PRICING AND ORDERING POLICY FOR DETERIORATING INVENTORY SYSTEM WITH PRICE SENSITIVE RAMPTYPE DEMAND, PRE AND POST DETERIORATION DISCOUNTS

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ABSTRACT
A trend of offering discounts on selling price is very casual nowadays. It is consider as promotional tools to boost the demand. Retailer dealing with non-instantaneous deteriorating items offers different price discounts to his customers, before and after deterioration starts. In this article, a policy is presented for non-instantaneous deteriorating items with price sensitive ramp type demand pattern. Pre deterioration discount is considered to be smaller than the post deterioration discount as per trend. Mathematical formulation is supported with a numerical examples and sensitivity with respect to different parameters is discussed.

KEYWORDS: Non-instantaneous Deterioration, Discounts on Selling Price, Price dependent ramp type demand.

RESUMEN
Una tendencia a ofrecer descuentos en el precio de venta es muy casual hoy en día. Se consideran como herramientas promocionales para impulsar la demanda. El comerciante que se ocupa de artículos deteriorados no instantáneos ofrece diferentes descuentos de precio a sus clientes, antes y después de que comience el deterioro. En este artículo, se presenta una política para artículos deteriorados no instantáneos con un patrón de demanda de rampa sensible al precio. El descuento por deterioro previo se considera más pequeño que el descuento por deterioro posterior a la tendencia. La formulación matemática se apoya con ejemplos numéricos y se discute la sensibilidad con respecto a diferentes parámetros.

PALABRAS CLAVE: Deteriroro No-instántaneo, Descuento en el precio de venta, precio dependiente del tipo de demanda de rampa

MSC: 90B05

1. INTRODUCTION

Deterioration is a process that prevents original usage of an item or degrades its quality. Deterioration may be observed as decay, dryness, evaporation, degradation, spoilage etc. Ghare and Schrader (1963) was the first to establish an inventory system with exponentially decayed items and deterministic demand. Ardalan (1994) presented temporary price discounts to design ordering policy. Wee and Yu (1997) developed temporary price discount models for items deteriorating exponentially. Nahmias (1982), Rafaat (1991), Shah and Shah (2000), Goyal and Giri (2001), Bakker et al. (2012) presented a rigorous survey on deteriorating inventory systems. However, most of researches assumed that the deterioration takes place as soon as the replenishment of an item in inventory system. In reality, most commodities maintain its quality or original conditions over a span. Normally, it is observed that food stuffs, firsthand vegetables and fruits have a short span of maintaining fresh quality, in which there is almost no spoilage. Whereas in items like volatile liquids, radio-active chemicals, trendy goods, electronic goods have more span of marinating their quality or freshness. Wu et al. (2006) and Ouyang et al. (2006) were first to incorporate this phenomenon in to the inventory system and they termed it as "non-instantaneous deterioration". Many researchers like, Ouyang et al. (2008), Wu et al. (2009), Jaggi and

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Verma (2010), Soni and Patel (2012), Shah et al. (2013), Dye (2013) and Wang et al. (2015) have discussed non-instantaneous deterioration in their study.
For the first time, Hill (1995) formulated an inventory model with ramp type demand rate. In case of ramp type demand rate, the rate of demand increases linearly at the beginning, then it goes constant until the end of replenishment cycle. Such demand pattern is mostly observed in new brand consumer goods which are likely to be introduced in the market. Most recent articles for ramp type demand are San-jose et al. (2017), Sharma et al. (2017), Chandra (2017) etc.
Here, pricing and ordering policy is discussed when retailer offers pre and post deterioration discounts in selling price. It is consider that retailer deals with an item having price sensitive and ramp type demand; shortages are not allowed; deterioration is non-instantaneous. Moreover, retailer offers a pre and post deterioration discounts on selling price to boost demand of an item. By examining the inventory system, an algorithm is proposed to define optimal pricing and ordering policy with aforesaid hypothesis. The article is outlined section wise. Section 2 deals with assumption and notations under consideration; Section 3 presents mathematical formulation of an inventory system. In support of this formulation, numerical examples are presented in Section 4, along with special cases. Sensitivity analysis is presented in Section 5 along with managerial insights. The learning is concluded in section 6.

