

ESTIMATORS FOR POPULATION VARIANCE USING AUXILIARY INFORMATION ON QUARTILE

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ABSTRACT

This paper suggests the generalized class of estimators, motivated by Sharma and Singh (2015), of finite population variance utilizing the known value of parameters related to an auxiliary variable such as quartile and its properties are studied in simple random sampling without replacement. The efficiency of proposed class of estimators is compared with some existing estimators discussed in literature and found that proposed generalized class of estimators is better than other existing estimators including usual unbiased estimator, estimators due to Isaki (1983), Das and Tripathi (1978) and estimators recently proposed by Singh and Pal (2016). An empirical as well as theoretical comparison is carried out to judge the performance of proposed class of estimators over other existing estimators of population variance using natural data set.

KEYWORDS: Auxiliary variable, Quartile, Simple random sampling, Bias, Mean Square Error.

MSC: 62D05

RESUMEN

En este paper se sugiere una clase generalizada de estimadores motivada por los resultados de Sharma and Singh (2015), la varianza de la población finita usa el valor conocido de parámetros relacionado con una variable auxiliar, como los cuartiles, y sus propiedades son estudiadas en el caso de muestreo simple aleatorio con reemplazo. La eficiencia de la clase de estimadores propuestos es comparada con la de algunos estimadores existentes, discutidos en la literatura y se halla que las clases propuestas de estimadores es mejor que otros existentes, como los estimadores insesgados debidos a Isaki (1983), Das y Tripathi (1978) y a estimadores propuestos recientemente por Singh y Pal (2016). Comparaciones empíricas así como teóricas llevan a cabo para juzgar el comportamiento de la clase propuesta de estimadores sobre otros estimadores existentes de la varianza de la población usando conjuntos de datos reales.

PALABRAS CLAVE: variable auxiliar, cuartil, muestreo simple aleatorio, sesgo, error cuadrático medio.

1. INTRODUCTION

It has been well recognised that use of suitable auxiliary information results in efficient estimators of population parameters of interest. Usually auxiliary information is available from the past experience, census or administrative data base. Sampling literature describes the procedure of improvement of estimators using auxiliary variable. Several ratio, product and regression method of estimation are good examples in this context (see Cochran (1977), Wolter (1985), Singh (2003)). Estimation of the finite population variance has important significance in various fields such as agriculture, industry, medical and biological sciences where we come across populations which are likely to be skewed. Variations are present in our daily life. It is a law of nature that no two things or individuals are exactly alike. For instance, a physician needs a full understanding of variation in the degree of human blood pressure, body temperature and pulse rate for

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adequate prescription (see Sharma and Singh (2014)). Several authors including Das and Tripathi (1978), Isaki (1983), Kadilar and Cingi (2006), Shabbir and Gupta (2007), Solanki et al. (2015), Sharma and Singh (2013, 2015) and recently Adichwal et al. (2015) suggested the procedure of variance estimation using auxiliary information. It is well known that the values of quartiles and their functions are not much affected by the extreme values or the presence of outliers in the population like other parameters such as variance, coefficient of variation and coefficient of kurtosis. Therefore it is advisable to use of quartiles or their functions as auxiliary information, keeping this in mind few authors including Subramani and Kumarapandiyan (2012, 2013), Singh et al. (2014), Khan and Shabbir (2013) and Singh and Pal (2016) considered the problem of variance estimation using information on the variance, quartiles, inter-quartile range, semi-quartile range and semi-quartile average of an auxiliary variable. This paper suggests the generalized class of estimators for estimation of population variance utilizing the information on variance and quartiles of auxiliary variable. The properties of the generalized class of estimators are shown under the large sample approximation. It has been seen theoretically as well as empirically that the proposed generalized class of estimators is more efficient than some other existing estimators.

Consider a finite population $U = U_1, U_2, \dots, U_N$ of N units and let y and x are study and auxiliary variables defined on U taking values (y_i, x_i) respectively on U_i 's ($i = 1, 2, \dots, N$). It is desired to estimate the population variance S_y^2 of the study variable y using information of auxiliary variable x . Let a simple random sample of size n be drawn without replacement from the finite population U . Here, we take n^{-1} instead of $(n^{-1} - N^{-1})$, i.e. we are ignoring the finite population correction (fpc) term throughout this paper.

