ANALYSIS OF EXPONENTIATED EXPONENTIAL MODEL UNDER STEP STRESS PARTIALLY ACCELERATED LIFE TESTING PLAN USING PROGRESSIVE TYPE-II CENSORED DATA

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ABSTRACT
Recently, partially accelerated life testing has become quite important in reliability and life testing studies. This paper discusses maximum likelihood estimation method in step-stress partially accelerated life tests when the lifetime of items under use condition follow the exponentiated exponential distribution. Based on progressively type-II censored samples; the point and interval estimations for the considered parameters and the tampering coefficient are obtained in closed forms. The observed Fisher information matrix is derived to calculate confidence intervals for the considered parameters. The performance of the resulting estimators of the developed model parameters is evaluated and investigated in terms of their biases and mean squared errors by using a Monte
Carlo simulation method.

KEYWORDs: Exponentiated exponential distribution; partially accelerated life testing; progressive type-II censoring; Fisher information matrix; Monte Carlo simulations.

MSC: 62N05, 62N01, 62N02, 65Y99.

RESUMEN
Recientemente. pruebas aceleradas parciales de vida se han convertido en casi importantes en estudios de fiabilidad y pruebas de vida. En este paper se comparan el método de estimación maximización contra el de verosimilitud en "step-stress partially accelerated life tests" cuando el tiempo de vida de los productos bajo la condición de uso se distribuyen "exponentiated exponential". Basados en muestras censuradas del tipo progresivo-II; se considera la estimación puntual y por intervalos de los parámetros considerados y el coeficiente de "tampering" son obtenidas en forma analítica. La observada matriz de información de Fisher es derivada para calcular intervalos confidenciales de los parámetros considerados. El comportamiento de los estimadores resultantes de los parámetros del modelo desarrollado es evaluado e investigado en términos de sus sesgos y errores cuadráticos medios usando el método de simulación Monte Carlo.

PALABRAS CLAVE: Distribución "exponentiated exponencial"; pruebas parciales aceleradas de vida; censura del tipo progresivo-II; matriz de información de Fisher; simulación de Monte Carlo.

1. INTRODUCTION

When the units of high reliability is being tested at normal stress or use condition, the test results a very few or no failure at the given constraints i.e. time, cost etc. So to get more and quick failures, we approach accelerated life test (ALT). In ALT, we put the units at higher stress than normal or use condition. The stresses applied in the test may be in the form of pressure, voltage, temperature, vibration, load, cycling rate etc. The data collected under stresses are used to estimate the life distribution at normal use condition. According to Nelson(1990) there are several ways by which stress can be applied into the life testing experiment. The common stresses are constant stress, step stress and progressive stress or linearly increasing stress. In constant stress test, each unit runs at a pre-specified stress level which does not vary with time. This means that every unit is subjected to only one stress level until the item fails or the test is terminated for any reason. While in step stress, the items are first put on some pre-specified level of stress and run for some

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specified time, if all the items do not fail then we change (increase) the stress level and the experiment continues until all the units get failed or censored at any specific point of time.

The main assumption in ALT is that the acceleration factor is known or mathematical model relating the lifetime of the units, exists which specifies the relationship between lifetime and stress(es). However there are some experiments in which neither acceleration factor is known nor such models exists or very hard to assume. For this case, partially accelerated life test is better option to choose. The assumption in PALT is that the model relating to the lifetime of the units between the mean life time and the stress is not known and cannot be assumed. The acceleration factor is also not known.

DeGroot and Goel(1979) have introduced the concept of step PALT in which an item is run first at use condition (normal stress) and if it does not fail for a pre-specified time τ, then it is put at accelerated condition until it gets failed or get censored at any specific time η. The remaining lifetime of the units are multiplied by the acceleration factor to overcome the deterioration in the lifetime at normal use condition and it is denoted by β. It is the ratio of the mean lifetime of the units at use condition to that at acceleration condition.

