that screening rate is greater than the demand rate. Zhou et al. (2016) also found a synergy economic order quantity model, in which the concepts of imperfect quality, inspection error and shortages with trade credit are considered. The dealings between the partners of the supply chain and consequences of the interaction is ignored in the above-mentioned papers. In the past few years, the game theory became a major tool for analyzing the interaction between the players in a buyer-seller supply chain models. This theory plays a decisive role in the field of supply chain problems whose aim is to develop supply chain policies with different assumptions with wide and varied perspectives. These policies demonstrate interaction between various channels in the supply chain to get effectual outcome.

Zhou et al. (2012) developed a specific condition in Supplier-Stackelberg game under inventory dependent demand, wherein, the credit policy increases the entire supply chain profit along with each member's profit. Further, Abad and Jaggi (2003) considered a supply chain model with trade credit policy offered to the buyer by the seller, in which the market demand was price sensitive. They developed non-cooperative and cooperative interaction between the players in the supply chain and find the optimal solution. Esmaeili et al.

(2009) explained several supply chain models, i.e. cooperative and non-cooperative games in which end demand not only depends on price sensitive but also on marketing expenditure. This work was extended by Esmaeili and Zeephongscul (2010) for the asymmetric information game, i.e. some information is known and private to the individual. In the similar scenario, Zhang and Panlop (2013) investigated non-cooperative models with the credit option by assuming the same demand function.

None of the researchers considered the effects of imperfect production on the supply chain model in the co-operative and non-cooperative environment with trade-credit financing. In this paper, a supply chain model has been developed with imperfect quality items, wherein the seller offers a fixed credit duration for the buyer as a payoff to influence supply to augment the order quantity. During the credit duration, the seller will maintain the cost of holding goods and would get the profit in terms of capital gain at a latter stage. The Capital cost of the seller is assumed to be a linear function of the length of the credit period (Abad and Jaggi (2003)).We presumed the end demand of the product depends upon the retail price. Optimal polices of the seller and buyer are obtained under co-operative and non-cooperative analogue. Co-operative relationship is established by a Pareto efficient solution method, and non-cooperative is obtained by Seller-Stackelberg approach. We will exhibit that in cooperative game buyer and seller could receive more benefit than to non-cooperative game.

This paper comprises of seven subdivisions, illustrating an introduction and Literature review at first subdivision. Notations and assumptions used in the paper from start to finish are discussed and contemplated at second. Third subdivision specifies systematically the non-cooperative Seller-Stackelberg mathematical model, in which seller behaves as leader and buyer behaves as a follower. In the fourth subdivision, we propose the co-operative game model with Pareto efficient approach. In fifth and six subdivisions, we provide numerical examples with sensitivity analysis to explain the significance of the theory of the paper. Last subdivision consists of conclusion with some suggestion for future research work.

Table 1 Contribution of different authors

Author(s)	**Supply chain**	**Inspection**	**Trade-credit policy**	**Non- cooperative game**	**Co-operative game**
Hayley and Higgins (1973)	√		√		
Kim et al. (1995)	√		√		
Hwang and Shinn (1997)	√		√		
Porteus (1986)	√	√			
Rosenblatt and Lee 1986)	√	√			
Schwaller (1988)	√	√			
Salameh and Jaber (2000)	√	√			
Cárdenas-Barrón (2000)	√	√			
Abad and Jaggi (2003)	√		√	√	√
Chung and Huang (2006)	√	√	√		
Jaber and Osman (2006)	√		√		
Wee (2007)	√	√			
Maddah and Jaber (2008)	√	√			
Esmaeili et al. (2009)	√			√	√
Zhou et al. (2012)	√		√	√	
Jaggi et al. (2013)	√	√	√		
Zhang and Panlop (2013)	√		√	√	
Zhou et al. (2016)	√	√	√		
Present paper	√	√	√	√	√

Notations and assumptions

Notations

The notations used are given below

Decision variables

c_b Buyer's unit purchasing cost ($/unit)

Q Order quantity of the buyer (units)

M Credit-period offered to the buyer by the seller (year)

p_b Buyer's retail price ($/unit)

Parameters

A_b Ordering cost of the buyer ($/order)

H_b Inventory carrying cost ($/unit/unit time)

T	Cycle time in years
I_s	Opportunity cost of capital of seller (\$/year)
A_s	Ordering cost of the seller (\$/order)
I	Inventory carrying cost (\$/year)
I_e	Interest earned rate for the buyer (\$/year)
I_p	Interest paid rate for the buyer (\$/year)
D	Annual demand rate (unit/year)
α	Percentage of defective items delivered by the seller to the buyer
λ	Buyer's screening rate (unit/year)
c_s	Cost of defective items per unit (\$/year), ($c_s < c_b$)
C	Seller's unit purchasing cost (\$/unit)

Assumptions

1) The annual demand is price sensitive i.e.$D = K * p^{-e}$ (Abad and Jaggi, 2003).
2) Shortages are not taken into consideration and planning horizon is infinite.
3) It is assumed that, there is $'\alpha'$ percentage defective items with uniform probability distribution in each lot (Jaggi et al., 2013)
4) To avoid shortages, the production rate is greater than the demand rate.
5) A definite credit period is offered by the seller for the buyer
6) Interest earned by the buyer, I_e and interest paid, I_p are considered equal (Zhang and Zeephoungsekul, 2013).
7) $I_s = a_1 + a_2 M, a_1 > 0, a_2 > 0,$ seller's opportunity cost of capital is assumed as a linear function of credit period. (Hayley and Higgins, 1973, Kim et al., 1995).
8) There is no carrying cost that links with lot size as the seller considers a lot to lot strategy.