Here \bar{Y} and \bar{X} are population means of study variable y and auxiliary variable x respectively. S_y^2 and S_x^2 are mean square error of study variable y and auxiliary variable x respectively. Q_i ($i=1,2,3$) denotes the i th quartile of auxiliary variable x . Some other notations are $Q_r = Q_3 - Q_1$; inter quartile range, $Q_d = (Q_3 - Q_1)/2$; population semi quartile range of auxiliary variable x , $Q_a = (Q_3 + Q_1)/2$; average of semi-quartile average of the auxiliary variable x , $\lambda_{40}^* = (\lambda_{40} - 1)$, $\lambda_{04}^* = (\lambda_{04} - 1)$,

$\lambda_{22}^* = (\lambda_{22} - 1)$; where, $\lambda_{rs} = \frac{\mu_{rs}}{\mu_{02}^{r/2} \mu_{20}^{s/2}}$ and $\mu_{rs} = \sum_{i=1}^N (y_i - \bar{Y})^r (x_i - \bar{X})^s / N$, (r,s) are non- negative integers.

Further to obtained the expression of biases and mean square errors of estimators, we define

$$s_y^2 = S_y^2(1 + e_0) \text{ and } s_x^2 = S_x^2(1 + e_1), \text{ such that } E(e_0) = E(e_1) = 0 \text{ and } E(e_0^2) = \left(\frac{\lambda_{40}^*}{n} \right),$$

$$E(e_1^2) = \left(\frac{\lambda_{04}^*}{n} \right), \text{ } E(e_0 e_1) = \left(\frac{\lambda_{22}^*}{n} \right).$$

The usual unbiased estimator of S_y^2 is given as

$$t_0 = s_y^2 = (n - 1)^{-1} \sum_{i=1}^n (y_i - \bar{y})^2 \tag{1.1}$$

where \bar{y} is the sample mean of study variable y .

When population variance S_x^2 of auxiliary variable x is known in advance, Isaki (1983) suggested a ratio type estimators

$$t_r = s_y^2 \left(S_x^2 / s_x^2 \right) \tag{1.2}$$

Here, it is important to mention that the estimators t_r due to Isaki (1983) is a member of Das and Tripathi (1978) class of estimators given as

$$t_{td} = s_y^2 \left(\frac{S_x^2}{s_x^2} \right)^\alpha \quad (1.3)$$

The usual difference estimator for population variance S_y^2 is given by

$$t_d = s_y^2 + d(S_x^2 - s_x^2) \quad (1.4)$$

where d is a constant to be determined such that the mean square error of t_d is minimum.

Recently, Singh and Pal (2016) envisaged the following class of estimators for population variance S_y^2 using information of auxiliary variable x

$$t_{skp} = w_1 s_y^2 + w_2 (S_x^2 - s_x^2) \left\{ \frac{\delta S_x^2 + \eta L^2}{\delta s_x^2 + \eta L^2} \right\} \quad (1.5)$$

Bias and minimum mean square error of estimator t_{skp} is given by

$$B(t_{skp}) = S_y^2 \left[w_1 \left\{ 1 + \left(\frac{\tau^*}{n} \right) (\tau^* \lambda_{04}^* - \lambda_{22}^*) \right\} + w_2 \left(\frac{r\tau^*}{n} \right) \lambda_{04}^* - 1 \right] \quad (1.6)$$

$$MSE(t_{skp}) = S_y^4 \left[1 - \frac{(b_2 b_4^2 - 2b_3 b_4 b_5 + b_1 b_3^2)}{(b_1 b_2 - b_3^2)} \right] \quad (1.7)$$

where,

$$b_1 = \left[1 + \frac{1}{n} (\lambda_{40}^* + 3\tau^{*2} \lambda_{04}^* - 4\tau^* \lambda_{22}^*) \right], b_2 = \left[\frac{r^2 \lambda_{04}^*}{n} \right], b_3 = \left[\frac{r}{n} (2\tau^* \lambda_{04}^* - \lambda_{22}^*) \right],$$

$$b_4 = \left[1 + \frac{\tau^*}{n} (\tau^* \lambda_{04}^* - \lambda_{22}^*) \right], b_5 = \left[\frac{r\tau^*}{n} \lambda_{04}^* \right], r = \frac{S_x^2}{S_y^2}, \tau^* = \frac{\delta S_x^2}{\delta S_x^2 + \eta L^2}$$

Here it is important to mention that Singh and Pal (2016) generated many estimators such as estimators due to Isaki (1983), Das and Tripathi (1978), Singh et al (2013) and Singh et al. (2015) from the class of estimators given in (1.5). The minimum mean square error of the class of estimators proposed by Singh and Pal (2016) is given in (1.7).