2. ASSUMPTION AND NOTATIONS

The following assumptions and notations are used to formulate proposed inventory system mathematically.

1. The inventory system under consideration deals with a single item. Replenishment rate is infinite and the lead time is zero or negligible. The length of planning horizon is infinite. Model does not possess shortages.
2. The function $I(t)$ represents level of an inventory at any instant of time t, $0 \le t \le T$ where T is cycle time.
3. The demand $R(P,t) = \begin{cases} a(1+b_1 t)P^{-\eta} & ; \quad 0 \le t \le \lambda \\ a(1+b_1\lambda)P^{-\eta} & ; \quad \lambda \le t \le T \end{cases}$, where $a > 0$ is scale of parameter of demand and $0 < b_1 < 1$ is rate of change of demand,
4. During the ordering cycle t_d is the time up to which the product has no deterioration. Thereafter, the product deteriorates with a constant rate $'\theta'$; $0 < \theta < 1$. Moreover, it is assumed that, deteriorated product is neither repaired nor replenished during the ordering cycle.
5. EOQ Q, is an initial level of stock to the inventory system.
6. We consider C is the purchase cost / unit; P is the selling price / unit; h is the holding cost / unit / time unit; A is an ordering cost per order; $Z(P,T)$ is an average profit of an inventory system per time unit.
7. To boost demand of an item, retailer offers a price discount to his customer. Here, we plot a general trend to offer different price discounts before and after the effect of deterioration. $\alpha_1 = (1-r_1)^{-\eta}$ is an effect of pre-deterioration discount over selling price on demand and $\alpha_2 = (1-r_2)^{-\eta}$ is an effect of post-deterioration discount over selling price on demand, is the effect of discounted selling price. r_1 is the pre-deterioration and r_2 is the post-deterioration discount rates over selling price P. As per trend $r_1 < r_2$.

3. MATHEMATICAL FORMULATION

Here it is assumed that retailer deals with non-instantaneous deteriorating items and offers different price discounts for fresh and deteriorated product. Under this assumption the differential equation associated with an inventory system can be derived as (1)

$$\frac{dI(t)}{dt} = \begin{cases} -R(P,t)(1-r_1)^{-\eta} & ; \quad 0 \le t \le t_d \\ -R(P,t)(1-r_2)^{-\eta} - \theta I(t) & ; \quad t_d \le t \le T \end{cases} \tag{1}$$

The deterioration starts at time t_d and demand of an item stabilize at time λ. Depending upon length of t_d and λ, two cases arises. **Case 1**: $0 \le t_d \le \lambda \le T$ **Case 2**: $0 \le \lambda \le t_d \le T$

Case 1: $0 \le t_d \le \lambda \le T$

$$\frac{dI(t)}{dt} = \begin{cases} -a(1+b_1 t)P^{-\eta}(1-r_1)^{-\eta} & ; \quad 0 \le t \le t_d \\ -a(1+b_1 t)P^{-\eta}(1-r_2)^{-\eta} - \theta I(t) & ; \quad t_d \le t \le \lambda \\ -a(1+b_1 \lambda)P^{-\eta}(1-r_2)^{-\eta} - \theta I(t) & ; \quad \lambda \le t \le T \end{cases} \tag{2}$$

Using boundary condition $I(T)=0$, continuity at $t=\lambda$ and $t=t_d$

$$I(t) = \begin{cases} I_A(t) & ; \quad 0 \le t \le t_d \\ I_B(t) & ; \quad t_d \le t \le \lambda \\ I_C(t) & ; \quad \lambda \le t \le T \end{cases} \tag{3}$$

Where,

$$I_{1A}(t) = \frac{1}{2}aP^{-\eta}(1-r_1)^{-\eta}(t_d - t)\left(2+b(t_d+t)\right) + \frac{1}{\theta^2}aP^{-\eta}(1-r_2)^{-\eta}\left(b\left(1-e^{\theta(\lambda-t_d)}\right) - \theta(bt_d+1) + \theta(b\lambda+1)e^{\theta(T-t_d)}\right)$$

$$I_{1B}(t) = \frac{1}{\theta^2}aP^{-\eta}(1-r_2)^{-\eta}\left(b\left(1-e^{\theta(\lambda-t)}\right) + \theta\left(e^{\theta(T-t)}(b\lambda+1) - (bt+1)\right)\right)$$