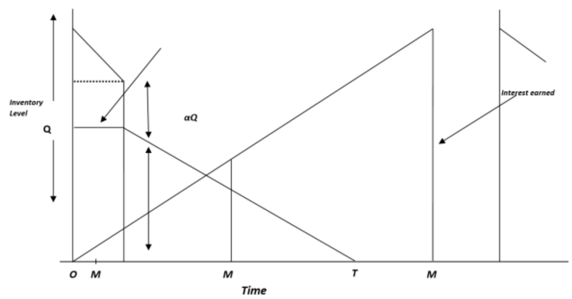

Figure 1. Inventory system with inspection and trade-credit for all the three cases (i) $M \leq t \leq T$ (ii) $t \leq M \leq T$ and (iii) $M \geq T$

2. MATHEMATICAL MODEL

This section develops mathematical formulations of buyer's, seller's, non-cooperative Seller-Stackelberg and cooperative game models to optimize the expected profits of each member of the supply chain.

2.1. The buyer's model

The buyer's main objective is to determine the order quantity and the selling price to maximize his expected profit. In this paper, the seller supplies the items to the buyer with unit price and offers credit period to the buyer. At the buyer's end, received lots Q units (assumed) goes through an inspection process and contain α percent defective items. Defective items and non-defective items separate from the entire received quantity, Q, at a rate of λ units per unit time by the screening process. At the end of screening time,$\boldsymbol{t} = Q/\lambda$, defective items, αQ, are sold at a discounted price c_s and non-defective items, (1-α)Q, are sold at a price p_b in a single lot. Therefore, the total revenue of the buyer is $p_b\ Q + c_s \alpha Q$. The purchasing cost of the buyer of Q quantity at a price c_b is c_bQ and ordering cost of the buyer is A_b, and inventory carrying cost will be equal to $(\frac{Q(1-\alpha)T}{2}+\frac{\alpha Q^2}{\lambda})H_b$, where $H_b = Ic_b$.

There are three possible cases:

(i) $M \le t \le T$ $(ii) t \le M \le T$ (iii) $M \ge T$

Case I : $\boldsymbol{M \le t \le T}$

The buyer gets an interest on the sales revenue generated during the time 0 to M at the rate I_e **.** Under this period the buyer has to settle the account and also arrange finance for remaining inventory stock during the time *M* to *T* and for defective items for the time period *M* to *t* at the specific rate of interest I_p to the seller. Buyer's earned interest for the inventory during the time 0 to *M* is $D\frac{M^2I_ec_b}{2}$ and buyer's paid interest for the inventory not sold after the credit period is $\frac{D(T-M)^2I_p\,c_b}{2} + c_sI_p\alpha Q(t-M)$.The total profit for the buyer is expressed as $TP_{b1}(p_b, Q)$.

$TP_{b1}(p_b, Q)$ = Sales Revenue - Purchasing cost - Ordering cost - Inventory carrying cost + Interest gain due to credit – Interest paid

$$=p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - A_b - \left(\frac{Q(1-\alpha)T}{2}+\frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2I_e\,c_b}{2} - \frac{D(T-M)^2I_p\,c_b}{2} - c_sI_p\alpha Q(t-M)$$

Put $=\frac{(1-\alpha)Q}{D}, t = \frac{Q}{\lambda}$, then buyer's profit becomes

$$TP_{b1}(p_b, Q) = p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - A_b - \left(\frac{Q^2(1-\alpha)^2}{2D}+\frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2I_e\,c_b}{2} - \frac{D\left(\frac{(1-\alpha)Q}{D}-M\right)^2I_p\,c_b}{2} - c_sI_p\alpha Q(\frac{Q}{\lambda}-M)$$

$$= p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - A_b - \left(\frac{Q^2(1-\alpha)^2}{2D}+\frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2I_e\,c_b}{2} - \frac{D}{2}\,(\frac{Q^2(1-\alpha)^2}{D^2} + M^2 - 2QM(1-\alpha)I_p\,c_b\,) - c_sI_p\alpha\frac{Q^2}{\lambda} + c_sI_pQM\alpha$$

Thus, the total expected buyer's profit is given by

$$E[TP_{b1}(p_b, Q)] = p_bE[1-\alpha]Q + c_sE[\alpha]Q - c_bQ - A_b - \left(\frac{Q^2E[(1-\alpha)^2]}{2D}+\frac{E[\alpha]Q^2}{\lambda}\right)Ic_b + D\frac{M^2I_ec_b}{2} - \left(\frac{Q^2E[(1-\alpha)^2]}{2D}+\frac{DM^2}{2} - QME[1-\alpha]\right)I_pc_b - c_sI_pE[\alpha]\frac{Q^2}{\lambda} + c_sI_pQME[\alpha]$$

$$= -\left[\left(\frac{E[(1-\alpha)^2]}{2D}+\frac{E[\alpha]}{\lambda}\right)Ic_b + \left(\frac{E[(1-\alpha)^2]}{2D}\right)I_pc_b + c_sI_p\frac{E[\alpha]}{\lambda}\right]Q^2 + \left[p_b(1-E[\alpha] + c_sE[\alpha] - c_b + M(1-E[\alpha]I_pc_b + I_pME[\alpha]\right]Q - A_b + D\frac{M^2I_ec_b}{2} - D\frac{M^2I_pc_b}{2}$$

By using, Renewal theory as used in Maddah and Jaber (2008), we have the expected total profit of the buyer per cycle

$$E[TP^c{}_{b1}(p_b, Q)] = \frac{E[TP_{b1}(p_b,Q)]}{E(T)} = \frac{D}{Q(1-E[\alpha])}E[TP_{b1}(p_b, Q)] = \frac{D}{Q(1-E[\alpha])}\left[-\left[\left(\frac{E[(1-\alpha)^2]}{2D}+\frac{E[\alpha]}{\lambda}\right)Ic_b + \left(\frac{E[(1-\alpha)^2]}{2D}\right)I_pc_b + c_sI_p\frac{E[\alpha]}{\lambda}\right]Q^2 + \left[p_b(1-E[\alpha] + c_sE[\alpha] - c_b + M(1-E[\alpha]I_pc_b + c_sI_pME[\alpha]\right]Q - A_b + D\frac{M^2I_ec_b}{2} - D\frac{M^2I_pc_b}{2}\right] \quad (1)$$