In life test experiment it is not always possible to observe all the information of the testing items, so the experiment may be terminated before all the units of test items get failed. This process is called censoring .The most common censoring schemes are Type-I (time) censoring and Type-II (failure) censoring.

In conventional Type-I censoring the test is terminated at a pre-specified time T while in conventional Type-II censoring the test is terminated at the time of the *r*-th failure (*r* is prefixed). But these censoring do not have flexibility to remove the items at points other than terminal point of the experiment. This leads us towards a more general censoring scheme i.e. progressive Type-II right censoring scheme. This scheme reduces the time and cost and it is very useful when items are being tested are expensive. It is described as follows: consider an experiment in which n identical items are being placed on the test to observe their lifetimes. Suppose their lifetimes are denoted as $(X_1, X_2, \cdots, X_n)$. At the time of first failure $X_{1:m:n}$, R_1 of the remaining $(n-1)$ units are removed randomly. Similarly at the time of second failure $X_{2:m:n}$, R_2 units are removed randomly from the surviving units $(n-2-R_1)$. Finally, at the time of *m*-th failure $X_{m:m:n}$ all the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ units are removed. The system $(R_1, R_2, \cdots, R_m)$ is fixed prior to the study. For more details on progressive type-II censoring one may refer Cohen (1963), Cohen and Norgaard (1977), Sarhan and Abuammoh (2008), Balakrishnan and Aggrawala (2000), Wu, Wu and Chan (2004).

A lot of literature is available on SSPALT analysis, for example, see Bhattacharya and Soejoeti (1989), Bai and Chung (1992), Bai, Chung and Chun (1993), Abdel Ghaly et al (2008),Abdel Ghaly et al (2007), Goel (1971), DeGroot and Goel (1979), Ismail (2006), Zarrin, S., Kamal M. and Saxena S.(2012), Kamal, M., Zarrin S., and Islam A.(2013). Ismail (2009) have considered progressively type-II censored data under optimal design of step stress life test. Shahab, Anwar and Islam (2015) recently studied the optimal design of step stress partially accelerated life test under progressive type-II censored data with random removals for Frechet distribution. Lone, Rahman and Islam (2016) used step stress partially accelerated life tests to estimate the parameters of Mukherjee-Islam distribution using time constraint data. Rahman, Lone and Islam (2016) extended the work and estimated the parameters from Mukherjee-Islam failure constraint data using SSPALT.

The rest of the paper is organised as follows: In section 2 we have described the model and test method. Assumptions are also made in this section. Section 3 deals with estimation technique. Maximum likelihood estimation technique is carried out to estimate the point and interval estimation of the parameters. Simulation study has been carried out in section 4. The conclusion of the paper is given in section 5.

2. MODEL DESCRIPTION AND TEST METHOD

Assumptions

(a) The lifetime of an item tested at both normal and accelerated condition follows exponentiated exponential distribution.
(b) The lifetime of an item under SSPALT is given as

$$Y = \begin{cases} T & 0 < T \le \tau \\ \tau + \beta^{-1}(T - \tau) & T > \tau \end{cases} \tag{1}$$

Where Y denotes the total lifetime of the item at SSPALT and T denotes the lifetime of the item at use condition, τ is the time at what stress is to be changed and acceleration factor $\beta > 1$.

(c) The lifetimes of the test items are independent and identically distributed.

Test Procedure

(a) All n independent items have been put on first at normal use condition and observe the failure times of the items.
(b) If all the items do not fail by the time τ, then level of stress is increased. Now observe the lifetimes of the items until the pre-specified time η. We terminate the test at time η.
(c) As the test started we got failures. At the time of the i^th^ failure we remove the R_i units from the remaining units. Finally at the time of m^th^ failure all the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ are removed from the test and test is terminated.