$$I_{1C}(t) = \frac{1}{\theta}aP^{-\eta}(1-r_2)^{-\eta}(b\lambda+1)\left(e^{\theta(T-t)}-1\right)$$

$$Q_1 = I(0) = I_{1A}(0) = \frac{1}{2}aP^{-\eta}(1-r_1)^{-\eta}t_d(2+bt_d) + \frac{1}{\theta^2}aP^{-\eta}(1-r_2)^{-\eta}\left(b\left(1-e^{\theta(\lambda-t_d)}\right) - \theta(bt_d+1) + \theta(b\lambda+1)e^{\theta(T-t_d)}\right) \tag{4}$$

The profit function for the inventory system consists following cost components:

1. Ordering cost OC: $OC = A$
2. Purchase cost PC: $PC_1 = CQ_1$
3. Inventory holding cost IHC_1: $IHC_1 = h\left(\int_0^{t_d} I_{1A}(t)dt + \int_{t_d}^{\lambda} I_{1B}(t)dt + \int_{\lambda}^{T} I_{1C}(t)dt\right)$
4. Sales revenue SR: $SR_1 = P(1-r_1)\left(\int_0^{t_d} R(P,t)dt\right) + P(1-r_2)\left(\int_{t_d}^{\lambda} R(P,t)dt + \int_{\lambda}^{T} R(P,t)dt\right)$

Total average profit per time unit is $Z_1(P,T) = \dfrac{SR_1 - OC - PC_1 - IHC_1}{T}$ (5)

Case 2: $0 \le \lambda \le t_d \le T$

$$\frac{dI(t)}{dt} = \begin{cases} -a(1+b_1 t)P^{-\eta}(1-r_1)^{-\eta} & ; \quad 0 \le t \le \lambda \\ -a(1+b_1 \lambda)P^{-\eta}(1-r_1)^{-\eta} & ; \quad \lambda \le t \le t_d \\ -a(1+b_1 \lambda)P^{-\eta}(1-r_2)^{-\eta} - \theta I(t) & ; \quad t_d \le t \le T \end{cases} \tag{6}$$

Using $I(T)=0$, continuity at $t=\lambda$ and $t=t_d$

$$I(t) = \begin{cases} I_A(t) & ; \quad 0 \le t \le \lambda \\ I_B(t) & ; \quad \lambda \le t \le t_d \\ I_C(t) & ; \quad t_d \le t \le T \end{cases} \tag{7}$$

Where,

$$I_{2A}(t) = aP^{-\eta}\left(\frac{1}{2}(1-r_1)^{-\eta}(t_d - t)(2+t_d+t) + \frac{1}{\theta}(1-r_2)^{-\eta}\left(e^{\theta(T-t_d)}-1\right)(b\lambda+1)\right)$$

$$I_{2B}(t)=\frac{1}{\theta}aP^{-\eta}(1+b\lambda)\left((1-r_1)^{-\eta}(t_d-t)\theta+(1-r_2)^{-\eta}\left(e^{\theta(T-t_d)}-1\right)\right)$$

$$I_{2C}(t)=\frac{1}{\theta}aP^{-\eta}(1-r_2)^{-\eta}(b\lambda+1)\left(e^{\theta(T-t)}-1\right)$$

$$Q_2=I(0)=I_{2A}(0)=aP^{-\eta}\left(\frac{1}{2}(1-r_1)^{-\eta}t_d(2+t_d)+\frac{1}{\theta}(1-r_2)^{-\eta}\left(e^{\theta(T-t_d)}-1\right)(b\lambda+1)\right) \tag{8}$$

As similar to Case 1 total profit function consists:

1. Ordering cost OC : $OC=A$
2. Purchase cost PC : $PC_2=CQ_2$
3. Inventory holding cost IHC : $IHC_2=h\left(\int_0^{\lambda}I_{2A}(t)dt+\int_{\lambda}^{t_d}I_{2B}(t)dt+\int_{t_d}^{T}I_{2C}(t)dt\right)$
4. Sales revenue SR : $SR_2=P(1-r_1)\left(\int_0^{\lambda}R(P,t)dt\int_{\lambda}^{t_d}R(P,t)dt\right)+P(1-r_2)\left(\int_{t_d}^{T}R(P,t)dt\right)$

