FRESH PRODUCE INVENTORY FOR TIME-PRICE AND STOCK DEPENDENT DEMAND

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ABSTRACT
The market demand for fresh produce product is rising regularly in today's health priority life. Basically, the demand for fresh produce products are also based on the measurement of freshness and the size of its shelf space for displaying the products which obviously fascinate more and more customers to purchase the product. Moreover, the expiration date and selling price for any deteriorating item plays a major role in the purchasing decision of a customer. Therefore, in this article, the market demand for fresh produce product is assumed to be a quadratic varying time function of its freshness, the selling price of item, the displayed stock in its shelf space and the expiration date. It may be gainful to preserve a high stock level at the end of the replenishment cycle, with this freshness-and-price-stock dependent demand. Therefore, varying the traditional way of zero ending inventory level to non-zero ending inventory is analyzed. So, the main objective of this article is to maximize the total profit by estimating the optimal selling price of the item, the optimal ending inventory level and the optimal replenishment cycle length, the optimal time to vacant the backroom and the optimal ordered quantity. The classical optimization technique is utilized for calculating the optimal values. Thereafter, using the concept of eigen-values of a Hessian matrix, we have proved the concave nature of the profit function. Finally, a numerical example along with the sensitivity analysis of decision variables by varying various inventory parameters are presented to validate the derived model and extracts significant managerial insights.

KEYWORDS: Deteriorating inventory, fresh produce products, shelf space, expiration date, stock- price-time-dependent demand.

MSC: 90B05

RESUMEN
La demanda del mercado de productos frescos está aumentando regularmente en la vida de prioridad de salud de hoy. Básicamente, la demanda de productos agrícolas frescos también se basa en la medición de la frescura y el tamaño de su espacio en los estantes para exhibir los productos que obviamente fascinan a más y más clientes para comprar el producto. Además, la fecha de vencimiento y el precio de venta de cualquier artículo deteriorado juegan un papel importante en la decisión de compra de un cliente. Por lo tanto, en este artículo, se supone que la demanda del mercado de producto de productos frescos es una función cuadrática de tiempo variable de su frescura, el precio de venta del artículo, el stock exhibido en su espacio de almacenamiento y la fecha de vencimiento. Puede ser beneficioso mantener un alto nivel de existencias al final del ciclo de reposición, con esta demanda dependiente de existencias de frescura y precio. Por lo tanto, se analiza la variación de la forma tradicional de nivel de inventario final cero a inventario final distinto de cero. Por lo tanto, el objetivo principal de este artículo es maximizar el beneficio total estimando el precio de venta óptimo del artículo, el nivel de inventario final óptimo y la duración óptima del ciclo de reposición, el tiempo óptimo para desocupar la trastienda y la cantidad ordenada óptima. La técnica de optimización clásica se utiliza para calcular los valores óptimos. A partir de entonces, utilizando el concepto de valores propios de una matriz de Hesse, hemos demostrado la naturaleza cóncava de la función de ganancia. Finalmente, se presenta un ejemplo numérico junto con el análisis de sensibilidad de las variables de decisión variando varios parámetros de inventario para validar el modelo derivado y extraer conocimientos gerenciales significativos.

PALABRAS CLAVE : Deterioro del inventario , productos producidos fresco, espacio en mostrador, fecha de expiración , demanda stock- price-time-dependiente

1. INTRODUCTION

Basically, the consumer's demand for fresh produce products like bread, milk, milk products, blood banks etc. are dependent on the age of the inventory which plays a major role in decision making of consumer's purchase, can be negatively impacted due to the damage of consumer's confidence on the product quality. Hence, the measurement of freshness of the product and the size of its shelf space for displaying the products which obviously fascinate more and more customers to purchase the product. Also, the expiration date

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intensely affects the product's demand rate, so it is to be treated as a main factor to estimate the freshness of a product. As the product approaches the expiration date, the customer's demand rate declines and tends to zero.

An inventory model was derived by Sarker *et al.* (1997) demonstrating the negative effect of ageing of stock on demand. Further an inventory model for perishable products was constructed by Hsu *et al.* (2006) including the expiration date. An inventory model dealing with the fresh produce products with stock-freshness condition dependent demand rate was developed by Bai and Kendall (2008). A model for deteriorating products by considering the measurement of product freshness in terms of the time remained until the expiration date was estimated by Herbon (2014). Many other researchers contributed significantly in the same field like Wang *et al.* (2014), Wu and Chan (2014), and Wu *et al.* (2014).

In practical situation, there are many aspects which influences the market demand rate like; selling price, stock, quality, time, and efforts in terms of either service and/or advertisement. Levin (1972) highlighted on maintaining greater displayed stock level could motivate to purchase more and more. Baker (1988) demonstrated an inventory model by expressing the demand pattern as a power function of displayed stock level. This happens due to the product's popularity and/or variety and its visibility to the consumer's. An inventory model with EOQ concept in which the total profit is maximized on fulfilling the constraints namely, budget and capacity of storage was developed by Sana and Chaudhari (2004) with the rate of demand together based on item obtainability and expenditures on advertisements. For deteriorating of items with the concept of partial back-logging and with a level of stock based rate of demand, and a bound on the extreme level of inventory, a model on inventory was represented by Min and Zhou (2009). Other model on inventory with rate of demand based on level of stock for items which are deteriorating in nature was proposed by Yang *et al.* (2010), permitting back-logging partially and including the inflation effect. An EOQ concept consisting of back-logging partially, in which the rate of demand is based on level of stock, and a manageable rate of deterioration determines the strategies of preservation and lot order size in optimum manner to rise the total profit to maximum, was formulated by Lee *et. al.* (2012).

