ON POWER OF THE CONTROL CHART FOR ZERO TRUNCATED BINOMIAL DISTRIBUTION UNDER MISCLASSIFICATION ERROR

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ABSTRACT
In this paper a mathematical investigation has been made on the effect of misclassification due to measurement error on power of the control chart for zero truncated binomial distribution (ZTBD):.Analytical formulas are obtained for calculating probabilities of errors of misclassification due to measurement error. The connection between apparent fraction defective (AFD) and true fraction defective (TFD) has been used to study the power of control chart. Expressions of average run length (ARL) and OC curve are also obtained.

KEYWORDS: Measurement error, misclassification, zero-truncated binomial distribution, power, average run length (ARL), OC curve.

MSC: 62P30

RESUMEN
En este paper una investigación matemática se desarrolló sobre el efecto de la mala clasificación debido al error de medición en la carta de control de la distribución Binomial truncada (ZTBD). Fórmulas analíticas son obtenidas para calcular las probabilidades de error de la clasificación debida al error de medición. La conexión entre la fracción aparente de defectuosos (AFD) y la verdadera (TFD) han sido usadas para estudiar la potencia de la carta de control.. Expresiones del average del largo de la corrida (ARL) y de la curva OC curva también son obtenidas.

1. INTRODUCTION

Statistical methods now-a-days have been successfully utilized to diverse manufacturing processes in industries to achieve desired quality levels of manufactured products with an optimum production cost. It is well-known within the industrial process, processes manufactured are frequently contaminated with measurement error which can lead to serious bias in the derived results. The nature and level of measurement error and its effect on the actual performance of various control charts can be overwhelming and studied by several researchers. For an up-to-date review see Maravelakis (2012), Sankle et al. (2012), Chakraborty and Khurshid (2013 a, b, 2014) and references therein.
To employ statistical techniques, inspections are performed on the finished products, throughout the time of production or once the production is done. The purpose is to verify that the production operations were carried out properly and that the production output meets the expectations of the conformance to a given set of requirements. In every single inspection system, there may be either of two potential types of errors: (i) a good (conforming) item to a specification may be misclassified as defective (nonconforming) or (ii) a defective (nonconforming) item may be misclassified as good (conforming): These kinds of errors are categorized as misclassification errors (or inspection errors) and are generally due to chance causes and can be estimated (Sankle and Singh, 2012):

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Misclassification is a particular type of measurement error. Clearly there is no coherent theory that embodies the key elements of misclassification error which is usually studied independently from measurement error, though there is clearly much intersection. Misclassification errors may considerably change the performance of (attribute) control charts, as has been investigated by several authors, including Dorris and Foote (1978), Case (1980), Schneider and Tang (1987), Suich (1988), Johnson et al. (1991), Singh and Sayyed (2001), Singh et al. (2002), Chen et al. (2011), Balamurali and Kalyanasundaran (2011): See also Chakraborty and Khurshid (2016):

In this paper, the power of the control chart for Zero-Truncated Binomial Distribution (ZTBD) is being studied by considering approximate expressions for calculating the probabilities of errors of misclassification due to measurement error. The relationship between apparent fraction defective (AFD) and true fraction defective (TFD) has been used to study the power of control chart.

The rest of paper is organized as follows: Section 2 delineates the ZTBD along with its mean and variance. In section 3 we state definitions, assumptions and notation used in this paper. Section 4 explains how to evaluate probabilities of misclassification. Eexpressions for the power of control chart for ZTBD under misclassification due to measurement error and OC curve are developed in Section 5. To study the sensitivity of the monitoring procedure the power of control chart in terms of ARL is explained in Section 6 with an example. Finally some results and its discussion and some conclusions are presented in Sections 7 and 8, respectively.