Total average profit per time unit is $Z_2(P,T)=\dfrac{SR_2-OC-PC_2-IHC_2}{T}$ (9)

From (5) and (9) the total average profit per time unit can be defined as follows:

$$Z(P,T)=\begin{cases}Z_1(P,T) & ; \quad 0\le t_d\le\lambda\\ Z_2(P,T) & ; \quad \lambda\le t_d\le T\end{cases} \tag{10}$$

To find optimal values of T and P, we use fundamental concepts of calculus. Consider equation,

$$\frac{\partial Z_i(P,T)}{\partial P}=0,\ i=1,2 \tag{11}$$

$$\frac{\partial Z_i(P,T)}{\partial T}=0,\ i=1,2 \tag{12}$$

By solving (11) and (12), find values of T and P, then use computational algorithm as under:

Computational algorithm

Step 1: Assign values to the parameters in proper units.

Step 2: If $0\le t_d\le\lambda$, then solve $\dfrac{\partial Z_1(P,T)}{\partial P}=0$ and $\dfrac{\partial Z_1(P,T)}{\partial T}=0$ to obtain T and P provided $\dfrac{\partial^2 Z_1}{\partial P^2}<0$ and $\dfrac{\partial^2 Z_1}{\partial P^2}\dfrac{\partial^2 Z_1}{\partial T^2}-\dfrac{\partial^2 Z_1}{\partial P\partial T}>0$. Find optimal values of Q and Z using equations (4) and (5) respectively.

Step 3: If $\lambda\le t_d\le T$, then solve $\dfrac{\partial Z_2(P,T)}{\partial P}=0$ and $\dfrac{\partial Z_2(P,T)}{\partial T}=0$ to obtain T and P provided $\dfrac{\partial^2 Z_2}{\partial P^2}<0$ and $\dfrac{\partial^2 Z_2}{\partial P^2}\dfrac{\partial^2 Z_2}{\partial T^2}-\dfrac{\partial^2 Z_2}{\partial P\partial T}>0$. Find optimal values of Q and Z using equations (8) and (9) respectively.

To illustrate, computational algorithm, examples were presented in section 4

4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

Example 1 (Case 1)

We consider values to the parameter in proper units. For $t_d=0.3$ years, $r_1=10\%$/year, $r_2=25\%$/year, $\theta=0.2$, $\eta=1.25$, $a=10000$ units, $b_1=0.2$, $A=\$1000$, $C=\$10$/unit, $h=\$2$/unit/time unit, $\lambda=0.8$ years. Solving $\dfrac{\partial Z_1(P,T)}{\partial P}=0$ and $\dfrac{\partial Z_1(P,T)}{\partial T}=0$ optimal values $T=2.7763$ years and $P=\$143.75$. Using these optimal values, $Q = 113.34$ units and $Z = \$1717.77$. Here, $\frac{\partial^2 Z_1}{\partial P^2}$ = -81.31<0 and $\frac{\partial^2 Z_1}{\partial P^2}\frac{\partial^2 Z_1}{\partial T^2} - \frac{\partial^2 Z_1}{\partial P \partial T} = 1.72 > 0$. Concavity shown in Figure 1 validates optimality of profit function. Figure 2 and Figure 3 presents variation of Z with respect to T and P respectively.