From the assumption (a) all the units follow exponentiated exponential distribution. So the pdf of EE is given as [see, Gupta and Kundu (2001)]

$$f_Y(y) = \alpha\lambda e^{-\lambda y}(1 - e^{-\lambda y})^{\alpha - 1}, \qquad y > 0,\ \alpha > 0 \quad and \quad \lambda > 0 \tag{2}$$

The cumulative distribution function is

$$F(y) = (1 - e^{-\lambda y})^{\alpha}, \qquad y, \alpha, \lambda > 0 \tag{3}$$

The reliability function of the EE distribution is given as

$$S(y) = 1 - (1 - e^{-\lambda y})^{\alpha}, \qquad y, \alpha, \lambda > 0 \tag{4}$$

From equation(1), the pdf of the lifetimes of the items are given as

$$f(y) = \begin{cases} 0 & 0 \le y \\ f_1(y) = f_Y(y) & 0 < y \le \tau \\ f_2(y) & \tau < y \le \eta \end{cases} \tag{5}$$

Where

$$f_2(y) = \alpha\lambda\beta e^{-\lambda[\tau + \beta(y - \tau)]}\left(1 - e^{-\lambda[\tau + \beta(y - \tau)]}\right)^{\alpha - 1} \tag{6}$$

and the survival functions are

$$S(y) = \begin{cases} 1 - (1 - e^{-\lambda y})^{\alpha} & 0 < y \le \tau \\ 1 - (1 - e^{-\lambda[\tau + \beta(\eta - \tau)]})^{\alpha}, & \tau < y \le \eta \end{cases} \tag{7}$$

3. ESTIMATION TECHNIQUE

While various methods for parameter estimation exist, maximum likelihood estimation (MLE) is one of the most widely used methods. It is very robust and has very good statistical properties. It can be applied to any probability distribution while other methods are somewhat restricted. The idea behind the maximum likelihood parameter estimation is to determine the estimates of the parameter that maximizes the likelihood of the sample data. Also, the MLEs have the desirable properties of being consistent and asymptotically normal for large samples. Here we have obtained the point and interval estimation of the acceleration factor and parameters of the exponentiated exponential distribution using progressive type-II censored data.

a. Point Estimation

After testing the lifetimes of the units, the observed values of the lifetimes are

$$y_{(1)} < \cdots < y_{(n_u)} \le \tau < y_{(n_u+1)} < \cdots < y_{(n_u+n_a-1)} < \eta$$

Where n_u and n_a are the number of subjects or items failed at normal conditions and accelerated conditions respectively. Let δ_{1i} and δ_{2i} be indicator function such that

$$\delta_{1i} = \begin{cases} 1 & y_{(i)} \le \tau \\ 0 & elsewhere \end{cases} \quad i = 1,2,\cdots,n \qquad \text{and} \qquad \delta_{2i} = \begin{cases} 1 & \tau < y_{(i)} \le \eta \\ 0 & elsewhere \end{cases} \quad i = 1,2,\cdots,n$$

For our convenience further we shall use y_i instead of $y_{(i)}$.

The likelihood function is given as

$$L(\theta) = \prod_{i=1}^{n} \left\{ [f_1(y_i).(S_1(y_i))^{R_i}]^{\delta_{1i}} [f_2(y_i).(S_2(y_i))^{R_i}]^{\delta_{2i}} \right\} \tag{8}$$

The log likelihood function of the above equation is written as

$$\ln L = n_0 \ln\alpha + n_0 \ln\lambda - \lambda\sum_{i=1}^{n}\delta_{1i}y_i + (\alpha-1)\sum_{i=1}^{n}\delta_{1i}\ln(1-e^{-\lambda y_i}) + \sum_{i=1}^{n}\delta_{1i}R_i\ln(1-(1-e^{-\lambda y_i})^{\alpha}) + n_a\ln\beta$$

$$-\lambda\sum_{i=1}^{n}\delta_{2i}\phi_i + (\alpha-1)\sum_{i=1}^{n}\delta_{2i}\ln(1-e^{-\lambda\phi_i}) + \sum_{i=1}^{n}\delta_{2i}R_i\ln(1-(1-e^{-\lambda\phi_\eta})^{\alpha}) \tag{9}$$

Where $n_0 = \sum_{i=1}^{n}\delta_{1i}$, $n_a = \sum_{i=1}^{n}\delta_{2i}$, $\phi_i = \tau + \beta(y_i - \tau)$ and $\phi_\eta = \tau + \beta(\eta - \tau)$

The first derivatives of the log likelihood function with respect to $\alpha, \beta \ and \ \lambda$ are taken respectively and equate them to zero.