2. THE ZERO-TRUNCATED BINOMIAL DISTRIBUTION (ZTBD)

The zero-truncated binomial distribution is a discrete distribution with probability mass function (pmf) given as follows

$$P(X=x)=\begin{cases}\dfrac{\binom{n}{x}p^x q^{n-x}}{1-q^n}, & x=1,2,\ldots,n;0<p<1,q=1-p\\ 0, & o.w.\end{cases} \tag{2.1}$$

The mean, the variance, and standard deviation for this distribution are (Johnson et al., 2005)

$$\mu=E(X)=\frac{np}{1-q^n}, \tag{2.2}$$

$$\sigma^2=Var(X)=\frac{1}{1-q^n}\left[npq+n^2p^2-\frac{n^2p^2}{1-q^n}\right], \tag{2.3}$$

and

$$\sigma=SD(X)=\sqrt{\frac{1}{1-q^n}\left[npq+n^2p^2-\frac{n^2p^2}{1-q^n}\right]}. \tag{2.4}$$

3. DEFINITIONS, ASSUMPTIONS AND NOTATION

In this section we state definitions, assumptions and notation used in this paper.

3.1. Assumptions

It is assumed that the measurements have been taken only to classify the production items into acceptable and rejectable units with certain specifications that can be expressed in terms of mean and standard deviation of the measurable quality characteristics.

The quality characteristic x is normally distributed with mean μ and standard deviation σ_p; thus

$$f(x) = \frac{1}{\sigma_p \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma_p}\right)^2 \right]. \tag{3.1}$$

The variable v denoting measurement error, is also normal with mean x and standard deviation σ_e

$$f(v) = \frac{1}{\sigma_e \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{v-x}{\sigma_e}\right)^2 \right]. \tag{3.2}$$

The units beyond $x = \mu \pm K\sigma_p$ are defective and the units within $x = \mu \pm K\sigma_p$ are non-defective where K is a constant for Shewhart control chart.

3.2. Notations

(i) TFD (true fraction defective) is the proportion of defective items when there is no error of misclassification and is denoted by P;
(ii) AFD (apparent fraction defective) is the proportion of defective items if error of misclassification is present is denoted by π.
If the misclassification error is zero then $AFD = TFD$.

4. EVALUATING THE PROBABILITIES OF MISCLASSIFICATION

Here we have classified the production process, after measurement into one of the two categories. They are either conforming (good) or non-conforming (defective) units. If P_1 is the probability of misclassification of a conforming unit and P_2 is the probability of misclassification of a non-conforming unit, then following Singh (1964) with the above mentioned assumptions (Section 3), P_1 and P_2 can be evaluated as follows:

$$P_1 = \int_{-K\sigma_p}^{K\sigma_p} f(x)\,dx\ [1 - \int_{-K\sigma_p}^{K\sigma_p} f(v)\,dv] \tag{4.1}$$

and

$$P_2 = \int_{K\sigma_p}^{\infty} f(x)\,dx \int_{-K\sigma_p}^{K\sigma_p} f(v)\,dv + \int_{-\infty}^{-K\sigma_p} f(x)\,dx \int_{-K\sigma_p}^{K\sigma_p} f(v)\,dv. \tag{4.2}$$

In fact P_1 and P_2 are the inspection risks, which are the type I and type II errors and take the values between 0 and 1.
The approximate expressions for P_1 and P_2 (Singh, 1964) are:

$$P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\} \tag{4.3}$$

and

$$P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\} \tag{4.4}$$

where $a = \sigma_e / \sigma_p$, $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}v^2\right] dv$, $h = \frac{K}{\sqrt{\left(\frac{\sigma_e}{\sigma_p}\right)^2 + 1}}$ and

$$T(h,a)=\frac{1}{\sqrt{2\pi}}\int_0^a \frac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2}\,dx .$$

Thus, P_1 (type I error) is the proportion of conforming units classified as a non-conforming ones and P_2 (type II error) is the proportion of non-conforming units classified as a conforming ones.

Our initial calculations (not furnished in this paper) show that $1.5 \le K \le 3$ and $\left(\sigma_e/\sigma_p\right) \le 0.5$ hold good for finding P_1 and P_2. Now if P denotes the incoming (true) fraction defective of the lots, then the expression of AFD (apparent fraction defective), following Lavin (1946) is denoted by π and is given by

$$\pi = P(1-P_2) + P_1(1-P). \tag{4.5}$$

For published material based on Lavin equation, see Collins et al. (1973), Collins and Case (1976), Johnson et al. (1991), Mittag and Rinne (1993) and Govindaraju and Jones (2015):

5. THE POWER OF THE CONTROL CHART FOR THE ZTBD

The data can often represented by a ZTBD if it consists at least a number or proportion of units having a specific attributes. In this section, we have developed the expression for the power of control chart for ZTBD under misclassification due to measurement error.