Firstly, Silver *et al.* (1969) have attempted for time varying demand rate, then after many scholars like Silver (1979), Chung *et al.* (1993, 1994), Bose *et al.* (1995), Hariga (1995), Lin, *et al.* (2000), Mehta *et al.* (2003, 2004), Shah *et al.* (2009), and Shah *et al.* (2014) have expressed rate of demand as time varying in terms of linear, exponential or quadratic, etc. in nature.

Moreover, an inventory system dealing with stock level based rate of demand, it may be profitable to maintain orders of more quantities and end with the positive on-hand stocks. An EOQ model with non-ending inventory and stock based demand was firstly developed by Urban (1992) by relaxing the terminal condition of zero. Same way, Yang (2014) investigated the gain in the total profit by relaxing the assumption of zero ending inventory in Pando et al. (2012) to non-zero ending inventory.

An inventory model for fresh produce with shelf space allocation and freshness condition based demand rate was established by Bai and Kendall (2008). Wu *et al.* (2016) proposed an inventory model for fresh produce increasing time varying trend demand depends on freshness, stock-level and expiration date. This article overcomes the limitations of the previous research work by introducing quadratic nature of time varying trended demand depending on freshness, stock level, and expiration-date as well as selling price of the fresh produce. This type of demand increases initially then after some time, it tends to decrease. Selling price is also a decision variable in the derived model. These two factors highlights the novelty of this paper.

This article deals with an inventory management problem of fresh product inventory having time-price and stock dependent demand rate. As such a fresh produce product is having a very small shelf-life and its usefulness or condition of freshness gradually deteriorates throughout its lifetime. In practical scenario, there is a limitation to display stocks in a shelf space, so this constraint is included in this article. The market demand rate of fresh produce product is assumed to be based on displayed stocks, selling price, and shelf space and expiration date. It is considered to be profitable to maintain a higher stock level at the end of the cycle for the stock dependent demand for a product. Therefore, varying the traditional assumption of zero ending inventory to a constant stock is considered.

Finally, an EOQ model for fresh produce product with quadratic time varying demand dependent on product freshness, selling price, stock level and expiration date is analyzed. The main objective of this article is to maximize the total profit function by estimating the optimal ending inventory level at the replenishment cycle length, optimal selling price, optimal replenishment cycle length, optimal order quantity and optimal time at which the backroom became vacant. The classical optimization technique is utilized for calculating the optimal values. Thereafter, using the concept of eigen-values of a Hessian matrix, we have proved the concave nature of the profit function. Finally, a numerical example along with the sensitivity analysis of decision variables by varying various inventory parameters are presented to validate the derived model and extracts significant managerial insights.

In section-2, notations and assumptions are considered, formulation of mathematical model is done in section-3, Numerical examples with sensitivity analysis are done in section-4 and finally conclusion and future scope are discussed in section-5.

2. NOTATIONS AND ASSUMPTIONS

a. Notations:

Parameters

A	Fixed ordering cost (dollar/order)
C	Unit purchasing price (in dollars)
H	Unit inventory holding cost per year (in dollars)
m	Expiration date (in years)
t	Time (in years)
s	Salvage value per unit (in dollars)
w	Available shelf space (in units)

Decision variables

T	Replenishment cycle length (in years)
E	Ending inventory level at time T (in units)
p	Unit selling price (in dollars)
Q	Optimum order size (in units)
t_1	Time period at which no more stock remains in the backroom (in years)

Functions

$Dt(t, p)$	Demand rate per unit at any time t and at any price p, which is a polynomial form in t, and concavely increasing in the number of displayed stocks
$I(t)$	Inventory level at time t (in units).
$TP(E,T, p)$	The total profit in each period (in dollars)
Q	Order size (in units)

Optimal values

T^*	Optimal replenishment cycle length (in years)
E^*	Optimal ending inventory level in units at time T (in units)
p^*	Optimal unit selling price (in dollars)
Q^*	Optimum order size (in units)
t_1^*	Optimal Time period at which no more stock remains in the backroom (in years)
TP^*	Optimal total profit per year (in dollars)

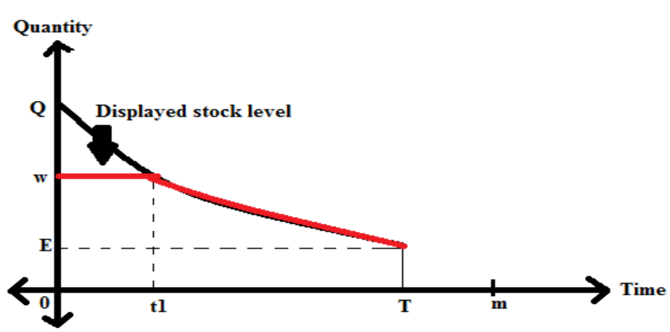

Figure 1: Graphical representation of the system

b. Assumptions:

1. Shortages and quantity discounts are impermissible.
2. As such the process of deterioration of fresh product is continuous in nature with its expiration date. Therefore, the freshness index is one at zero time period and then gradually decays over time periods, and approaches closer to zero when it is tending to the expiration date m. Hence, assuming the freshness index at time t is given by,

$$f(t)=\frac{m-t}{m}, \quad 0\le t\le m \tag{1}$$

3. The retailer obtains Q units and displays w units on the shelf with the rest of the products $(ie.\ Q-w\ units)$ stored in the backroom at time zero. When sales are made, stocks in the backroom are shifted to the shelf space until no more stocks in the backroom at time t_1 as demonstrated in figure-1. Hence, during this time period $[0,t_1]$, the shelf space is occupied fully and the demand rate depends on the quadratic trend, price and the freshness index. Let us assume the demand rate at time t, with p as the selling price per unit, where, $a>0$ is a scale demand, $0\le b<1$ denotes the linear rate of change of demand with respect to time, $0\le c<1$ denotes the quadratic rate of change of demand and η,λ are the mark up for selling price and available shelf space respectively, is given by,

$$Dt(t,p)=a(1+bt-ct^2)w^{\lambda}p^{-\eta}f(t); \qquad 0\le t\le t_1<T$$
$$where,\ a>0,\ 0\le b\le 1,\ 0\le c\le 1,\ \eta>1,\ 0\le\lambda<1 \tag{2}$$

4. When the demand depends on the stock level and time-varying freshness, it would be more desirable and profitable to keep fresh products and higher on hand displayed stocks. (i.e., non-zero ending inventory). Therefore, assuming that the ending inventory level $E\ge 0$.
5. The shelf space is only partially stocked and the demand rate depends on its freshness and the displayed units for the time period t_1 to T. Once the ending inventory level reaches to E units at the replenishment cycle time T, the retailer sells those E units at a salvage price s per unit, receives a new order quantity Q units, and starts a new replenishment cycle. Assuming the demand rate at time t, during this time period as,

$$Dt(t,p)=a(1+bt-ct^2)\left[I(t)\right]^{\lambda}p^{-\eta}f(t); \qquad t_1\le t<T$$
$$where,\ a>0,\ 0\le b\le 1,\ 0\le c\le 1,\ \eta>1,\ 0\le\lambda<1 \tag{3}$$

6. Without loss of generality, we may assume that the order quantity Q is greater than or equal to the shelf space w $(i.e.\ Q\ge w)$ or reducing the shelf space to Q units to fully utilize the shelf space as $t_1\ge 0$.
7. Replenishment occurs instantaneously.

3. **MATHEMATICAL MODEL FORMULATION:**

On the basis of the above mentioned assumptions, the inventory level $I(t)$, at time t, during the time period $[0,t_1]$ is computed by the following differential equation:

$$\frac{dI(t)}{dt} = -\left[a\left(1+bt-ct^2\right)w^{\lambda}p^{-\eta}\left(\frac{m-t}{m}\right)\right] \quad ; \quad 0 \le t \le t_1$$
(4)

With the boundary condition $I(t_1) = w$. Solving the differential equation (4) with $I(0) = Q$ we obtain,

$$I(t) = \frac{-ap^{-\eta}w^{\lambda}\left[\frac{1}{4}ct^4 + \frac{1}{3}(-cm-b)t^3 + \frac{1}{2}(bm-1)t^2 + mt\right]}{m} + Q, \quad 0 \le t \le t_1$$
(5)

Substituting t_1 into equation (5), and using $I(t_1) = w$ and $t_1 < T \le m$, we get t_1 as the positive root of the following equation,

$$\left[\frac{1}{4}ct_1^4 + \frac{1}{3}(-cm-b)t_1^3 + \frac{1}{2}(bm-1)t_1^2 + mt_1\right] - \frac{(Q-w)m}{w^{\lambda}p^{-\eta}a} = 0 \tag{6}$$

Therefore, the order quantity is,

$$Q = I(0) = w + \frac{ap^{-\eta}w^{\lambda}\left[\frac{1}{4}ct_1^4 + \frac{1}{3}(-cm-b)t_1^3 + \frac{1}{2}(bm-1)t_1^2 + mt_1\right]}{m} \ge w \tag{7}$$

In the same way, the inventory level $I(t)$ at time t during the time period $[t_1, T]$ is computed by the following differential equation:

$$\frac{dI(t)}{dt} = -\left[a\left(1+bt-ct^2\right)\left[I(t)\right]^{\lambda}p^{-\eta}\left(\frac{m-t}{m}\right)\right] \quad ; \quad t_1 \le t \le T, \tag{8}$$

With the boundary condition $I(t_1) = w$. Solving the differential equation (8) with $I(T) = E$ we obtain,

$$I(t) = \left\{(1-\lambda)\left[\frac{ap^{-\eta}}{m}\left[\begin{array}{l}\frac{1}{4}c\left(T^4-t^4\right) + \frac{1}{3}(-cm-b)\left(T^3-t^3\right) \\ +\frac{1}{2}(bm-1)\left(T^2-t^2\right) + m(T-t)\end{array}\right]\right] + E^{1-\lambda}\right\}^{\frac{1}{(1-\lambda)}}, \quad t_1 \le t \le T, \tag{9}$$

Refer to Appendix 1, for the detailed derivation. Substituting t_1 into equation (9), and using $I(t_1) = w$ and $t_1 < T \le m$, and rearranging terms, we get t_1 as the positive root of the following equation,

$$\left\{\begin{array}{l}\left[\frac{1}{4}ct_1^4 + \frac{1}{3}(-cm-b)t_1^3 + \frac{1}{2}(bm-1)t_1^2 + mt_1\right] + \frac{m\left(w^{1-\lambda} - E^{1-\lambda}\right)}{ap^{-\eta}(1-\lambda)} \\ -\left[\frac{1}{4}cT^4 + \frac{1}{3}(-cm-b)T^3 + \frac{1}{2}(bm-1)T^2 + mT\right]\end{array}\right\} = 0 \tag{10}$$

From equation (6) and equation (10), we obtain the order quantity as,

$$Q = w + \frac{w^{\lambda}p^{-\eta}a}{m}\left[\frac{1}{4}cT^4 + \frac{1}{3}(-cm-b)T^3 + \frac{1}{2}(bm-1)T^2 + mT\right] - \frac{m\left(w^{1-\lambda} - E^{1-\lambda}\right)}{ap^{-\eta}(1-\lambda)} \ge w \tag{11}$$

Assuming $t_1 = \alpha T$ where α be the proportionality constant.