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n_0}{\alpha} + \sum_{i=1}^{n}\delta_{1i}\ln A_i - \sum_{i=1}^{n}\delta_{1i}R_i\ln A_i A_i^{\alpha}(1-A_i^{\alpha})^{-1} + \sum_{i=1}^{n}\delta_{2i}\ln B_i - \sum_{i=1}^{n}\delta_{2i}R_i\ln C\, C^{\alpha}(1-C^{\alpha})^{-1} = 0$$

(10)

$$\frac{\partial \ln L}{\partial \beta} = \frac{n_a}{\beta} - \lambda\sum_{i=1}^{n}\delta_{2i}(y_i-\tau) + \lambda(\alpha-1)\sum_{i=1}^{n}\delta_{2i}(y_i-\tau)e^{-\lambda\phi_i}B_i^{-1} - \alpha\lambda(\eta-\tau)e^{-\lambda\phi_\eta}C^{\alpha-1}(1-C^{\alpha})^{-1} = 0 \tag{11}$$

$$\frac{\partial \ln L}{\partial \lambda} = \frac{n_0}{\lambda} - \sum_{i=1}^{n}\delta_{1i}y_i + (\alpha-1)\sum_{i=1}^{n}\delta_{1i}y_i e^{-\lambda y_i}A_i^{-1} - \alpha\sum_{i=1}^{n}\delta_{1i}R_i y_i e^{-\lambda y_i}A_i^{\alpha-1}(1-A_i^{\alpha})^{-1} - \sum_{i=1}^{n}\delta_{2i}\phi_i$$

$$+(\alpha-1)\sum_{i=1}^{n}\delta_{2i}\phi_i e^{-\lambda\phi_i}B_i^{-1} - \alpha\phi_\eta e^{-\lambda\phi_\eta}C^{\alpha-1}(1-C^{\varepsilon})\sum_{i=1}^{n}\delta_{2i}R_i = 0 \tag{12}$$

Where $A_i = 1-e^{-\lambda y_i}$, $B_i = 1-e^{-\lambda\phi_i}$ and $C = 1-e^{-\lambda\phi_\eta}$

Here we have obtained a system of nonlinear equations (10)- (12) which are not in closed form. Their mathematical solutions are not possible so we use iterative procedure to solve them. Newton-Raphson iterative technique is used to solve the above system.

b. Interval Estimation

In this subsection the confidence interval is estimated and it is approximate confidence intervals of the parameters. It is possible only because the asymptotic distribution property of maximum likelihood estimators of the unknown parameters $\Omega = (\alpha, \beta, \lambda)$. The asymptotic distribution of the maximum likelihood estimators of Ω is given as follows:

$$((\hat{\alpha}-\alpha),(\hat{\beta}-\beta),(\hat{\lambda}-\lambda)) \rightarrow N(0, I^{-1}(\alpha,\beta,\lambda)),$$

Where $I(\alpha,\beta,\lambda)$ is the Fisher information matrix of the unknown parameters $\Omega=(\alpha,\beta,\lambda)$ and its inverse $I^{-1}(\alpha,\beta,\lambda)$ is variance-covariance matrix. The elements of the Fisher information matrix are given here as follows:

$$I=\begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \alpha^2} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda} \\ -\frac{\partial^2 \ln L}{\partial \beta \partial \alpha} & -\frac{\partial^2 \ln L}{\partial \beta^2} & -\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} \\ -\frac{\partial^2 \ln L}{\partial \lambda \partial \alpha} & -\frac{\partial^2 \ln L}{\partial \lambda \partial \beta} & -\frac{\partial^2 \ln L}{\partial \lambda^2} \end{bmatrix}$$