Kanazuka (1986) has proved that the power of detecting the change of process for the control chart, P_d can be found by

$$P_d = P\left\{X \ge UCL\right\} + P\left\{X \le LCL\right\} \tag{5.1}$$

where UCL and LCL are upper and lower control limits respectively.
Thus, under misclassification the Shewhart's control limits for ZTBD are

$$\pi\left[1-(1-\pi)^n\right]^{-1} \pm K\left[(n^{-2})\left\{1-(1-\pi)^n\right\}^{-1}\left\{n\pi(1-\pi)+n^2\pi^2-\frac{n^2\pi^2}{1-(1-\pi)^n}\right\}\right]^{1/2} \tag{5.2}$$

Normally we choose $K=3$ as it will give no false alarm with probability of at least 99.73%. Hence, the power of the control chart under misclassification error is

$$P_d = \left[1-\frac{\sum_{x_e=1}^{UCL-1}\binom{n}{x_e}\pi^{x_e}(1-\pi)^{n-x_e}}{\left\{1-(1-\pi)^n\right\}}\right] + \frac{\sum_{x_e=1}^{LCL}\binom{n}{x_e}\pi^{x_e}(1-\pi)^{n-x_e}}{\left\{1-(1-\pi)^n\right\}} \tag{5.3}$$

where x_e is the number of apparent fraction defectives observed by the inspector.

The operating characteristic (OC) curve, under misclassification, depicts the probability that a sample fraction defective x_e/n, will fall within control limits as a function of the error process fraction defective π is given by (Kanazuka, 1986)

$$P_e(\pi) = \sum_{x_e=LCL}^{UCL}\binom{n}{x_e}\pi^{x_e}(1-\pi)^{n-x_e}. \tag{5.4}$$

6. THE AVERAGE RUN LENGTH (ARL) FOR THE ZTBD UNDER MISCLASSIFICATION ERROR

To study the sensitivity of the monitoring procedure, one can also study the ARL which is the average number of points that must be plotted before a point indicates an out of control condition when operating is statistical control. Thus under misclassification error, the ARL is $ARL = [P_d]^{-1}$ where P_d is the probability that a single point exceeds the control limits.

In this case, one can interpret the results of the power of the control chart in terms of ARL just by reversing equation (5.3), rather than drawing conclusions based on P_d.

Example and Illustration

Consider the data for fraction defective, where 4 samples, each of size 15 were inspected and number of defectives along with proportions of defectives are obtained as follows:

Table 1: A hypothetical example

Sample #	Number of defects (d_i)	Fraction defectives $p_i = (d_i/n)$
1	1	0.07
2	4	0.27
3	2	0.13
4	5	0.33

Here overall sample proportion of defectives is $p = \overline{p} = 0.2$. For our analysis we have kept $\hat{p} = \overline{p} = \dfrac{\sum_{i=1}^{4} p_i}{4} = 0.2$, the overall sample proportion of defective fixed and the values of n being changed in different situations to see the effect of the sample size of on the power of the control chart.

7. RESULTS AND DISCUSSION

This section provides results of our computation. To obtain the values of the power of the control chart (P_d) under misclassification error, we first obtain $\pi = P(1-P_2) + P_1(1-P)$ based on the approximate expressions P_1 and P_2 (Equations 4.3 and 4.4):

Table 2: Values of $T(h,a) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^a \dfrac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2}dx$, $\Phi(h)$, $P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\}$ and $P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\}$

Table 2-A: $K = 1.5$ and $\Phi(K) = 0.9332$

$a = \sigma_e/\sigma_p$	$h = \dfrac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.5	1.34	0.07039360	0.9099	0.16408720	0.11748720
0.4	1.39	0.05503907	0.9177	0.12557814	0.09457814
0.3	1.44	0.04001047	0.9251	0.08812094	0.07192094
0.25	1.46	0.03294319	0.9279	0.07118638	0.06058638
0.20	1.47	0.02635462	0.9292	0.05670924	0.04870924
0.15	1.48	0.01970635	0.9306	0.04201270	0.03681270
0.10	1.49	0.01305494	0.9319	0.02740988	0.02480988
0.05	1.50	0.00646451	0.9332	0.01292902	0.01292902