2. THE SUGGESTED GENERALISED CLASS OF ESTIMATORS

We propose a generalized family of estimators for population variance of the study variable y , as

$$t_m = \left\{ w_1 s_y^2 \left(\frac{S_x^2}{s_x^2} \right)^\alpha \exp \left(\frac{\eta (S_x^2 - s_x^2)}{\eta (S_x^2 + s_x^2) + 2\lambda} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2 \quad (2.1)$$

where w_1 and w_2 are suitable constants to be determined such that MSE of t_m is minimum, η and λ are either real numbers or the functions of the known parameters of auxiliary variables such as quartiles, coefficient of variation C_x , skewness $\beta_{1(x)}$, kurtosis $\beta_{2(x)}$ and correlation coefficient ρ (see Sharma and Singh (2015)).

A set of new estimators generated from (2.1) using suitable values of w_1, w_2, α, η and λ are listed in Table 2.1.

Table 2.1: Set of estimators generated from the class of estimators t_m

Subset of proposed estimator	W_1	W_2	α	η	λ
$t_0 = S_y^2$ (usual unbiased estimator)	1	0	0	0	1
$t_r = S_y^2 \left(\frac{S_x^2}{s_x^2} \right)$ (due to Isaki (1983) & Das and Tripathi (1978))	1	0	1	0	1
$t_d = S_y^2 + d(S_x^2 - s_x^2)$ (difference estimator)	1	-1	0	0	1
$t_{m1} = \left\{ s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{(S_x^2 - s_x^2)}{(S_x^2 + s_x^2) + 2} \right) \right\} + (1 - w_1) S_x^2$	1	0	1	1	1
$t_{m2} = \left\{ w_1 s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{Q_1 (S_x^2 - s_x^2)}{Q_1 (S_x^2 + s_x^2) + 2Q_3} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2$ Q_3	W_1	W_2	1		Q_1
$t_{m3} = \left\{ w_1 s_y^2 \exp \left(\frac{Q_1 (S_x^2 - s_x^2)}{Q_1 (S_x^2 + s_x^2) + 2Q_3} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2$ Q_3	W_1	W_2	0		Q_1
$t_{m4} = \left\{ s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{Q_1 (S_x^2 - s_x^2)}{Q_1 (S_x^2 + s_x^2) + 2} \right) \right\}$	1	0	1		Q_1 1
$t_{m5} = \left\{ s_y^2 \exp \left(\frac{Q_d (S_x^2 - s_x^2)}{Q_d (S_x^2 + s_x^2) + 2} \right) \right\}$	1	0	0		Q_d 1
$t_{m6} = \left\{ s_y^2 \exp \left(\frac{Q_1 (S_x^2 - s_x^2)}{Q_1 (S_x^2 + s_x^2) + 2Q_d} \right) \right\}$ Q_d	1	0	0		Q_1
$t_{m7} = \left\{ w_1 s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{\rho (S_x^2 - s_x^2)}{\rho (S_x^2 + s_x^2) + 2Q_1} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2$ Q_1	W_1	W_2	1		ρ
$t_{m8} = \left\{ w_1 s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{\rho (S_x^2 - s_x^2)}{\rho (S_x^2 + s_x^2) + 2Q_3} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2$ Q_3	W_1	W_2	1		ρ
$t_{m9} = \left\{ w_1 s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{\rho (S_x^2 - s_x^2)}{\rho (S_x^2 + s_x^2) + 2Q_d} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2$ Q_d	W_1	W_2	1		ρ
$t_{m10} = \left\{ w_1 s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{\rho (S_x^2 - s_x^2)}{\rho (S_x^2 + s_x^2) + 2Q_a} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2$ Q_a	W_1	W_2	1		ρ
$t_{m11} = \left\{ w_1 s_y^2 \left(\frac{S_x^2}{s_x^2} \right) \exp \left(\frac{C_y^2 (S_x^2 - s_x^2)}{C_y^2 (S_x^2 + s_x^2) + 2Q_1} \right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2$ Q_1	W_1	W_2	1		C_y^2