Case II : $\boldsymbol{t} \le M \le T$

In this case, sales revenue of the buyer, purchasing cost, ordering cost and inventory carrying cost of the buyer will be same as in case I, interest gain due to credit period and interest paid will be different. In this case, the buyer not only earns interest at rate I_e on the revenue generated from the average sales for the period 0 to M but also earns interest from revenue generated by the sales of defective items, αQ, at discounted price for the time period *t* to M but pays interest at a rate I_p for the period *M* to *T*. Interest gain due to credit for the period 0 to M is $D\frac{M^2 I_e p_b}{2} + c_s I_e \alpha Q(M-t)$ and interest paid in the period M $to\ \ T$ is $\frac{D(T-M)^2 I_p c_b}{2}$. The total profit for the buyer is expressed as $\mathrm{T}P_{b2}(p_b, Q)$.

$\mathrm{T}P_{b2}(p_b, Q)$ = Sales Revenue - Purchasing cost - Ordering cost - Inventory carrying cost+ Interest gain due to credit – Interest paid

$$\mathrm{T}P_{b2}(p_b, Q) = p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - A_b - \left(\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2 I_e p_b}{2} + c_s I_e \alpha Q(M-t) - \frac{D(T-M)^2 I_p c_b}{2}$$

Put $T = \frac{(1-\alpha)Q}{D}, t = \frac{Q}{\lambda}$

Thus, the total expected buyer's profit is given by

$$\mathrm{T}P_{b2}(p_b, Q) = p_b(1-E[\alpha])Q + c_sE[\alpha]Q - c_bQ - A_b - \left(\frac{Q^2E[(1-\alpha)^2]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2 I_e p_b}{2} + c_s I_e E[\alpha]Q(M - \frac{Q}{\lambda}) - \frac{1}{2}D\left(\frac{(1-E[\alpha])Q}{D} - M\right)^2 I_p\, c_b$$

By using, Renewal theory as used in Maddah and Jaber (2008), the buyer's expected total profit per cycle,

$$E[TP^c{}_{b2}(p_b, Q)] = \frac{E[TP_{b2}(p_b, Q)]}{E(T)}$$

$$= \frac{D}{Q(1-E[\alpha])}\left[p_b(1-E[\alpha])Q + c_sE[\alpha]Q - c_bQ - A_b - \left(\frac{Q^2[E(1-\alpha)^2]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{M^2 I_e p_b}{2} + c_s I_e E[\alpha]Q(M - \frac{Q}{\lambda}) - \frac{1}{2}D\left(\frac{(1-E[\alpha])Q}{D} - M\right)^2 I_p\, c_b\right] \quad (2)$$

***Case III* : $\boldsymbol{t \le T \le M}$**

In this case buyer's total earned interest is $D\frac{T^2 I_e p_b}{2} + p_b I_e DT(M-T) + c_s I_e \alpha Q(M-t)$ and no interest is payable by the buyer to the seller, thus his paid interest is equal to zero. The total profit for the buyer is expressed as $\mathrm{T}P_{b3}(p_b, Q)$.

$\mathrm{T}P_{b3}(p_b, Q)$ = Sales Revenue - Purchasing cost - Ordering cost - Inventory carrying cost + Interest gain due to credit

$$=p_b(1-\alpha)Q + c_s\alpha Q - c_bQ - A_b - \left(\frac{Q(1-\alpha)T}{2} + \frac{\alpha Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{T^2 I_e p_b}{2} + p_b I_e DT(M-T) + c_s I_e \alpha Q(M-t)$$

Put $T = \frac{(1-\alpha)Q}{D}, t = \frac{Q}{\lambda}$, thus, the total expected buyer's profit is given by

$$\mathrm{T}P_{b3}(p_b, Q) = p_b(1-E[\alpha])Q + c_sE[\alpha]Q - c_bQ - A_b - \left(\frac{Q^2[E(1-\alpha)^2]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b + D\frac{\frac{(E[(1-\alpha)^2])Q}{D^2}^2 I_e\, p_b}{2} + p_b I_e D[\frac{(1-E[\alpha])Q}{D}M + \frac{(E[(1-\alpha)^2])Q}{D^2}^2] + c_s I_e E[\alpha]Q(M - \frac{Q}{\lambda})$$

By using, Renewal theory as used in Maddah and Jaber (2008), the buyer's expected total profit per cycle

$$E[TP^c{}_{b3}(p_b, Q)] = \frac{E[TP_{b3}(p_b, Q)]}{E(T)}$$

$$= \frac{D}{Q(1-E[\alpha])}\left[p_b(1-E[\alpha])Q + c_sE[\alpha]Q - c_bQ - A_b - \left(\frac{Q^2E[(1-\alpha)^2]}{2D} + \frac{E[\alpha]Q^2}{\lambda}\right)\mathrm{I}c_b + \frac{1}{2}\frac{(E[(1-\alpha)^2])Q}{D}^2 I_e\, p_b + p_b I_e D\left[\frac{(1-E[\alpha])Q}{D}M + \frac{(E[(1-\alpha)^2])Q}{D^2}^2\right] + c_s I_e E[\alpha]Q\left(M - \frac{Q}{\lambda}\right)\right] \quad (3)$$

Under the assumption, $I_e = I_p$, the mathematical expression for buyer's expected profit per cycle for case I, case II and case III (Zhang *et al.* 2013) are same and denoted by $E[TP^c{}_b(p_b, Q)]$.