The elements of the Fisher information matrix are

$$\frac{\partial^2 \ln L}{\partial \alpha^2}=-\frac{n_0}{\alpha^2}-\sum_{i=1}^{n}\delta_{2i}R_iA_i^{\alpha}\left(\frac{\ln A_i}{1-A_i^{\alpha}}\right)^2-\sum_{i=1}^{n}\delta_{2i}R_iC^{\alpha}\left(\frac{\ln C}{1-C^{\alpha}}\right)^2$$

$$\frac{\partial^2 \ln L}{\partial \beta^2}=-\frac{n_a}{\beta^2}-\lambda^2(\alpha-1)\sum_{i=1}^{n}\delta_{2i}\left(\frac{y_i-\tau}{B_i}\right)^2 e^{-\lambda\phi_i}$$
$$-\alpha\lambda^2(\eta-\tau)^2 e^{-\lambda\phi_\eta}\,\frac{C^{\alpha-1}[C^{\alpha-1}-1+(\alpha-1)e^{-\lambda\phi_\eta}C^{-1}+e^{-\lambda\phi_\eta}C^{\alpha-1}]}{(1-C^{\alpha})^2}\sum_{i=1}^{n}\delta_{2i}R_i$$

$$\frac{\partial^2 \ln L}{\partial \lambda^2}=-\frac{n_0}{\lambda^2}-(\alpha-1)\sum_{i=1}^{n}\delta_{1i}y_i^2e^{-\lambda y_i}A_i^{-2}-(\alpha-1)\sum_{i=1}^{n}\delta_{1i}\phi_i^2e^{-\lambda\phi_i}B_i^{-2}$$
$$-\alpha\sum_{i=1}^{n}\delta_{1i}R_iy_i^2e^{-\lambda y_i}\,\frac{A_i^{\alpha-1}[A_i^{\alpha}-1+(\alpha-1)e^{-\lambda y_i}A_i^{-1}+e^{-\lambda y_i}A_i^{\alpha-1}]}{(1-A_i^{\alpha})^2}$$
$$-\alpha\phi_\eta^2e^{-\lambda\phi_\eta}\,\frac{C^{\alpha-1}[C^{\alpha}-1+(\alpha-1)e^{-\lambda\phi_\eta}C^{-1}+e^{-\lambda\phi_\eta}C^{\alpha-1}]}{(1-C^{\alpha})^2}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \beta}=\lambda\sum_{i=1}^{n}\delta_{2i}(y_i-\tau)e^{-\lambda\phi_i}B_i^{-1}-\lambda(\eta-\tau)e^{-\lambda\phi_\eta}C^{\alpha-1}(1-C^{\alpha}+\alpha\ln C)(1-C^{\alpha})^{-1}\sum_{i=1}^{n}\delta_{2i}R_i$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \lambda}=-\sum_{i=1}^{n}\delta_{2i}(y_i-\tau)+(\alpha-1)\sum_{i=1}^{n}\delta_{2i}(y_i-\tau)e^{-\lambda\phi_i}(1-\lambda\phi_i-e^{-\lambda\phi_i})A_i^{-2}$$
$$-\sum_{i=1}^{n}\delta_{2i}R_i\alpha(\eta-\tau)e^{-\lambda\phi_\eta}C^{\alpha-2}(1-\lambda\phi_\eta-e^{-\lambda\phi_\eta}+\lambda\alpha\phi_\eta e^{-\lambda\phi_\eta}-C^{\alpha}+e^{-\lambda\phi_\eta}C^{\alpha}+\lambda C^{\alpha}\phi_\eta)(1-C^{\alpha})^{-2}$$