The total profit of the system is computed with the following components:

1. Sales revenue:

The sales revenue per unit time is given by,

$$SR = \frac{p\left(Q-E\right)}{T} \tag{12}$$

2. Salvage value:

The salvage value per unit time is given by,

$$SV = \frac{1}{T}sE \tag{13}$$

3. Purchasing cost:

The purchasing cost per unit time is given by,

$$PC = \frac{CQ}{T} \tag{14}$$

4. Ordering cost:

The ordering cost per unit time is given by,

$$OC = \frac{A}{T} \tag{15}$$

5. Holding cost:

The holding cost during $\left[0, t_1\right]$ is,

$$H_1 = h\int_0^{t_1} I\left(t\right)dt$$

$$H_1 = h\int_0^{t_1}\left[-\frac{ap^{-\eta}w^{\lambda}}{m}\left[\frac{1}{4}ct^4 + \frac{1}{3}\left(-cm-b\right)t^3 + \frac{1}{2}\left(bm-1\right)t^2 + mt\right]\right]dt + hQt_1 \tag{16}$$

The holding cost during $\left[t_1, T\right]$ is,

$$H_2 = h\int_{t_1}^{T} I\left(t\right)dt$$

$$H_2 = h\int_{t_1}^{T}\left\{\left(1-\lambda\right)\left[\frac{ap^{-\eta}}{m}\left[\begin{array}{l}\frac{1}{4}c\left(T^4-t^4\right)+\frac{1}{3}\left(-cm-b\right)\left(T^3-t^3\right)\\ +\frac{1}{2}\left(bm-1\right)\left(T^2-t^2\right)+m\left(T-t\right)\end{array}\right]\right]+E^{1-\lambda}\right\}^{\frac{1}{(1-\lambda)}}dt \tag{17}$$

The integration of (17) seems to be too complicated to get an explicitly analytical solution. However, the holding cost of H_2 is relatively small to the overall profit. For simplicity, we may use a simple approximation to calculate it as given below. The average inventory level during $\left[t_1, T\right]$ approximately equals to $\frac{(w+E)}{2}$.

Thus, the holding cost during the time period $\left[t_1, T\right]$ approximately equals to,

$$H_2 \approx \frac{h}{2}\left(w+E\right)\left(T-t_1\right) \tag{18}$$

Therefore, the total holding cost from equation (16) and equation (18) is given by,

$$H = H_1 + H_2 = \left[h\int_0^{t_1} \left[-\frac{ap^{-\eta}w^{\lambda}}{m} \left[\frac{1}{4}ct^4 + \frac{1}{3}\left(-cm-b\right)t^3 + \frac{1}{2}\left(bm-1\right)t^2 + mt \right] \right] dt + hQt_1 \right] + \frac{h}{2}\left(w+E\right)\left(T-t_1\right) \tag{19}$$

Now the total profit per unit time of the inventory system with freshness, selling price and-stock dependent demand is calculated as,

$$TP\left(T,E,p\right) = \frac{1}{T}\left(SR + SV - PC - OC - HC\right)$$

Therefore, from equation (12) to equation (15) and equation (19). We get,

$$TP\left(T,E,p\right) = \frac{1}{T}\left(\begin{array}{l} p\left(Q-E\right) + sE - CQ - A \\ -\left[h\int_0^{t_1} \left[-\frac{ap^{-\eta}w^{\lambda}}{m} \left[\frac{1}{4}ct^4 + \frac{1}{3}\left(-cm-b\right)t^3 + \frac{1}{2}\left(bm-1\right)t^2 + mt \right] \right] dt + hQt_1 \right] \\ +\frac{h}{2}\left(w+E\right)\left(T-t_1\right) \end{array} \right) \tag{20}$$

Now, to maximize the total profit stated in equation (20), we apply the below stated necessary and sufficient condition:

$$\frac{\partial TP}{\partial T} = 0, \frac{\partial TP}{\partial E} = 0, \frac{\partial TP}{\partial p} = 0 \tag{21}$$

To check the concavity of the total profit function of obtained solution, we adopt the below stated algorithm,
Step 1: Assigning the inventory parameters some specific hypothetical values.
Step 2: Obtaining the solutions by solving simultaneous equations stated in equation (21), utilizing the mathematical software Maple XVIII.
Step 3: Computing all the Eigen values of below stated hessian matrix H at the optimal point obtained from equation (21),

$$H = \begin{bmatrix} \frac{\partial^2 TP}{\partial T^2} & \frac{\partial^2 TP}{\partial TE} & \frac{\partial^2 TP}{\partial Tp} \\ \frac{\partial^2 TP}{\partial ET} & \frac{\partial^2 TP}{\partial E^2} & \frac{\partial^2 TP}{\partial Ep} \\ \frac{\partial^2 TP}{\partial pT} & \frac{\partial^2 TP}{\partial pE} & \frac{\partial^2 TP}{\partial p^2} \end{bmatrix}$$

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f all of the eigenvalues are negative, it is said to be a negative-definite matrix. Then the profit function is concave down then stop.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1 Numerical example

In this section, a numerical example is provided to illustrate the derived mathematical model.
Example: Assuming the following values in developed model.