Table 2-B: $K = 1.75$ and $\Phi(K) = 0.9599$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.5	1.57	0.04915861	0.9418	0.11641722	0.08021722
0.4	1.63	0.03763664	0.9484	0.08677328	0.06377328
0.3	1.68	0.02722368	0.9535	0.06084736	0.04804736
0.25	1.70	0.02237561	0.9554	0.04925122	0.04025122
0.20	1.72	0.01759671	0.9573	0.03779342	0.03259342
0.15	1.73	0.01315372	0.9582	0.02800744	0.02460744
0.10	1.74	0.008706727	0.9591	0.018213454	0.016613454
0.05	1.75	0.004304809	0.9599	0.008609618	0.008609618

Table 2-C: $K = 2.0$ and $\Phi(K) = 0.9772$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	1.79	0.03308721	0.9633	0.08007442	0.05227442
0.40	1.86	0.02471443	0.9686	0.05802886	0.04082886
0.30	1.92	0.01745997	0.9726	0.03951994	0.03031994
0.25	1.94	0.01433163	0.9738	0.03206326	0.02526326
0.20	1.96	0.01125022	0.9750	0.02470044	0.02030044
0.15	1.98	0.008244447	0.9761	0.017588894	0.015388894
0.10	1.99	0.00545368	0.9767	0.01140736	0.01040736
0.05	2.00	0.002692772	0.9772	0.005385544	0.005385544

Table 2-D: $K = 2.25$ and $\Phi(K) = 0.9878$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.02	0.020711060	0.9783	0.050922120	0.031922120
0.40	2.09	0.015359960	0.9817	0.036819920	0.024619920
0.30	2.16	0.010555770	0.9846	0.024311540	0.017911540
0.25	2.18	0.008656291	0.9854	0.019712582	0.014912582
0.20	2.20	0.006785243	0.9861	0.015270486	0.011870486
0.15	2.23	0.004852139	0.9871	0.010404278	0.009004278
0.10	2.24	0.003208399	0.9875	0.006716798	0.006116798
0.05	2.25	0.001582424	0.9878	0.003164848	0.003164848

Table 2-E: $K = 2.50$ and $\Phi(K) = 0.9938$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.24	0.012561410	0.9875	0.031422820	0.018822820
0.40	2.33	0.008819303	0.9901	0.021338606	0.013938606
0.30	2.40	0.006015980	0.9918	0.014031960	0.010031960
0.25	2.43	0.004810231	0.9925	0.010920462	0.008320462
0.20	2.45	0.003766157	0.9929	0.008432314	0.006632314
0.15	2.48	0.002681432	0.9934	0.005762864	0.004962864
0.10	2.49	0.001772875	0.9936	0.003745750	0.003345750
0.05	2.50	0.008734041	0.9938	0.001746808	0.001746808

Table 2-F: $K = 2.75$ and $\Phi(K) = 0.9970$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.47	0.0070560530	0.9932	0.017912106	0.010312106
0.40	2.56	0.0049003870	0.9948	0.012000774	0.007600774
0.30	2.64	0.0032323650	0.9959	0.007564730	0.005364730
0.25	2.67	0.0025776230	0.9962	0.005955246	0.004355246
0.20	2.70	0.0019622790	0.9965	0.004424558	0.003424558
0.15	2.72	0.0014301680	0.9967	0.003160336	0.002560336

0.10	2.74	0.0009200657	0.9969	0.001940131	0.001740131
0.05	2.75	0.0004528698	0.9970	0.000905740	0.000905740

Table 2-G: $K = 3.00$ and $\Phi(K) = 0.9987$

$a = \sigma_e / \sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	2.69	0.003860856	0.9964	0.010021712	0.005421712
0.40	2.79	0.002578258	0.9974	0.006456516	0.003856516
0.30	2.88	0.001637422	0.9980	0.003974844	0.002574844
0.25	2.91	0.001302675	0.9982	0.003105350	0.002105350
0.20	2.94	0.000988815	0.9984	0.002277630	0.001677630
0.15	2.97	0.000698636	0.9985	0.001597271	0.001197271
0.10	2.99	0.000448474	0.9986	0.000996947	0.000796947
0.05	3.00	0.000220578	0.9987	0.000441156	0.000441156

Note: The function $T(h,a)$ has been tabulated by Owen (1956) and Smirnov and Bolsev (1962): Interested readers may obtain a simple QBASIC program from the first author.