Demand is assumed to be price sensitive, $D = Kp_b{}^{-e}$

Thus, buyer's expected profit can be expressed as follows

$$E[TP^c{}_b(p_b, Q)] = Kp_b{}^{-e}\left[p_b - A_1Q - A_2 - \frac{A_3}{Q}\right] - c_bA_4Q \quad (4)$$

Let

$A_1 = \frac{E[\alpha]Ic_b}{(1-E[\alpha])\lambda} + \frac{c_s I_e E[\alpha]}{(1-E[\alpha])\lambda}$, $A_2 = \frac{c_b}{(1-E[\alpha])} - \frac{c_s E[\alpha]}{(1-E[\alpha])} - \frac{c_s I_e E[\alpha] M}{(1-E[\alpha])} - c_b I_p M$, $A_3 = \frac{A_b}{(1-E[\alpha])}$, $A_4 = \frac{E[(1-\alpha)^2]}{2\,(1-E[\alpha])}(\mathrm{I} + I_p)$ The first order condition with respect to p_b for a fixed Q, which gives the unique value of p_b and that maximize $E[TP^c{}_b(p_b, Q)]$. The first order conditions yield,

$$p_b = \frac{e}{e-1}\left(A_1 Q + A_2 + \frac{A_3}{Q}\right),\ e \geq 1 \tag{5}$$

Where A_1,A_2, and A_3 are defined by the equation (4)
Substituting the value of p_b into equation (4),

$$E[TP^c{}_b(p_b(Q), Q)] = \frac{K}{e}\left(\frac{e}{e-1}\left[A_1 Q + A_2 + \frac{A_3}{Q}\right]\right)^{-e+1} - c_b A_4 Q \tag{6}$$

The first order condition $E[TP^c{}_b(p_b(Q), Q)]$ given in equation (6) with respect to Q gives the result,

$$Q = \frac{kA_3 {p_b}^{-e}}{c_b A_4 + kA_1 {p_b}^{-e}} \tag{7}$$

And second order condition of $E[TP^c{}_b(p_b(Q), Q)]$ with respect to Q

$$\frac{\partial^2}{\partial Q^2} E[TP^c{}_b(p_b(Q), Q)] = \frac{e^2}{(e-1)p_b}\left(A_1 - \frac{A_3}{Q^2}\right)^2 - 2\frac{A_3}{Q^3} \tag{8}$$

$$where\ \ p_b = \frac{e}{e-1}\left(A_1 Q + A_2 + \frac{A_3}{Q}\right)$$

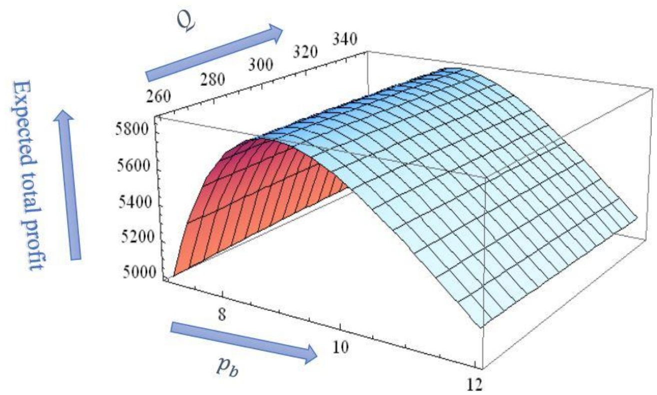

Figure 2. Buyer's expected total profit with respect to p_b and Q

$$\frac{\partial^2 E[TP^c{}_b(p_b,Q)]}{\partial {p_b}^2} > 0\ ,\ \frac{\partial^2 E[TP^c{}_b(p_b,Q)]}{\partial Q^2} > 0 \ \text{and}\ \left[\frac{\partial^2 E[TP^c{}_b(p_b,Q)]}{\partial {p_b}^2}\right]\left[\frac{\partial^2 E[TP^c{}_b(p_b,Q)]}{\partial Q^2}\right] - \left[\frac{\partial^2 E[TP^c{}_b(p_b,Q)]}{\partial p_b \partial Q}\right]^2 < 0$$

It's quite difficult to prove the concavity of the above expected total profit function defined in equation (4) analytically.
Thus, expected total profit $E[TP^c{}_b(p_b, Q)]$ in equation (4) is concave function with respect to p_b and Q for $e \geq 1$ is shown with the help of the graph (fig 2). Further, first and second derivatives are defined in the Appendix A in the end of the paper.

2.2. Seller's model

The motive of the seller is to find his optimal policies which are selling price, c_b, and credit period, M, to maximize his total expected profit. The sales revenue generated by seller is $c_b Q$, purchasing cost per year is CQ and ordering cost per year is denoted by A_s. Opportunity cost per year is $I_s c_b\ (1-\alpha)MQ$ and cycle length of the supply chain is T= $(1-\alpha)Q/D$. There is no inventory cost carrying cost since seller follows lot to lot strategy. The total profit for the seller is expressed as $\mathrm{T}P_s(c_b, M)$.
The total profit function of the seller is
Seller profit = Sales Revenue - Purchasing cost - Ordering cost - Opportunity cost

$$\mathrm{Tp}_s(c_b, M) = c_b Q - CQ - A_s - (a_1 + a_2 M)\big((1-\alpha)\big)c_b M\, Q$$

Expected profit of seller per cycle is

$E[\mathrm{TP}^c{}_s(c_b,M)] = \frac{D}{Q(1-E[\alpha])}[c_bQ - CQ - A_s - (a_1 + a_2 M)((1-E[\alpha]))c_bM\, Q\,] = D[\frac{1}{(1-E[\alpha])}(c_b - C - \frac{A_s}{Q}) - (a_1 + a_2 M)c_bM\,]$ (9)