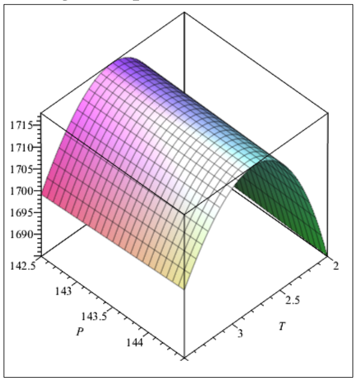

Figure 1 Profit/time unit

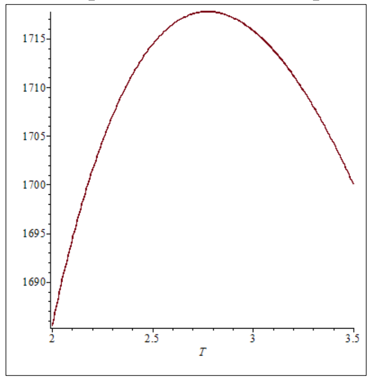

Figure 2 $T \rightarrow Z$ for fixed P

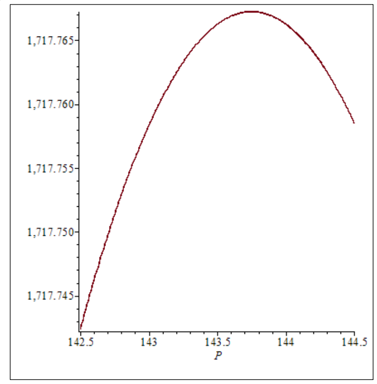

Figure 3 $P \rightarrow Z$ for fixed T

Example 2 (Case 2)

We consider values to the parameter in proper units. For $t_d = 0.3$ years, $r_1 = 10\%$/year, $r_2 = 25\%$/year, $\theta = 0.2$, $\eta = 1.25$, $a = 10000$ units, $b_1 = 0.2$, $A = \$1000$, $C = \$10$/unit, $h = \$2$/unit/time unit, $\lambda = 0.2$ years. Solving $\frac{\partial Z_2(P,T)}{\partial P} = 0$ and $\frac{\partial Z_2(P,T)}{\partial T} = 0$ optimal values $T = 3.3295$ years and $P = \$163.47$. Using these optimal values, $Q = 112.19$ units and $Z = \$1473.79$. Here, $\frac{\partial^2 Z_2}{\partial P^2}$ = -0.020<0 and $\frac{\partial^2 Z_2}{\partial P^2}\frac{\partial^2 Z_2}{\partial T^2} - \frac{\partial^2 Z_2}{\partial P \partial T} = 0.81 > 0$. Concavity shown in Figure 4 validates optimality of profit function. Figure 5 and Figure 6 presents Z with respect to T and P respectively.

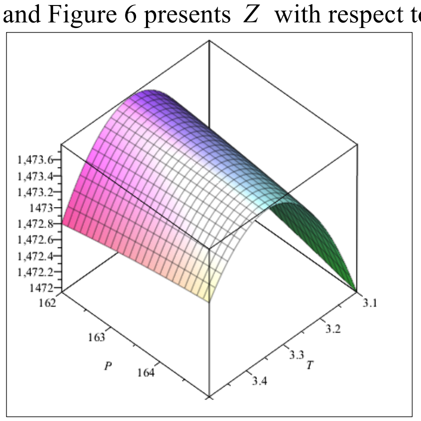

Figure 4 Profit/time unit

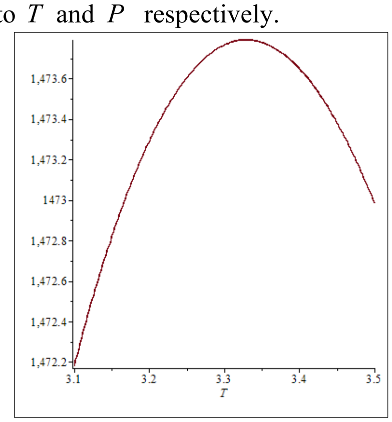

Figure 5 $T \rightarrow Z$ for fixed P

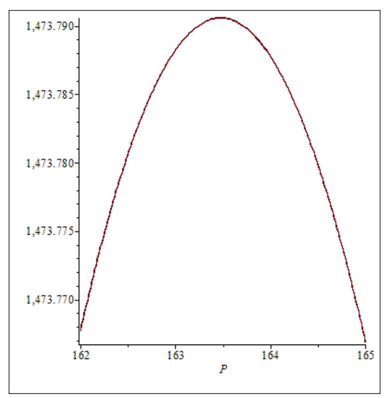

Figure 6 $P \rightarrow Z$ for fixed T

5. SENSITIVITY ANALYSIS

Sensitivity with respect to different parameters is studied as under:

Table I: Variation with respect to t_d

t_d	T	P	Z	Q
0.25	3.4053	167.79	1463.22	113.36
0.30	3.3296	163.48	1473.79	112.19
0.35	3.2530	159.20	1484.61	111.01

Table II: Variation with respect to r_1

r_1	T	P	Z	Q
0.05	3.1958	158.06	1488.32	110.17
0.10	3.3296	163.48	1473.79	112.19
0.15	3.4602	168.87	1459.83	114.10