$$\frac{\partial^2 \ln L}{\partial \alpha \partial \lambda}=\sum_{i=1}^{n}\delta_{1i}y_ie^{-\lambda\phi_i}A_i^{-1}-\sum_{i=1}^{n}\delta_{1i}y_ie^{-\lambda y_i}A_i^{\alpha-1}(1-A_i^{\alpha}+\alpha\ln A_i)(1-A_i^{\alpha})^{-2}+\sum_{i=1}^{n}\delta_{2i}\phi_ie^{-\lambda\phi_i}B_i^{-1}$$
$$-\phi_\eta e^{-\lambda\phi_\eta}C^{\alpha-1}(1-C^{\alpha}+\alpha\ln C)(1-C^{\alpha})^{-2}\sum_{i=1}^{n}\delta_{2i}R_i$$

Thus the two sided confidence intervals for α, β and λ are approximated for $100(1-\gamma)\%$ respectively, as follows

$$\hat{\alpha} \pm Z_{\gamma/2}\sqrt{I_{11}^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})}, \qquad \hat{\beta} \pm Z_{\gamma/2}\sqrt{I_{11}^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})}, \qquad and \qquad \hat{\lambda} \pm Z_{\gamma/2}\sqrt{I_{11}^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})}$$

Where $Z_{\gamma/2}$ is the upper $(\gamma/2)th$ percentile of a standard normal distribution.

3. SIMULATION PROCEDURE

Since theoretically it is not easy to compare the performance of the different methods or censoring schemes, the simulation study is performed to compare different sampling schemes for different parameter values by Monte Carlo simulations technique. The term different sampling schemes means for different sets of R_i and for different η values.

Specifically, in this section the comparison of the performance of the MLEs is studied in terms of their biases and mean square errors (MSEs) for different choices of α, β and λ. The asymptotic confidence intervals of 95% are also constructed on the basis of assumptions on asymptotic distribution of the ML estimators.

Three progressive censoring schemes are considered here

$$Scheme1: R_1 = \cdots = R_{m-1} = 0 \ and \ R_m = n-m$$

$$Scheme2: R_1 = n-m \ \ and \ \ R_2 = \cdots = R_m = 0$$

$$Scheme3: R_1 = \cdots = R_{m-1} = 1 \ and \ R_m = n-2m+1$$

For each scheme, the biases, MSEs and confidence intervals are estimated for 10000 replications and tabulated as 1-4.

Table 1: The Mean value of MLEs with bias and MSE of the parameters $(\alpha = 0.8, \beta = 1.2, \lambda = 1.25)$ for different sample sizes under progressive type-II censoring

(n,m)	Schemes	Estimate of α			Estimate of β			Estimate of λ		
		MLE	Bias	MSE	MLE	Bias	MSE	MLE	Bias	MSE
(20,10)	1	0.751	0.508	0.456	1.403	0.496	0.458	1.193	0.569	0.527
	2	0.612	0.563	0.506	1.518	0.536	0.510	1.404	0.643	0.599
	3	0.721	0.539	0.485	1.441	0.531	0.478	1.118	0.593	0.560
(20,15)	1	0.782	0.486	0.409	1.347	0.488	0.436	1.206	0.505	0.459
	2	0.721	0.560	0.538	1.386	0.521	0.499	1.389	0.563	0.503
	3	0.763	0.531	0.443	1.273	0.513	0.464	1.147	0.541	0.476
(35,25)	1	0.821	0.462	0.394	1.381	0.304	0.273	1.351	0.539	0.479
	2	0.737	0.547	0.430	1.481	0.458	0.384	1.548	0.586	0.530
	3	0.759	0.493	0.414	1.397	0.419	0.328	1.383	0.550	0.503
(35,30)	1	0.789	0.390	0.329	1.261	0.286	0.217	1.320	0.423	0.351
	2	0.841	0.458	0.375	1.473	0.373	0.305	1.498	0.487	0.426
	3	0.767	0.418	0.347	1.360	0.340	0.289	1.423	0.470	0.428
(50,35)	1	0.813	0.378	0.263	1.339	0.232	0.193	1.224	0.377	0.301
	2	0.862	0.399	0.358	1.394	0.314	0.279	1.108	0.460	0.396
	3	0.785	0.396	0.319	1.406	0.263	0.207	1.116	0.409	0.372
(50,40)	1	0.809	0.253	0.203	1.297	0.192	0.153	1.209	0.282	0.217
	2	0.758	0.323	0.294	1.316	0.278	0.234	1.112	0.367	0.280
	3	0.773	0.289	0.259	1.287	0.216	0.186	1.119	0.327	0.273