$a = 50000\,units, b = 15\%, c = 1\%, h = \$0.2/unit, C = \$20/unit, \lambda = 0.1, m = 0.25\,years,$
$A = \$100/unit, s = \$15/unit, w = 20\,units, \alpha = 0.2, \eta = 1.1.$

Inventory Parameters	Decision Variables	Percentage variation of Decision Variables				
		-20%	-10%	0	10%	20%

Solution: By following the above stated algorithm

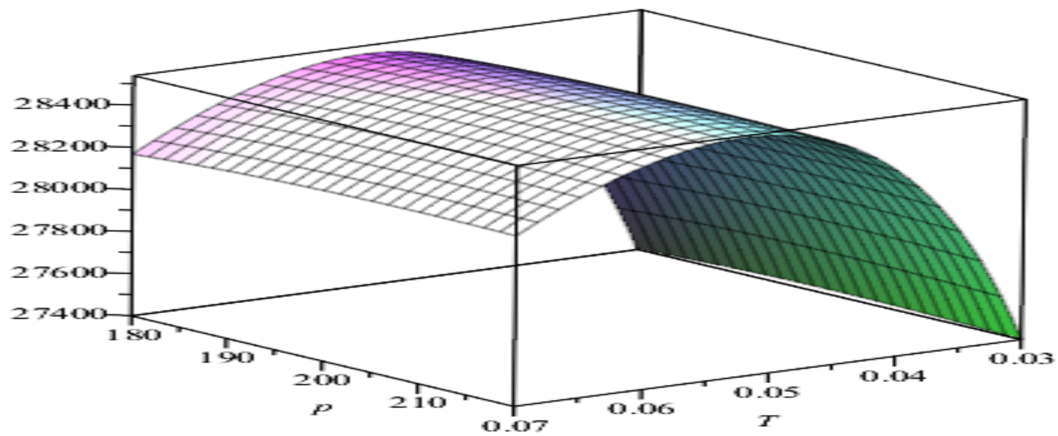

Figure 2: Concavity of Profit function

The optimal ending inventory level E*= 15.3416 units;

The optimal cycle time is $T^* = 0.0519\ years \approx 18\ days$;

The optimal Selling price $p^* = \$\,206.3119$;

The total profit per year TP $= \$\,28538.5363$;

The optimal order quantity Q* = 24.2439 units;

The optimal time to vacant the backroom $t_1^* = 0.01038\ years \approx 3\ days$.

For checking the concavity of total profit function, we check the nature of the Eigen values of hessian matrix.

$$H = \begin{bmatrix} \frac{\partial^2 TP}{\partial T^2} & \frac{\partial^2 TP}{\partial TE} & \frac{\partial^2 TP}{\partial Tp} \\ \frac{\partial^2 TP}{\partial ET} & \frac{\partial^2 TP}{\partial E^2} & \frac{\partial^2 TP}{\partial Ep} \\ \frac{\partial^2 TP}{\partial pT} & \frac{\partial^2 TP}{\partial pE} & \frac{\partial^2 TP}{\partial p^2} \end{bmatrix}$$

$$H = \begin{bmatrix} -2699394.473 & -2.331983877 & 19.63846150 \\ -2.331983877 & -24.01892908 & 0.5175479438 \\ 19.63846150 & 0.5175479438 & -0.09533938026 \end{bmatrix}$$

All the three Eigen values are computes as,

$$\lambda_1 = -2.6993*10^6 < 0, \lambda_2 = -24.0301 < 0, \lambda_3 = -0.0840 < 0$$

Therefore, all the three Eigen values of Hessian matrix are negative. So, the profit function is concave down in nature as shown in the figure-2