Table III: Variation with respect to r_2

r_2	T	P	Z	Q
0.20	3.2576	140.98	1644.34	121.40
0.25	3.3296	163.48	1473.79	112.19
0.30	3.4229	192.17	1308.98	103.39

Table IV: Variation with respect to θ

θ	T	P	Z	Q
0.1	3.6283	159.13	1508.11	120.04
0.2	3.3296	163.48	1473.79	112.19
0.3	3.0915	167.58	1471.30	98.24

Table V: Variation with respect to a

a	T	P	Z	Q
9000	3.5797	171.10	1297.46	105.57
10000	3.3296	163.48	1473.79	112.19
11000	3.1152	157.12	1652.21	118.35

Table VI: Variation with respect to b_1

b_1	T	P	Z	Q
0.15	3.3460	163.93	1457.42	111.50
0.20	3.3296	163.48	1473.79	112.19
0.25	3.3134	163.03	1490.17	112.88

Table VII: Variation with respect to A

A	T	P	Z	Q
900	3.0932	156.48	1504.93	107.11
1000	3.3296	163.48	1473.79	112.19
1100	3.5552	170.35	1444.74	116.81

Table VIII: Variation with respect to C

C	T	P	Z	Q
8	3.1122	132.84	1549.43	132.54
10	3.3296	163.48	1473.79	112.19
12	3.5082	194.28	1412.73	97.26

Table IX: Variation with respect to h

h	T	P	Z	Q

1	3.5822	149.61	1532.63	138.86
2	3.3296	163.48	1473.79	112.19
3	3.1630	176.93	1424.74	94.70

Here, values of C, P, h, A, Z are in \$, t_d, λ, T are in years, θ, b_1, r_1, r_2 are rates of variations.

Managerial Insights:

The following managerial issues are observed.

(1) From Table I, one may observe that increase in delay period t_d decreases the length of ordering cycle and reduces the selling price per unit and suggests keeping a stock little bit less. Over all, at the end of the inventory cycle the average profit of an inventory system increases.

(2) From Table II, it is observed that increase in pre deterioration discount rates r_1 suggests increase in selling price and ordering quantity, hence the length of ordering cycle increases too. Moreover, as pre deterioration rate increases the average profit per time unit decreases.

(3) Again, from Table III, it is observed that increase in post deterioration discount rates r_2 suggests increase in selling price and ordering quantity, hence the length of ordering cycle increases too. Moreover, as post deterioration rate increases the average profit per time unit decreases. Here, inventory system is highly sensitive over the discounts offered on selling price after deterioration starts.

(4) From Table IV, as rate of deterioration θ increases the length of an ordering cycle decreases and hence, the ordering quantity and average profit per time unit decreases. Moreover, suggested to keep selling price more to sustain with profit margin.

(5) From Table V and VI, increase in demand a and rate of variation in demand b_1 reduces the length of ordering cycle and selling price. But this results in to gain in the average profit of an inventory system with a bit higher stock level.

(6) From Table VII, increase in ordering cost A increases the length of ordering cycle and ordering quantity, but reduces the average profit per time unit. It is suggested to keep selling price a bit more to sustain with profit margin.

(7) From Table VIII, increase in purchase cost C increases length of ordering cycle, selling price and suggests ordering less which reduces the average profit per time unit.

(8) From Table IX, increase in holding cost h per unit per time unit decreases length of ordering cycle and increases selling price and suggests ordering less which reduces the average profit per time unit.

6. CONCLUSION

An optimal pricing and ordering policy is presented from the retailer's point of view, where deterioration of a product is non-instantaneous and retailer offers different price discounts on selling price before and after deterioration starts. Demand of an item is observed to be of ramp type and sensitive with selling price. Mathematical formulation is presented and illustrated with support of numerical examples. Sensitivity analysis with respect to various parameters is done and strategic options were outlined using sensitivity analysis. It is observed that offering pre and post deterioration discount rates in selling prices helps to sustain with the average profit per time unit of an inventory system. One may extend this model by assuming demand to be selling price sensitive and trapezoidal type in nature. As various parameters associated with inventory system are uncertain, one may discuss this model in fuzzy environment to deal with uncertainty.

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