The simulation study is carried out according to the following algorithm:

(1) Specify the value of n and m.

(2) Specify the values of the parameters α, λ and acceleration factor β.

(3) Generate a random sample of size n from uniform distribution U(0,1) and then use inverse transformation method and obtain $y = -\ln(1-u^{1/\alpha})/\lambda$ using Eq.(3).

(4) Progressive type-II censored data is used to compute the MLEs of the unknown parameters. Newton-Raphson method is applied to solve the nonlinear equations in terms of α, β and λ.

(5) Compute the average values of biases and MSEs of the parameters and acceleration factor for all the sample sizes.

(6) Step 3-5 are replicated 10000 times with different values of n , m , α , β and λ .

Table 2: The Mean value of MLEs with bias and MSE of the parameters $(\alpha = 0.8, \beta = 1.3, \lambda = 1.25)$ for different sample sizes under progressive type-II censoring

(n,m)	Schemes	Estimate of α			Estimate of β			Estimate of λ		
		MLE	Bias	MSE	MLE	Bias	MSE	MLE	Bias	MSE
(20,10)	1	0.823	0.471	0.429	1.228	0.568	0.524	1.185	0.379	0.338
	2	0.859	0.516	0.490	1.167	0.673	0.598	1.352	0.446	0.385
	3	0.836	0.493	0.448	1.201	0.634	0.564	1.120	0.413	0.347
(20,15)	1	0.783	0.459	0.399	1.438	0.547	0.467	1.237	0.358	0.276
	2	0.829	0.487	0.452	1.498	0.590	0.523	1.372	0.406	0.367
	3	0.790	0.472	0.436	1.440	0.554	0.492	1.342	0.375	0.328
(35,25)	1	0.787	0.378	0.314	1.502	0.439	0.372	1.290	0.351	0.293
	2	0.763	0.456	0.367	1.462	0.519	0.432	1.483	0.379	0.354
	3	0.772	0.423	0.356	1.432	0.482	0.402	1.357	0.367	0.337
(35,30)	1	0.803	0.370	0.284	1.395	0.425	0.356	1.339	0.283	0.231
	2	0.821	0.393	0.347	1.527	0.462	0.418	1.421	0.349	0.293
	3	0.813	0.375	0.314	1.467	0.441	0.377	1.445	0.316	0.247
(50,35)	1	0.765	0.246	0.184	1.364	0.325	0.268	1.241	0.238	0.187
	2	0.748	0.279	0.264	1.497	0.369	0.338	1.372	0.276	0.261
	3	0.756	0.268	0.216	1.398	0.347	0.295	1.364	0.254	0.238
(50,40)	1	0.795	0.173	0.134	1.285	0.278	0.229	1.169	0.173	0.137
	2	0.823	0.236	0.179	1.458	0.353	0.287	1.395	0.214	0.196
	3	0.809	0.207	0.140	1.392	0.313	0.253	1.261	0.189	0.167

The results are summarized in table 1-4. The 10000 replications are used to avoid the randomness. From the table it is observed that the biases and MSEs are decreasing as the values of sample size is increases for all cases. Scheme2 only have a slightly larger biases and MSEs than scheme 1 and scheme 3 because of heavy censoring in the early stage of the experiment. The confidence intervals are also getting narrower as the sample size increases.