$$\begin{aligned}
t_{m12} &= \left\{ w_1 s_y^2 \exp\left(\frac{C_y^2 (S_x^2 - s_x^2)}{C_y^2 (S_x^2 + s_x^2) + 2Q_a}\right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2 & W_1 & \quad W_2 & \quad 0 & \quad C_y^2 \\
& \quad Q_a \\
t_{m13} &= \left\{ w_1 s_y^2 \exp\left(\frac{C_y^2 (S_x^2 - s_x^2)}{C_y^2 (S_x^2 + s_x^2) + 2Q_d}\right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2 & W_1 & \quad W_2 & \quad 0 & \quad C_y^2 \\
& \quad Q_d \\
t_{m14} &= \left\{ w_1 s_y^2 \exp\left(\frac{C_y^2 (S_x^2 - s_x^2)}{C_y^2 (S_x^2 + s_x^2) + 2Q_r}\right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2 & W_1 & \quad W_2 & \quad 0 & \quad C_y^2 \\
& \quad Q_r \\
t_{m15} &= \left\{ w_1 s_y^2 \left(\frac{s_x^2}{S_x^2}\right) \exp\left(\frac{C_x^2 (S_x^2 - s_x^2)}{C_x^2 (S_x^2 + s_x^2) + 2\rho}\right) \right\} + w_2 s_x^2 + (1 - w_1 - w_2) S_x^2 & W_1 & \quad W_2 & \quad -1 & \quad C_x^2 \quad \rho
\end{aligned}$$

Expressing (2.1) in terms of e's, we have

$$t_m = w_1 S_y^2 (1 + e_0)(1 + e_1)^{-\alpha} \exp\{-ke_1(1 + ke_1)^{-1}\} + w_2 S_x^2 (1 + e_1) + (1 - w_1 - w_2) S_x^2 \quad (2.2)$$

where, $k = \frac{\eta S_x^2}{2(\eta S_x^2 + \lambda)}$. Up to the first order of approximation we have,

$$(t_m - S_y^2) = [(w_1 - 1)b + w_1 S_y^2 \{e_0 - ae_1 + de_1^2 - ae_0 e_1\} + w_2 S_x^2 e_1] \quad (2.3)$$

taking expectation both sides we get the bias of estimator t_m

$$\text{Bias}(t_m) = (w_1 - 1)b - \frac{S_y^2}{n} w_1 (d\lambda_{04}^* - a\lambda_{22}^*) \quad (2.4)$$

where $a = (\alpha + k)$, $b = (S_y^2 - S_x^2)$ and $d = \left\{ \frac{3}{2} k^2 + \alpha k + \frac{\alpha(\alpha + 1)}{2} \right\}$

Squaring both sides of equation (2.3) and neglecting terms of e's having power greater than two, we have

$$(t_m - S_y^2)^2 = \left[(1 - 2w_1)b^2 + w_1^2 \{b^2 + S_y^2 (e_0^2 + a^2 e_1^2 - 2ae_0 e_1)\} + w_2^2 S_x^4 e_1^2 + 2w_1 w_2 S_y^2 S_x^2 (e_0 e_1 - ae_1^2) \right] \quad (2.5)$$

Taking expectations both sides, we get the MSE of the estimator t_m to the first order of approximation as

$$\text{MSE}(t_m) = \left[(1 - 2w_1)b^2 + 2bw_1 A + w_1^2 B + w_2^2 C + 2w_1 w_2 D \right] \quad (2.6)$$

where,

$$A = \frac{S_y^2}{n} (d\lambda_{04}^* - a\lambda_{22}^*), \quad B = b^2 + \frac{S_y^2}{n} \{S_y^2 (\lambda_{40}^* + a^2 \lambda_{04}^* - 2a\lambda_{22}^*) - 2ab\lambda_{22}^* + 2bd\lambda_{04}^*\}$$

$$C = \frac{S_x^4}{n} \lambda_{04}^*, \quad D = \frac{S_y^2 S_x^2}{n} (\lambda_{22}^* - a\lambda_{04}^*)$$

The optimum values of w_1 and w_2 are obtained by minimizing (2.6) and is given by

$$w_1^* = \frac{b^2 C}{(BC - D^2)} \quad \text{and} \quad w_2^* = \frac{-b^2 D}{(BC - D^2)} \quad (2.7)$$

Substituting the optimal values of W_1 and W_2 in equation (2.6) we obtain the minimum MSE of the estimator t_m as

$$MSE_{\min}(t_m) = b^2 \left[1 - \frac{bC(b-2A)}{(BC-D^2)} \right] \quad (2.8)$$

thus we can establish the following theorem

Theorem 2.1. To the first degree of approximation $MSE(t_m) \geq b^2 \left[1 - \frac{bC(b-2A)}{(BC-D^2)} \right]$ with equality

holding if

$$W_1 = W_1^*$$

$$W_2 = W_2^*$$

where, W_i^* ($i=1,2$) are given in equation (2.6).