a	E	15.3014	15.3234	15.3416	15.3571	15.3703
	T	0.0579	0.0547	0.0519	0.0495	0.0474
	p	204.4861	205.4812	206.3119	207.0184	207.6286
	Q	23.2211	23.7479	24.2439	24.7139	25.1617
	t_1	0.0115	0.0109	0.0103	0.0099	0.0094
	TP	22146.8896	25332.1117	28538.5363	31763.0488	35003.2294
b	E	15.3407	15.3412	15.3416	15.3421	15.3426
	T	0.0517	0.0518	0.0519	0.0519	0.0520
	p	206.2682	206.2900	206.3119	206.3337	206.3555
	Q	24.2163	24.2301	24.2439	24.2577	24.2716
	t_1	0.0103	0.0103	0.0103	0.0103	0.0104
	TP	28514.5743	28526.5483	28538.5363	28550.5382	28562.5542
c	E	15.3417	15.3417	15.3416	15.3416	15.3416
	T	0.0519	0.0519	0.0519	0.0519	0.0519
	p	206.3137	206.3128	206.3119	206.3109	206.3100
	Q	24.2450	24.2445	24.2439	24.2433	24.2427
	t_1	0.0103	0.0103	0.0103	0.0103	0.0103
	TP	28539.0788	28538.8076	28538.5363	28538.2651	28537.9940
h	E	15.3401	15.3401	15.3401	15.3401	15.3401
	T	0.0519	0.0519	0.0519	0.0519	0.0519
	p	206.3085	206.3085	206.3085	206.3085	206.3085
	Q	24.2423	24.2423	24.2423	24.2423	24.2423
	t_1	0.0103	0.0103	0.0103	0.0103	0.0103
	TP	28537.4091	28537.4091	28537.4091	28537.4091	28537.4091
C	E	18.7806	16.8571	15.3416	14.0795	12.9913
	T	0.0409	0.0469	0.0519	0.0561	0.0599
	p	175.2469	192.3158	206.3119	218.1349	228.2991
	Q	27.4419	25.6824	24.2439	23.0176	21.9440
	t_1	0.0081	0.0093	0.0103	0.0112	0.0119
	TP	30753.8818	29547.4415	28538.5363	27664.7347	26890.7769
λ	E	14.2720	14.8606	15.3416	15.7416	16.0791
	T	0.0530	0.0525	0.0519	0.0512	0.0506
	p	202.9685	204.7915	206.3119	207.5961	208.6933
	Q	22.9605	23.6521	24.2439	24.7613	25.2223
	t_1	0.0106	0.0105	0.0103	0.0102	0.0101
	TP	26723.2739	27613.7175	28538.5363	29497.3881	30490.36054
m	E	15.3016	15.3235	15.3416	15.3570	15.3701
	T	0.0462	0.0491	0.0519	0.0545	0.0570
	p	204.4453	205.4617	206.3119	207.0366	207.6638
	Q	23.2004	23.7366	24.2439	24.7268	25.1891

	t_1	0.0092	0.0098	0.0103	0.0109	0.0114
	TP	27659.4652	28135.2491	28538.5363	28886.1647	29189.9563
A	E	15.3268	15.3346	15.3416	15.3481	15.3541
	T	0.0490	0.0505	0.0519	0.0532	0.0546
	p	205.6146	205.9781	206.3119	206.6198	206.9051
	Q	23.8236	24.0376	24.2439	24.4429	24.6354
	t_1	0.0098	0.0101	0.0103	0.0106	0.0109
	TP	28934.5529	28733.7691	28538.5363	28348.4153	28163.0215
s	E	12.6785	14.0018	15.3416	16.6966	18.0753
	T	0.0572	0.0547	0.0519	0.0485	0.0445
	p	191.6880	199.2591	206.3119	212.4549	217.1916
	Q	23.1100	23.6659	24.2439	24.8388	25.4480
	t_1	0.0114	0.0109	0.0103	0.0097	0.0089
	TP	27768.9835	28126.1396	28538.5363	29016.8592	29577.0961
w	E	12.3094	13.8274	15.3416	16.8522	18.3592
	T	0.0500	0.0509	0.0519	0.0528	0.0537
	p	208.4152	207.3404	206.3119	205.3247	204.3745
	Q	20.6453	22.4505	24.2439	26.0264	27.7991
	t_1	0.0100	0.0101	0.0103	0.0105	0.0107
	TP	28172.8431	28372.2051	28538.5363	28678.3024	28796.2928
α	E	15.3418	15.3417	15.3416	15.3416	15.3415
	T	0.0519	0.0519	0.0519	0.0519	0.0519
	p	206.3094	206.3107	206.3119	206.3130	206.3142
	Q	24.2442	24.2440	24.2439	24.2437	24.2436
	t_1	0.0083	0.0093	0.0103	0.0114	0.0124
	TP	28538.5742	28538.5552	28538.5363	28538.5182	28538.50098

Table 1: Sensitivity Analysis of the decision variables on varying various inventory parameters

4.2. Sensitivity analysis on the optimal inventory policy

In this part, the sensitivity analysis of the decision variables with respect to various inventory parameters is carried out. Table-1 demonstrates the values of decision variables on varying the various inventory parameters from case-2 and case-4 respectively in the range -20% to 20%. From table-1 the following observations are extracted;Sensitivity analysis of basic market scale demand (a):

In order to maintain the freshness of the products and adequate quantity to be displayed for higher sales, the ending inventory level at time *T*, ordered quantity increases gradually along with simultaneous shortening of replenishment cycle length with a slight drop in the time period point at which no more stock remains in the backroom is observed with a bit hike in selling price resulting in the total profit gain of the firm, with the variation of scale demand.

(a) Sensitivity analysis of linear rate of change of demand (b):

With the variation in linear rate of change of demand, there are increments in ending inventory level at time *T* as well as in ordered quantity. Lengthening of replenishment cycle length as well as the enlargement of time period point at which no more stock remains in the backroom are witnessed with a higher selling price, the total profit rises.

(b) Sensitivity analysis of quadratic rate of change of demand (c):

(c) There is a decrement in each decision variable with the variation in quadratic rate of change of demand. The ending inventory level, the ordered quantity decreases with the decline in replenishment cycle length as well as in time period at which no more stock remains in the backroom with the drop in selling price, therefore, the total profit of the firm falls.

(d) Sensitivity analysis of holding cost (h):

(e) With the variation in holding cost, each decision variable values remains unchanged. Therefore, firm's total profit remains unaltered.

(f) Sensitivity analysis of unit purchasing cost (C):

(g) With the variation of the purchasing cost, there is a decrement in ending inventory level at time T as well as in ordered quantity, along with the lengthening of replenishment cycle length, a hike in time period to vacant the backroom is seen with vastly uplifting the selling price but finally, the firm's total profit declines.