Table 2 provides the values of P_1 and P_2 for different combinations of $T(h,a)$ and $\Phi(h)$ for a fixed K. It has been observed from Table 2 (A-G) that for a fixed K, the values of P_1 and P_2 show a decreasing trend if the measurement error $a = \sigma_e / \sigma_p$ decreases. Also observe that for any given $a = \sigma_e / \sigma_p$ the values of P_1 are greater than P_2 when $h \cong K$ then $P_1 = P_2$.

Table 3: Relationship between $TFD\,(=P)$ and $AFD\,(=\pi)$ for different values of $a = \sigma_e / \sigma_p$ for fixed K

$K = 2.0$

P	π	π	π
	$a = 0.5$	$a = 0.10$	$a = 0.15$
0	0.005385	0.011407	0.017589
0.01	0.015280	0.021119	0.027259
0.02	0.025170	0.030971	0.036929
0.03	0.035116	0.040753	0.046600
0.04	0.044955	0.050535	0.056270
0.05	0.054846	0.060317	0.065940

Table 3 shows the relationship between true fraction defective (TFD) and apparent fraction defective (AFD). It is observed that for a fixed K and $a = \sigma_e / \sigma_p$ as the values of the true fraction defective (P) increase, the values of π i.e., apparent (observed) fraction defective also increase. For fixed P, as the values of measurement error $a = \sigma_e / \sigma_p$ increase, there is considerable increase in the values of π. Complete representation of the Table 2 is shown in Figure 1.

Table 4 depicts the effect of K on probabilities of misclassification of conforming units (P_1) and non-conforming units (P_2): For fixed $a = \sigma_e / \sigma_p$, if we increase the value of K, there is a decreasing trend for P_1 but for fixed K, the values of P_1 increase as $a = \sigma_e / \sigma_p$ is increased. Figure 2 (A and B) shows the graphic representation between K and probabilities of misclassification. One can also calculate P_1 and P_2 from the graphs (Figure 2) by knowing the standard deviation σ_e of measurement error (which assumes same for all the values of K) as given in Singh (1964).

Figure 1: Relationship between apparent fraction defective (AFD) and true fraction defective (TFD)

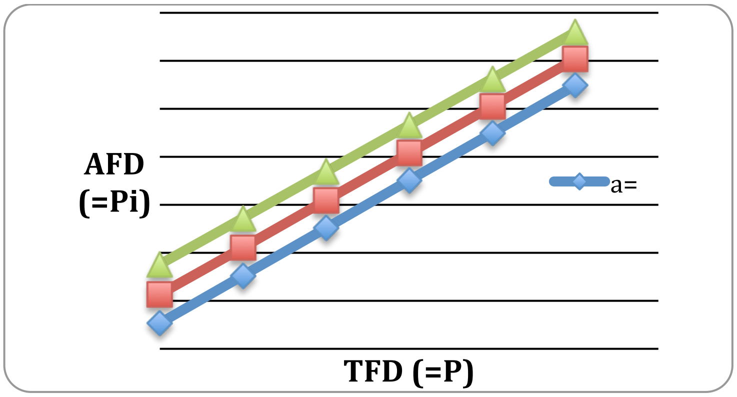


Figure 2 (A): The effect of K on probabilities of misclassification of conforming units (P_1)

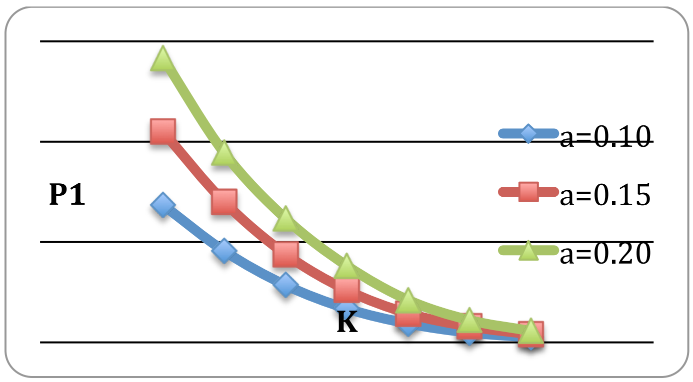


Figure 2 (B): The effect of K on probabilities of misclassification of conforming units (P_2)