3. EFFICIENCY COMPARISONS

Comparison of usual unbiased estimator and proposed estimator is

$$V(s_y^2) - MSE_{\min}(t_m) = \frac{S_y^4}{n} \lambda_{40}^* - b^2 \left[1 - \frac{bC(b-2A)}{(BC-D^2)} \right] > 0 \quad (3.1)$$

From equations (1.7) and (2.8) we have

$$MSE(t_{skp}) - MSE_{\min}(t_m) > 0 \Rightarrow S_y^4 \left[1 - \frac{b_2 b_4^2 - 2b_3 b_4 b_5 + b_1 b_5^2}{(b_1 b_2 - b_3^2)} \right] - (S_y^2 - S_x^2) \left[1 - \frac{bC(b-2A)}{(BC-D^2)} \right] > 0 \quad (3.2)$$

If conditions (3.1) and (3.2) holds, then proposed class of estimators t_m is better than the usual unbiased estimator s_y^2 and minimum mean square error of the class of estimators proposed by Singh and Pal (2016).

3.1. Empirical Study

Data Statistics: The performance of the suggested estimators t_{mi} ($i=1,2,\dots,15$) which have been generated from proposed generalized class of estimators t_m are evaluated along with other existing estimators discussed in literature for the population data set, given in table (4.1), taken from Murthy (1967). Percent relative efficiency (PRE's) of the suggested estimators t_{mi} ($i=1, 2,\dots,15$) is calculated and summarized in

table (4.2) with respect to usual unbiased estimator S_y^2 using the formula : $PRE(t_m, S_y^2) = \frac{MSE(S_y^2)}{MSE(t_m)} \times 100$

To illustrate the efficiency of proposed generalized class of estimators in the application, we consider the following population data set.

Table 4.1: The population parameter of data set

N	80	C_y	0.3542	Q_1	5.1500
n	20	S_x	8.4563	Q_2	10.300
\bar{Y}	51.8264	C_x	0.7507	Q_3	16.975
\bar{X}	11.2646	λ_{04}	2.8664	Q_r	11.825

ρ	0.9413	λ_{40}	2.2667	Q_d	5.9125
S_y	18.3569	λ_{22}	2.2209	Q_a	11.0625

Table 4.2: PRE's of different estimators with respect to S_y^2

Estimators	PRE	Estimators	PRE
S_y^2	100.00	t_{m7}	282.94
t_{skp}	(183, 353)*	t_{m8}	297.49
t_r	296.40	t_{m9}	284.07
t_{md}	155.91	t_{m10}	290.94
t_{m1}	106.17	t_{m11}	313.72
t_{m2}	279.75	t_{m12}	435.40
t_{m3}	410.93	t_{m13}	431.13
t_{m4}	105.03	t_{m14}	435.70
t_{m5}	283.06	t_{m15}	420.17
t_{m6}	279.47		

* Percent relative efficiency of members of class of estimators suggested by Singh and Pal (2016) 183 to 353.

Analysing table 4 we observe that almost all the estimators $t_i, (i = 3,5,7,8, \dots, 15)$ which are the members of the proposed generalised class of estimators t_m performs better than the usual unbiased estimator, ratio type estimator (due to Isaki (1983)), product and difference estimator discussed in this paper. We also observe that the efficiency of the members $t_j, (j = 12,13,14,15)$ among the other members of proposed class of estimators t_m are high and better than the class of estimators and its members recently proposed by Singh and Pal (2016) along with other estimators considered here. It is worth mentioning that the estimators based on the auxiliary information related to quartile, function of quartile or other functions of auxiliary information are more efficient than the one which does not utilize the suitable auxiliary information.

4. CONCLUSIONS

In this article we have suggested a generalized class of estimators for the population variance S_y^2 of study variable y when information is available on an auxiliary variable x such as coefficient of variation, coefficient of kurtosis, quartiles and their functions, in simple random sampling without replacement (SRSWOR). In addition, some known estimators of population variance such as usual unbiased estimator, ratio estimators and difference type estimator are found to be members of the proposed generalized class of estimators. Some new members are also generated from the proposed generalized class of estimators utilizing the information on different functions of auxiliary information. We have determined the biases and mean square errors of the proposed class of estimators up to the first order of approximation. The proposed generalized class of estimators are advantageous in the sense that the properties of the estimators, which are members of the proposed class of estimators, can be easily obtained from the proposed generalized class. Thus this study unifies properties of several estimators of population variance. In theoretical as well as in empirical efficiency comparisons assessed on known natural data set, it has been shown that the proposed generalized

class of estimators are more efficient than the usual unbiased estimator, ratio estimator, product estimator, difference estimator and estimators proposed by Singh and Pal(2016) for estimating the population variance.

RECEIVED: OCTOBER, 2017

REVISED: FEBRUARY, 2018

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