(h) Sensitivity analysis of available shelf space (λ):

(i) If mark-up for available shelf space is varied, the ending inventory level and ordered quantity increases but shortening of replenishment cycle length as well as fall in time period to vacant the backroom is observed. But a hike in selling price results in higher profit margin of the firm.

(j) Sensitivity analysis of expiration date (m):

(k) With the variation in expiration date of product, there are increments in ending inventory level at time T as well as in ordered quantity. Lengthening of replenishment cycle length as well as the enlargement of time period point to vacant the backroom are witnessed with a higher selling price results the total profit gain of the firm.

(l) Sensitivity analysis of fixed ordering cost (A):

(m) With respect to the increment in ordering cost, there are increments in ending inventory level as well as in ordered quantity. Lengthening of replenishment cycle length as well as the elaboration of time period point to vacant the backroom are seen with a higher selling price but finally declines the total profit gain of the firm.

(n) Sensitivity analysis of proportionality constant of replenishment cycle length (α):

(o) There is a decrement in ending inventory level at time T as well as in ordered quantity, with shortening slightly of replenishment cycle length and a hike in time period to vacant the backroom, but selling price increases. Finally, the firm's total profit declines with the variation of proportionality constant of replenishment cycle length.

(p) Sensitivity analysis of Salvage value (s):

(q) High increment in ending inventory level and ordered quantity with shortening in replenishment cycle length and a fall in time period to vacant the backroom are observed. With a hike in selling price, the firm's total profit grows, with the variant salvage values.

(r) Sensitivity analysis of available shelf space (w):

(s) If available shelf space is varied, the ending inventory level and ordered quantity increases highly but shortening of replenishment cycle length as well as fall in time period to vacant the backroom is observed. But a hike in selling price results in higher profit margin of the firm.

4. CONCLUSION

The market demand for fresh produce product is rising significantly in today's health priority life. Basically, the demand for fresh produce products are also based on the measurement of freshness and the size of its shelf space for displaying the products which obviously fascinate more and more customers to purchase the product. Moreover, the expiration date and selling price for any deteriorating item plays a major role in the purchasing decision of a customer. Therefore, in this article, the market demand for fresh produce product is assumed to be a quadratic varying time function of its freshness, the selling price of item, the displayed stock in its shelf space and the expiration date. It may be gainful to preserve a high stock level at the end of the replenishment cycle, with this freshness-and-price-stock dependent demand.

Therefore, varying the traditional way of zero ending inventory level to non-zero ending inventory. So, then fulfilling the main objective of this article to maximize the total profit by estimating the optimal selling price of the item, the optimal ending inventory level and the optimal replenishment cycle length, the optimal time to vacant the backroom and the optimal ordered quantity. The classical optimization technique is utilized for calculating the optimal values. Thereafter, using the concept of eigen-values of a Hessian matrix, we have proved the concave nature of the profit function. Finally, a numerical example along with the sensitivity analysis of decision variables by varying various inventory parameters are presented to validate the derived model and extracts significant managerial insights. The illustration shows the gain in total profit by variation of inventory parameters a , b , λ , m , s , W and loss in profit gain by variation in c , C , A , α .

The derived model can be further extended by utilizing the concept of trade credit and/or including the constraint of shortages, partial backlogging; also to hike the total profit of the firm efforts for advertising and/or service investment can be used.

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REFERENCES

[1] BAI, R., and KENDALL, G. (2008): A model for fresh produce shelf-space allocation and inventory management with freshness-condition-dependent demand. **INFORMS Journal on Computing**, 20, 78–85.

[2] BAKER, R.C., and URBAN, T.L. (1988). A deterministic inventory system with inventory-dependent demand rate. **Journal of the Operational Research Society**, 39 , 823–831.

[3] BOSE, S., GOSWAMI, A., and CHAUDHURI, K.S. (1995). An EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting. **Journal of the Operational Research Society**, 46, 771–782.

[4] CHUNG, K.J., and TING, P.S. (1993). A heuristic for replenishment of deteriorating items with a linear trend in demand. **Journal of the Operational Research Society**, 44, 1235–1241.

[5] CHUNG, K.J., and TING, P.S. (1994). On replenishment schedule for deteriorating items with time proportional demand. **Production Planning and Control**, *5*, 392–396.

[6] HARIGA, M. (1995), An EOQ model for deteriorating items with shortages and time-varying demand. **Journal of the Operational Research Society**, 46, 398–404.

[7] HERBON, A. (2014). Dynamic pricing vs. acquiring information on consumers' heterogeneous sensitivity to product freshness. **International Journal of Production Research**, 52, 918–933.

[8] HSU, P.H., WEE, H.M., and TENG, H.M. (2006). Optimal lot sizing for deteriorating items with expiration date. **Journal of Information and Optimization Sciences and Industrial Engineering**, 27, 271–286.

[9] KHANRA, S., and CHAUDHURI, K.S. (2003). A note on order level inventory model for deteriorating item with time dependent quadratic demand. **Computers Operations Research**, 30, 1901–1916.

[10] LEE, Y.P. and DYE, C.Y. (2012). An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. **Computers and Industrial Engineering**, 63, 474-482.

[11] LEVIN, R.I., MCLAUGHLIN, C.P., LAMONE, R.P., and KOTTA, J.F. (1972). **Productions/operations management: Contemporary policy for managing operating systems**, 373, NY: McGraw-Hill, New York.