First order condition with respect to M for fixed c_b results in $\frac{\partial}{\partial M}E[\mathrm{TP}^c{}_s(c_b,M)] = -D[a_1c_b + 2a_2\, c_bM]$

And second order differentiation yield the results, $\frac{\partial^2}{\partial M^2}E[\mathrm{TP}^c{}_s(c_b,M)] = -\,2a_2\,c_b\,D < 0$

Given expected profit function$E[\mathrm{TP}^c{}_s(c_b,M)]$ is concave for a fixed c_b

By equation (9), it can be easily seen that $E[\mathrm{TP}^c{}_s(c_b,M)]$ is linear in c_b, thus, selling price of the seller is unbounded and denoted by $c_b{}^*$ and seller have to set the selling price by setting zero profit i.e. $E(\mathrm{TP}^c{}_s(c_b,M)) = 0$, we have $c_{b_0} = \frac{4a_2\,(c+\frac{A_s}{Q})}{4a_2+a_1{}^2(1-E[\alpha])}$, $c_b{}^* = T\,c_{b_0} = T\,\frac{4a_2\,(C+\frac{A_s}{Q})}{4a_2+a_1{}^2(1-E[\alpha])}$, (for some$T > 1$) can be gained through the mediation with the buyer.

2.3. The non-cooperative Stackelberg model

The non-cooperative Stackelberg strategic game structure is used to establish the correlation among the partners of the supply chain. In this model, two players, seller and buyer interact with each other. One player performs as the leader and take initiative to first move and another player act as follower, move sequentially and shows his best response based on available information. The intent of the leader is to plan the best policies based on the best response given by the follower as to maximize his gain.

The Seller-Stackelberg model

In this model, the seller performs as a leader and the buyer performs as a follower. The seller moves first and offers selling price, c_b , and credit period, M, to the buyer. Grounded on the seller's first move, the buyer chooses his best strategy for determining the optimal selling price, p_b, and order quantity, Q, which is defined in the buyer's model by the equation (5) and equation (7). The seller's aim is to maximize his gain based on the decision variables of the buyer p_b and Q. Now, the problem reduces to

Max $E(\mathrm{TP}^c{}_s(c_b,M))$

Subject to the conditions

$$p_b = \frac{e}{e-1}(A_1Q + A_2 + \frac{A_3}{Q}) \tag{10}$$

$$Q = \frac{kA_3p_b{}^{-e}}{c_bA_4+kA_1p_b{}^{-e}} \tag{11}$$

$where\ \ A_1 = \frac{E[\alpha]Ic_b}{(1-E(\alpha))\lambda} + \frac{c_sI_eE[\alpha]}{(1-E(\alpha))\lambda}, A_2 = \frac{c_b}{(1-E[\alpha])} - \frac{c_sE[\alpha]}{(1-E[\alpha])} - \frac{c_sI_eE[\alpha]M}{(1-E[\alpha])} - c_bI_pM\ \ , A_3 = \frac{A_b}{(1-E[\alpha])}\ , A_4 = \frac{E[(1-\alpha)^2]}{2\,(1-E[\alpha])}(\mathrm{I} + I_p)$

From equation (11) gives

$$p_b = (\frac{k(A_3 - A_1Q^2)}{c_bA_4Q^2})^{1/e} \tag{12}$$

From equation (10) and equation (12), we get

$$M = \left(\frac{c_b}{(1-E[\alpha])} - \frac{c_sE[\alpha]}{(1-E[\alpha])} - \frac{e-1}{e}p_b + A_1Q + \frac{A_3}{Q}\right) / (\frac{c_sI_eE[\alpha]}{(1-E[\alpha])} + c_bI_p) \tag{13}$$

Substituting the values of p_b and M into seller's problem, this problem becomes

$$\mathrm{Max}\ E(\mathrm{TP}^c{}_s(c_b,Q)) = \mathrm{K}\,p_b{}^{-e}[\frac{1}{(1-E[\alpha]))}(c_b - C - \frac{A_s}{Q}) - (a_1 + a_2 M)c_bM\,] \tag{14}$$

Where,

$$p_b = (\frac{k(A_3 - A_1Q^2)}{c_bA_4Q^2})^{1/e}, M = \left(\frac{c_b}{(1-E[\alpha])} - \frac{c_sE[\alpha]}{(1-E[\alpha])} - \frac{e-1}{e}p_b + A_1Q + \frac{A_3}{Q}\right) / (\frac{c_sI_eE[\alpha]}{(1-E[\alpha])} + c_bI_p)$$

$E[\mathrm{TP}^c{}_s(c_b,Q)]$ is non-linear objective function. Solve the above problem for different optimal values of Q and c_b.

2.4. The co-operative game

The co-operative games are the games in which both the players of the supply chain work together with an objective to maximize their profit. The Pareto efficient solution is one of the approach to solve such type of games. It is a state in which one player can't perform well off without making another player's worse off. Such co-operation is carried out by taking the joint optimization of the weighted sum of the seller's and the buyer's profit function. In this approach objective is to optimize the profits of buyer and seller by determining retailer price, p_b, selling price of the seller, c_b, trade credit, M, offered by the seller and lot size, Q. The Pareto efficient solution can be determined by maximizing the joint weighted sum of buyer and seller's expected profit (Esmaeili *et al*., 2009).

$$E[JTP_{sb}] = \mu E[\mathrm{TP}^c{}_s] + (1-\mu)E[\mathrm{TP}^c{}_b], \qquad 0 \le \mu \le 1 \tag{15}$$

$$E[JTP_{sb}] = \mu\, D[\frac{1}{(1-E\,[\alpha])}(c_b - C - \frac{A_S}{Q}) - (a_1 + \, a_2\, M)c_b M\,] + (1-\mu)Kp_b{}^{-e}\left[p_b - A_1 Q - A_2 - \frac{A_3}{Q}\right] - c_b A_4 Q$$