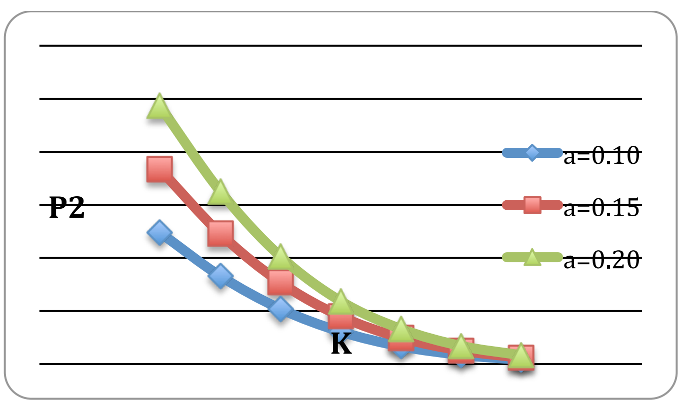


Table 4: Probabilities of misclassification of conforming units (P_1) and non conforming units (P_2) for different values of K and $a = \sigma_e / \sigma_p$.

K	$a = 0.10$		$a = 0.15$		$a = 0.20$	
	P_1	P_2	P_1	P_2	P_1	P_2
1.50	0.027409880	0.024809880	0.042012700	0.036812700	0.056709240	0.048709240
1.75	0.018213454	0.016613454	0.028007440	0.024607440	0.037793420	0.032593420

2.00	0.011407360	0.010407360	0.017588894	0.015388894	0.024700440	0.020300440
2.25	0.006716798	0.006116798	0.010404278	0.009004278	0.015270486	0.011870486
2.50	0.003745750	0.003345750	0.005762864	0.004962864	0.008432314	0.006632314
2.75	0.001940131	0.001740131	0.003160336	0.002560336	0.004424558	0.003424558
3.00	0.000996947	0.000796947	0.001597271	0.001197271	0.002277630	0.001677630

Table 5 (A-F) shows the different values of power of control chart (P_d) for the corresponding values of π. Here, we observe how the power curve (P_d) changes for different values of n, K, $a = \sigma_e / \sigma_p$, UCL and LCL. From the Table 5 (A, B, C) it is observed that values of P_d go on decreasing as we increase K (K=1.5 to K=3) for fixed $a = \sigma_e / \sigma_p$ and $P_1 = P_2$. Also no change in the values of P_d being observed if there is marginal increase in the values of $a = \sigma_e / \sigma_p$ for fixed n and fixed K.

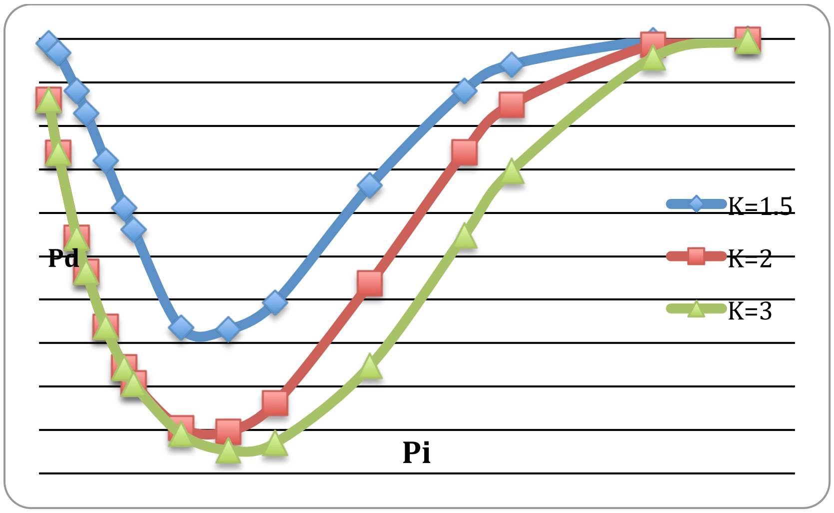
Table 5: Power of control chart (P_d) for ZTBD under misclassification (due to measurement error)