[12] LIN, C., TAN, B., and LEE, W.C. (2000). An EOQ model for deteriorating items with time-varying demand and shortages. **International Journal of Systems Science**, 31, 394–400.

[13] MEHTA, N.J., and SHAH, N.H. (2003). An inventory model for deteriorating items with exponentially increasing demand shortages under inflation and time discounting. **Investigación Operational**, 23, 103–111.

[14] MEHTA, N.J., and SHAH, N.H. (2004). An inventory model for deteriorating items with exponentially decreasing demand shortages under inflation and time discounting. **Industrial Engineering Journal**, 33, 19–23.

[15] MIN, J. and ZHOU, Y. W. (2009). A perishable inventory model under stock-dependent selling rate and shortage-dependent partial backlogging with capacity constraint, **International Journal of Systems Science**, 40, 33-44.

[16] PANDO, V., GARCIA-LAGUNAA, J., SAN-JOSE, L.A., and SICILIA, J. (2012). Maximizing profits in an inventory model with both demand rate and holding cost per unit time dependent on the stock level. **Computers and Industrial Engineering**, 62, 599–608.

[17] SANA S., and CHAUDHARI K.S. (2004). A stock-review EOQ model with stock-dependent demand, quadratic deterioration rate, **Advanced Modeling and Optimization**, 6, 25-32.

[18] SARKER, B. R., MUKHERJEE, S., and BALAN, C.V. (1997). An order-level lot size inventory model with inventory-level dependent demand and deterioration. **International Journal of Production Economics**, 48, 227–236.

[19] SILVER, E.A., AND MEAL, H.C. (1969). A simple modification of the EOQ for the case of a varying demand rate. **Production and Inventory Management**, 10, 52–65.

[20] SILVER, E.A. (1979). A simple inventory replenishment decision rule for a linear trend in demand. **Journal of the Operational Research Society**, 30, 71–75.

[21] SHAH, N.H., GOR, A.S., and JHAVERI, C. (2009). Integrated optimal solution for variable deteriorating inventory system of vendor-buyer when demand is quadratic. **Canadian Journal of Pure and Applied Sciences**, 3, 713–717.
[22] SHAH, N.H., and SHAH, B.J. (2014). EPQ model for time-declining demand with imperfect production process under inflationary conditions and reliability. **International Journal of Operations Research**, 11(3), 91–99.
[23] URBAN, T.L. (1992). An inventory model with an inventory level- dependent demand rate and relaxed terminal conditions. **Journal of the Operational Research Society**, 43, 721–724.
[24] YANG, C.T. (2014). An inventory model with both stock dependent demand rate and stock-dependent holding cost rate. **International Journal of Production Economics**, 155, 214–221.
[25] WANG, W.C., TENG, J.T., and LOU, K.R. (2014). Seller's optimal credit period and cycle-time in supply chain for deteriorating items with maximum lifetime. **European Journal of Operational Research**, 232, 315–321.
[26] WU, J., and CHAN, Y.L. (2014). Lot-sizing policies for deteriorating items with expiration dates and partial trade credit to credit-risk customers. **International Journal of Production Economics**, 155, 292–301.
[27] WU, J., OUYANG, L.Y., CARDENAS-BARRON, L.E., and GOYAL, S.K. (2014). Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. **European Journal of Operational Research**, 237, 898–908.
[28] WU, J., CHANG, C. T., CHENG, M. C., TENG, J. T., and AL-KHATEEB, F. B. (2016). Inventory management for fresh produce when the time-varying demand depends on product freshness, stock level and expiration date. **International Journal of System Science: Operations Logistics**, 3, 138-147.
[29] YANG, H.L., TENG, J.T., and CHERN, M.S. (2010). An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages, **International Journal of Production Economics**, 123, 8-19.

Appendix-1: Solution of differential equation (8):

Re-arranging (8) we get,

$$\left[I(t)\right]^{-\lambda} dI(t) = -\left[a\left(1+bt-ct^2\right)p^{-\eta}\left(\frac{m-t}{m}\right)\right]dt \tag{A1}$$

Taking integration on both sides of (A1) yields,

$$\frac{1}{(1-\lambda)}\left[I(t)\right]^{1-\lambda} = \frac{-ap^{-\eta}\left[\frac{1}{4}ct^4+\frac{1}{3}(-cm-b)t^3+\frac{1}{2}(bm-1)t^2+mt\right]}{m}+C \tag{A2}$$

Substituting $I(T) = E$ into (A2), and re-arranging terms, we have

$$C = \frac{1}{(1-\lambda)}\left[E\right]^{1-\lambda} + \frac{ap^{-\eta}\left[\frac{1}{4}cT^4+\frac{1}{3}(-cm-b)T^3+\frac{1}{2}(bm-1)T^2+mT\right]}{m} \tag{A3}$$

Finally, substituting (A3) into (A2), re-arranging the terms, we get,

$$I(t) = \left\{(1-\lambda)\left[\frac{ap^{-\eta}}{m}\left[\begin{array}{l}\frac{1}{4}c\left(T^4-t^4\right)+\frac{1}{3}(-cm-b)\left(T^3-t^3\right)\\+\frac{1}{2}(bm-1)\left(T^2-t^2\right)+m(T-t)\end{array}\right]\right]+E^{1-\lambda}\right\}^{\frac{1}{(1-\lambda)}} \tag{A4}$$