, where A_1, A_2, A_3 and A_4 are defined by the equation (4)

Taking first order condition for maximizing $E[JTP_{sb}]$ with respect to c_b , gives the result

$$\mu = \frac{w_1}{w_1 - w}, \tag{16}$$

$$w = \left(\frac{D}{1-E[\alpha]} - s(a_1 + \, a_2\, M)MD\right),\ w_1 = I_p MD - D\frac{E[\alpha]IQ}{(1-E[\alpha])\lambda} - \frac{E[(1-\alpha)^2]Q}{2\,(1-E[\alpha])}\left(\mathrm{I} + I_p\right) - \, D\frac{1}{(1-E[\alpha])}$$

And the first order condition with respect to Q, p_b *and* M yield the following results

$$Q = \sqrt{\left(\frac{\mu A_S D}{(1-E[\alpha])} + (1-\mu)A_3 D\right)/(1-\mu)(A_1 D + c_b A_4)} \tag{17}$$

$$p_b \ = \frac{e}{e-1}\frac{(w_3(1-\mu) - \mu w_2)}{(1-\mu)}\ , \tag{18}$$

Where, $w_2 = \frac{1}{(1-E[\alpha])}[c_b - C - \frac{A_S}{Q}] - (a_1 + \, a_2\, M)c_b M, \quad w_3 = A_1 Q + A_2 + \frac{A_3}{Q}$

$$M = \frac{1}{2\mu a_2 c_b}\left((1-\mu)\left(\frac{c_S I_e E[\alpha]M}{(1-E[\alpha])} + c_b I_p M\right) - \, \mu a_1 c_b\right) \tag{19}$$

3. NUMERICAL EXAMPLES

An example is shown to show the effect of the defective items and credit financing in the Seller- Stackelberg game model. Input parameters in this example are taken from two papers, Abad and Jaggi (2003) and Jaggi et al. (2013) which are given below. Suppose $C = \$\,3$ units, $A_b = \$40$, $A_s = \$300, a_1 = 0.08, a_2 = 0.06, I_e = 0.16, I_p = \ 0.16,\ c_s = 5\ , \mathrm{I} \ = \ 0.12, e = 2.5\ , \mathrm{K} = 400000, \lambda \ = \ 175200$ unit/year. The fraction of imperfect quality item, α, uniformly distributed on $(a, b), 0 < a < b < 1 \ \ i.e., \alpha \sim \mathrm{U(a,b)}$ with $\alpha \sim \mathrm{U(a,b)}$, $\mathrm{E}[\alpha] = \frac{\mathrm{a+b}}{2}$ and $E[(1-\alpha)^2]$ can be determined with the formula

$E[(1-\alpha)^2] = \int_a^b (1-\alpha)^2 f(\alpha)d\alpha = \frac{a^2+ab+b^2}{3} + 1 - a - b$, the expected value of α is $E[\alpha] = 0.02$,

$E[(1-\alpha)^2]$ = 0.960, where a = 0 and b = 0.04. Equation (14) give the results, $Q = 294$ units and $c_b = \$5.757$. Equations (12) and (13) yields, $p_b = \$8.940$ and $M = 0.583$ years. With these results, the end demand, $D \ = 1674$ units. To avoid shortages during the screening time, we check the condition $\mathrm{E}[\alpha] \le \left(1 - \frac{D}{\lambda}\right)$. Here, $1 - \frac{D}{\lambda} = 0.990$. and $\mathrm{E}[\alpha] = 0.02$ i.e. $\mathrm{E}[\alpha] \le \left(1 - \frac{D}{\lambda}\right)$. The buyer's expected profit, $E[(\mathrm{TP}^c{}_b]$= *\$5753* and seller's profit, $E[(\mathrm{TP}^c{}_s]$= *\$2319*.The cycle length, *T= 0.171* years (from equation (4) and equation (9)).

Example 2

An example is shown to show the effect of the defective items and credit financing in the cooperative game model. We assumed the same values of all parameters as defined in Example 1, except $c_s = 3$. Under the cooperative approach suppose seller and buyer agree at $c_b = \$4.4/$ unit through negotiation. Then using equations (16), (17), (18), and (19), we obtained μ =0*.503*, $p_b = \$5.061$ /unit, *Q=2007*/ unit, *M=.667* years. The end demand, D = 6942 units. To avoid shortages during the screening time, we check the condition $\mathrm{E}[\alpha] \le \left(1 - \frac{D}{\lambda}\right)$. Here, $1 - \frac{D}{\lambda} = 0.960$. and $\mathrm{E}[\alpha] = 0.02$ i.e. $\mathrm{E}[\alpha] \le \left(1 - \frac{D}{\lambda}\right)$. The seller's profit $E[(\mathrm{TP}^c{}_s]$= \$*6413* and the buyer's profit $E[(\mathrm{TP}^c{}_b]$= \$*6340*. It is quite apparent from the numerical example, that selling price is less, demand and order quantity is more in co-operative a game in comparison to the Seller-Stackelberg game, which gains more profit to each player.

Sensitivity analysis

In this part sensitivity analysis is performed to investigate the effect of three parameters, price elasticity, *e*, fraction of defective items, α , and interest gain, I_e , on $c_b, p_b, M, D, Q, E[\mathrm{TP}^c{}_s], E[\mathrm{TP}^c{}_b]$ in the non-cooperative Seller-Stackelberg game model and Co-operative game model. The results are figure out in the following tables 2-7.