	Table: 5 A $a = \sigma_e / \sigma_p = 0.05$ $n = 15$, $K = 1.5$ $P_1 = P_2 = 0.01292902$ $UCL = 6, LCL = 1$	**Table:5 B** $a = \sigma_e / \sigma_p = 0.05$ $n = 15$, $K = 2$ $P_1 = P_2 = 0.005385544$ $UCL = 5, LCL = 0$	**Table: 5 C** $a = \sigma_e / \sigma_p = 0.05$ $n = 15$, $K = 3$ $P_1 = P_2 = 0.0004411564$ $UCL = 7, LCL = 0$
π	P_d	P_d	P_d
0.05	0.6816	0.6816	0.6814
0.10	0.4349	0.4349	0.4331
0.15	0.2717	0.2717	0.2541
0.20	0.2000	0.2000	0.1411
0.25	0.2181	0.2181	0.0852
0.30	0.3104	0.3104	0.0809
0.35	0.4490	0.4490	0.1260

	Table: 5 D $a = \sigma_e / \sigma_p = 0.05$ $n = 20$, $K = 1.5$ $P_1 = P_2 = 0.01292902$ $UCL = 7, LCL = 2$	**Table: 5 E** $a = \sigma_e / \sigma_p = 0.50$ $n = 20$, $K = 1.5$ $P_1 = 0.1640872$, $P_2 = 0.1174872$ $UCL = 9, LCL = 3$	**Table: 5 F** $a = \sigma_e / \sigma_p = 0.50$ $n = 20$, $K = 3$ $P_1 = 0.010021712$, $P_2 = 0.005421712$ $UCL = 8, LCL = 1$
π	P_d	P_d	P_d
0.05	0.8823	0.9752	0.5882
0.10	0.6349	0.8487	0.3080
0.15	0.4037	0.6349	0.1484
0.20	0.2846	0.4147	0.0909
0.25	0.3033	0.2637	0.1233
0.30	0.4270	0.2198	0.2347
0.35	0.5956	0.2818	0.4010

But if we increase the size of the sample (Table 5D) for fixed K and $P_1 = P_2$ there is a change in the values of P_d. The values of the power (P_d) is less if the size of the sample is larger for fixed $a = \sigma_e / \sigma_p$. It is also understood from the Table 5 (E and F), that the values of the power (P_d) is more, if n increased along with the value of $a = \sigma_e / \sigma_p$.

Figure 3: Relationship between π and P_d for the Table 5 (A, B, C) for different values of K.



Graphical representation for some values of P_d for the Table5 (A, B, C) is shown in Figure 3 for $K = 1.5, 2, 3$. **Table 5:** Power of control chart (P_d) for ZTBD under misclassification (due to measurement error)

Table 6 (A): Values of ARL
($n = 15$)

	π						
K	0.05	0.10	0.15	0.20	0.25	0.30	0.35
1.50	1.47	2.30	3.68	5	4.59	3.22	2.23
3.00	1.47	2.91	3.94	7.09	11.74	12.36	7.94

Table 6 (B): Values of ARL
($n = 20$)

		π						
K	a	0.05	0.10	0.15	0.20	0.25	0.30	0.35
1.50	0.05	1.13	1.58	2.48	3.51	3.29	2.34	1.68
2.00	0.50	1.03	1.18	1.58	2.41	3.79	4.55	3.55
3.00	0.50	1.70	3.25	6.74	11.0	8.11	4.26	2.49

Table 6 for fixed values of $n = 15, 20$, $K = 1.5, 3$ and $\pi = 0.05(0.05)3.5$ shows that the values of ARL decreases till $\pi = 3$, after that values of ARL increases.

8. CONCLUSIONS

This study communicates explicit formulas on the effect of misclassification due to measurement error on power of the control chart for zero truncated binomial distribution (ZTBD): Analytical formulas are achieved for calculating probabilities of errors of misclassification due to measurement error. Numerical results reveal that for a fixed K, the values of P_1 and P_2 show a decreasing trend if the measurement error $a = \sigma_e / \sigma_p$ decreases. It also shows that for a fixed K and $a = \sigma_e / \sigma_p$ as the values of the true fraction defective (P) increase, the values of π i.e., apparent (observed) fraction defective also increase. Power of control chart (P_d) for the corresponding values of π decreases as

K (K =1.5 to K =3) for fixed $a = \sigma_e / \sigma_p$ and $P_1 = P_2$ is increased. The result clearly shows that misclassification error lessens the consumer's risk, P_2 and increase the producer's risk, P_2.

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