Table 3: Confidence intervals of the estimators $(\alpha = 0.8, \beta = 1.2, \lambda = 1.25)$ at confidence level 0.95

(n,m)	Schemes	Estimate of α		Estimate of β		Estimate of λ	
		LCL	UCL	LCL	UCL	LCL	UCL
(20,10)	1	0.347	1.289	0.753	1.879	0.638	1.965
	2	0.305	1.327	0.736	1.883	0.617	1.987
	3	0.362	1.238	0.760	1.852	0.635	1.970
(20,15)	1	0.447	1.139	0.793	1.799	0.720	1.837
	2	0.436	1.194	0.743	1.832	0.678	1.950
	3	0.452	1.104	0.786	1.783	0.705	1.857
(35,25)	1	0.487	1.118	0.859	1.758	0.787	1.753
	2	0.468	1.120	0.817	1.784	0.733	1.776
	3	0.473	1.119	0.871	1.772	0.747	1.758
(35,30)	1	0.507	1.014	0.936	1.674	0.845	1.679
	2	0.493	1.114	0.874	1.738	0.798	1.736
	3	0.502	1.007	0.903	1.692	0.826	1.699
(50,35)	1	0.523	1.115	0.960	1.630	0.873	1.612
	2	0.503	1.127	0.947	1.689	0.849	1.662
	3	0.518	1.118	0.962	1.646	0.862	1.633
(50,40)	1	0.637	0.988	0.986	1.594	0.913	1.578
	2	0.614	1.004	0.958	1.602	0.878	1.632
	3	0.632	0.992	0.971	1.583	0.896	1.618

Table 4: Confidence intervals of the estimators $(\alpha = 0.8, \beta = 1.3, \lambda = 1.25)$ at confidence level 0.95

(n,m)	Schemes	Estimate of α		Estimate of β		Estimate of λ	
		LCL	UCL	LCL	UCL	LCL	UCL
(20,10)	1	0.394	1.342	0.787	1.818	0.673	1.906
	2	0.338	1.322	0.746	1.892	0.631	1.956
	3	0.361	1.336	0.773	1.839	0.659	1.927
(20,15)	1	0.478	1.106	0.806	1.725	0.738	1.795
	2	0.418	1.176	0.748	1.782	0.681	1.873
	3	0.457	1.148	0.790	1.747	0.724	1.839
(35,25)	1	0.518	1.102	0.870	1.738	0.773	1.704
	2	0.480	1.141	0.828	1.785	0.718	1.757

	3	0.497	1.124	0.856	1.751	0.765	1.718
(35,30)	1	0.543	1.002	0.967	1.619	0.860	1.623
	2	0.514	1.107	0.889	1.684	0.783	1.707
	3	0.532	1.009	0.938	1.647	0.847	1.660
(50,35)	1	0.589	1.128	0.993	1.580	0.910	1.573
	2	0.546	1.259	0.955	1.627	0.867	1.628
	3	0.574	1.196	0.983	1.602	0.895	1.598
(50,40)	1	0.679	0.936	1.008	1.535	0.963	1.499
	2	0.610	0.991	0.947	1.583	0.891	1.572
	3	0.662	0.975	0.989	1.559	0.926	1.534

4. CONCLUSION

In this paper, we have considered exponentiated exponential progressive type-II censored data under SSPALT. Maximum likelihood estimators have been estimated numerically using Newton-Raphson method. From the above observed results, it can be seen that the biases and MSEs of the parameters are decreasing with increasing sample size, n, and the confidence intervals become narrower with the increasing values of n. The performance of the testing plans and model assumptions are usually evaluated by the properties of the maximum likelihood estimates of model parameters. Hence from the numerical results we can conclude that estimates of the acceleration factor and parameters are more suitable and stable with relatively small biases and MSEs with increasing sample size. Therefore the assumptions made by us are fulfilled and test design is robust.

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