Table 2. Sensitivity analysis of Seller-Stackelberg game with respect to e

e	2.0	2.2	2.4	2.6	2.8
C_b	6.763	6.248	5.893	5.643	5.466
Q	336	325	306	281	253
p_b	12.480	10.601	9.382	8.575	8.018
M	0.619	0.607	0.596	0.571	0.541
T	0.128	0.143	0.162	0.184	0.211
D	2568	2220	1856	1498	1177
Buyer's profit	15714	10417	7007	4725	3180
Seller's profit	6261	4286	2867	1858	1146

Table 3. Sensitivity analysis Seller-Stackelberg game with respect to α

α	0.005	0.010	0.015	0.020	0.025
C_b	5.817	5.797	5.778	5.757	5.738
Q	283	287	290	294	297
p_b	9.070	9.017	8.989	8.940	8.916
M	0.557	0.572	0.572	0.583	0.581
T	0.174	0.173	0.173	0.171	0.172
D	1614	1638	1651	1674	1685
Buyer's profit	5628	5679	5705	5753	5777
Seller's profit	2257	2277	2298	2319	2341

Table 4. Sensitivity analysis Seller-Stackelberg game with respect to I_e

$I_e = I_p$	0.11	0.12	0.13	0.14	0.15
C_b	5.429	5.483	5.543	5.608	5.678
Q	327	319	312	305	299
p_b	9.094	9.084	9.057	9.037	8.999
M	0.174	0.254	0.341	0.416	0.494
T	0.199	0.194	0.189	0.183	0.178
D	1604	1608	1620	1629	1646
Buyer's profit	5634	5638	5658	5671	5701
Seller's profit	2337	2318	2306	2302	2306

Table 5. Sensitivity analysis of cooperative game with respect to *e*

E	2.0	2.2	2.4	2.6	2.8
C_b	4.4	4.4	4.4	4.4	4.4
Q	2519	2315	2109	1911	1721
p_b	6.021	5.533	5.194	4.944	4.760
M	0.680	0.674	0.669	0.664	0.658
T	0.224	0.242	0.269	0.299	0.333
D	11034	9280	7671	6273	5067
Buyer's profit	21221	13152	8111	4917	2899
Seller's profit	10435	8713	7132	5760	4585

Table 6. Sensitivity analysis of cooperative game with respect to α

α	0.005	0.010	0.015	0.020	0.025
C_b	4.4	4.4	4.4	4.4	4.4
Q	2004	2005	2006	2007	2009
p_b	5.068	5.066	5.063	5.061	5.059
M	0.655	0.659	0.663	0.667	0.672
T	0.288	0.287	0.285	0.283	0.282
D	6918	6928	6935	6942	6949
Buyer's profit	6442	6407	6374	6340	6306
Seller's profit	6314	6349	6381	6413	6446

Table 7. Sensitivity analysis of cooperative game with respect to I_e

$I_e = I_p$	0.11	0.12	0.13	0.14	0.15
C_b	4.4	4.4	4.4	4.4	4.4
Q	2177	2135	2097	2063	2034
p_b	5.169	5.158	5.142	5.121	5.093
M	0.232	0.316	0.401	0.488	0.576
T	0.324	0.316	0.308	0.300	0.292
D	6585	6620	6672	6740	6833
Buyer's profit	4422	4716	5049	5428	6751
Seller's profit	7848	7597	7332	7048	5849

The effect of fraction of defective items, α , in non-cooperative (Seller-Stackelberg) and Co-operative game (Pareto efficient solution concept) on decision variables c_b, p_b, M, Q is also shown through graph.

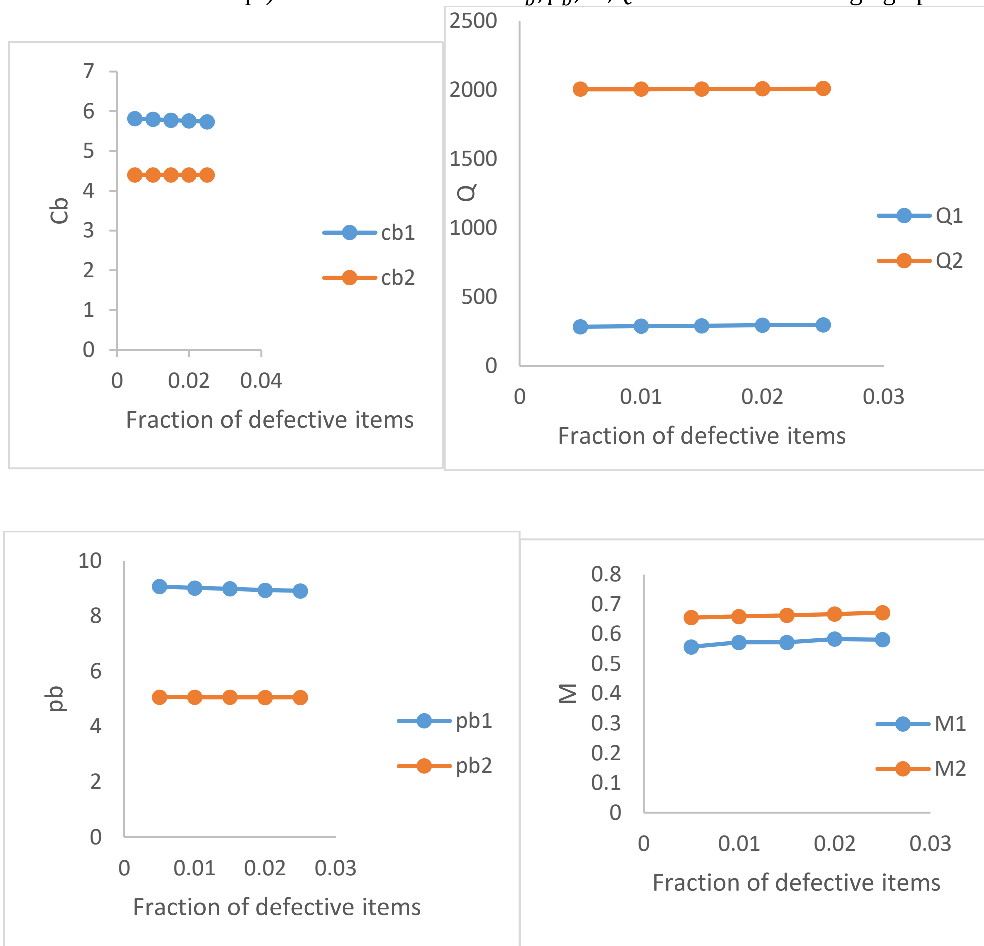

Figure 3. The effect of defective items, α, on c_b, p_b, M, Q

Observations

1.It is evident from Table 2 that when value of e increases, product market demand and selling price decreases considerably, concurrently buyer's profit decreases. The findings indicates, a negligible decrement (almost constant) in the value of credit period. It also shows that the decision to offer credit is insensitive to the price elasticity of the end demand, but the cycle length increases, which results in a decrease in the demand and seller's expected profit.
2.It is depicted from Table 3 that when the fraction of imperfect quality items is increased respectively order quantity also increases. With an increased fraction of defective items, the recurrent orders are placed by the buyer which results in an increase in profit of both the players.
3.It is also observed from the Table 4 that increase in I_e simultaneously increases in credit period, indicating to higher earning of interest by buyer which results in higher expected profit. When I_e increase, order quantity decreases, which results in less seller's profit.
4.It can be seen from the Table 5 that as price of elasticity increases, the selling price decreases significantly and optimal order quantity also decreases, which leads to a decrement in seller's and buyer's expected profits. Selling price is higher in the non-cooperative game with respect to the co-operative game. Whereas, optimal order quantity is less in the non-cooperative game than as in the co-operative game. Findings also suggest that both the seller and buyer are getting more profit at co-operative game as compared to non-cooperative game.
5.Further, Table 6 shows a slight increase in the optimal order quantity and marginal decreases in the cycle length with respect to increase in the fraction of defective items. On the other hand, retailer price decreases, which results in the decrement of buyer's expected profit.

6.If we compare the Table 3 with Table 6, we find that order quantity in the co-operative game is more than in the non-cooperative game where selling price is less in cooperative game. The profit of both the players in the co-operative game is higher than in the non-cooperative game whenever the fraction of defective items increases.

7.It is also observed from the Table 7 that increase in I_e, indicating to higher earning of interest by buyer which results in higher expected profit. When I_e increases, order quantity decreases, which results in a decrement in seller's profit.

8.The figure shows from Table 4 and Table 7 that order quantity is more in cooperative game than to non-cooperative game. Hence seller would prefer cooperative game as, he earns more profit in this game.

4. CONCLUSIONS

In this paper, we developed supply chain model with imperfect quality items, where end demand is price sensitive and unit price charged by the seller and the length of the credit period are considered as decision variables. Seller-Stackelberg under non-cooperative and Pareto efficient solution under cooperative game theoretic approaches are discussed, analyzed and discussed at length. The present models formulated in this paper provide a framework to the seller for managing pricing and credit policy. Foremost, this study displays the impact of trade credit financing on the expected profit of players under supply chain and finding shows that both the partners of supply chain enhance their profits by using credit financing policy. Later, it is assumed that screening rate is more than the demand rate. This assumption allows the buyer to fulfill the demand, out of the items which are found to be of perfect quality, along with the screening process. Results clearly help the buyer to determine his ordering policy. It is distinctly visible from the numerical results that order quantity is increasing due to imperfect production of the items which effects the expected profit of both the players. It can also be seen that both the players attained more profit in cooperative game approach as compared to non-cooperative game approach. When the production rate is less than the demand rate, then shortages may be considered in the future model. The current work can also be extended to the situations where there is one seller offering different credit periods to multiple buyers with different demand functions. Further, this work can be extended for three level supply chain with human errors.

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APPENDIX A

Expected profit function for buyer is given by

$E[TP^c{}_b(p,Q)] = Kp_b{}^{-e}\left[p_b - A_1Q - A_2 - \frac{A_3}{Q}\right] - c_bA_4Q$, where A_1, A_2, A_3 and A_4 are defined by equation (4)

First and second derivative with respect to p_b, Q are given below

$\frac{\partial E[TP^c{}_b(p,Q)]}{\partial p_b} = kp_b{}^{-e} - kep_b{}^{-e-1}(p_b - A_1Q - A_2 - \frac{A_3}{Q}) = (1-e)kp_b{}^{-e} + kep_b{}^{-e-1}(A_1Q + A_2 + \frac{A_3}{Q})$

$\frac{\partial^2 E[TP^c{}_b(p,Q)]}{\partial p_b{}^2} = e(e-1)kp_b{}^{-e-1} - ke(e+1)p_b{}^{-e-2}(A_1Q + A_2 + \frac{A_3}{Q})$, $\frac{\partial E[TP^c{}_b(p,Q)]}{\partial Q} = kp_b{}^{-e}\left(-A_1 + \frac{A_3}{Q^2}\right) - c_bA_4$, $\frac{\partial^2 E[TP^c{}_b(p,Q)]}{\partial Q^2} = kp_b{}^{-e}\left(-\frac{2A_3}{Q^3}\right)$, $\frac{\partial^2 E[TP^c{}_b(p,Q)]}{\partial p_b \partial Q} = ekp_b{}^{-e-1}(A_1 - \frac{A_3}{Q^2})$

Where A_1, A_2, A_3 and A_4 are defined by equation 4.