DECISION SUPPORT MODEL WITH PRICE AND TIME INDUCED DEMAND UNDER MULTILEVEL CONDITIONAL DELIVERIES

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ABSTRACT
In this paper an inventory model for decaying goods with time and selling price induced quadratic demand is considered to determine optimal cycle time, optimal purchase quantity and minimum total cost of the inventory system. The model is developed and solved analytically by considering four stage conditional deliveries associated with different credit periods and discounting. Numerical illustrations and sensitivity analysis are provided to deduce managerial insights. Findings suggest that the inventory manager can be more benefited by making the values of holding cost, selling price and cash discount relatively higher that will reduce the total cost of the inventory system.

KEYWORDS: Replenishment, Decaying goods, Inventory, Conditional delivery.

MSC: 90B05

RESUMEN
En este documento se considera un modelo de inventario para productos en descomposición con el tiempo y la demanda cuadrática inducida por el precio de venta para determinar el tiempo de ciclo óptimo, la cantidad de compra óptima y el costo total mínimo del sistema de inventario. El modelo se desarrolla y resuelve analíticamente considerando entregas condicionales de cuatro etapas asociadas con diferentes períodos de crédito y descuentos. Se proporcionan ilustraciones numéricas y análisis de sensibilidad para deducir conocimientos gerenciales. Los hallazgos sugieren que el administrador de inventario puede beneficiarse más al hacer que los valores de costo de mantenimiento, precio de venta y descuento en efectivo sean relativamente más altos, lo que reducirá el costo total del sistema de inventario.

PALABRAS CLAVE: Re-abastecimiento, Productos Deteriorables, Inventario, Despacho Condicional

1. INTRODUCTION

The classical inventory economic order quantity model is based on the assumption that the supplier is paid immediately. However in practice to attract more sales, suppliers frequently undergo both price and time induced demand and so provides the retailers incentives like multistage conditional deliveries to motivate faster payment and stimulate sales.

Deterioration is a continuous process that decreases the effectiveness of goods when stored for a long time. So decision makers adopt various techniques for disposal of decaying items with minimum inventory cost. Managing the inventory of perishable items helps in smooth functioning of an enterprise or business organization.

Goyal (1985) first developed an inventory model under condition of permissible delay in payment. Aggarwal and Jaggi(1995)generalized Goyal's model for deteriorating items. Ouyang et al. (1999) developed an EOQ model for perishable items with partial backlogging and trade credits. Das et al. (2011) considered an inventory model with time varying demand under permissible delay in payments.

Tripathy and Pradhan (2012) developed the inventory model for three parameter weibull deterioration with permissible delay in payment associated with salvage value. Lou and Wang (2013) derived optimal trade credit and order quantity with consideration of default risk. Roy et al. (2013) introduced an economic production quantity model under permissible delay with time proportional deterioration. Guchhait et al. (2014) made the inventory policy with variable demand under trade credit.

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Teng et al. (2014) developed inventory model under two level trade credits. Sarkar et al. (2014) developed the inventory model for selling price and time dependent demand pattern. Tripathy and Pandey(2015) introduced backlogging in permissible delay period with time dependent demand. Muniappan et al. (2015) derived an EOQ model for deteriorating items with inflation and time value of money considering delay in payment. Sarkar and Saren(2015) derived trade credit policy with variable deterioration for fixed lifetime products. Tayal et al. (2016) presented an integrated production inventory model for perishable products with investment in preservation technology.

M. Rameswari and R.Uthaya Kumar (2018) developed an integrated inventory model for deteriorating items with price dependent demand under two level trade credit policy. Tripathy and Bag (2018) formulated decision support model with default risk under conditional delay. Tripathi et al. (2018) adorned their research work by using stock dependent demand under various trade credits.

In this paper a replenishment model is framed for deteriorating items with time and selling price induced quadratic demand. Four levels of conditional deliveries are considered and individual cases are developed accordingly. The objective of the study is the optimization of the cost function to reduce economic losses. The rest of the paper is developed as follows. Notations and assumptions are placed in section 2. Mathematical formulation and optimal solution are derived in section 3. Numerical illustrations for the proposed inventory model are demonstrated in section 4. Section 5 reports the variation of system parameters on changing the core parameters of the model. Finally the conclusion and future research scopes are mentioned in section 6.

Table 1 Contribution of different authors

Authors	Demand	Demand type	Number of stages	Deterioration
Goyal(1985)	Constant	Linear	2	Absent
Agarwal&Jaggi(1995)	Time dependent	Linear	2	Present
Das et al.(2011)	Time dependent	Linear	3	Absent
C.K.Tripathy&L.M.Pradhan(2012)	Constant	Constant	2	Present
Teng, Yang, Chern(2014)	Time dependent	Linear	2	Absent
Shah et al.(2014)	Time and credit period dependent	Quadratic	1	Present
S.Tayal et al.(2016)	Time dependent	Exponential	2	Present
P.K.Tripathy and S.Pradhan(2012)	Time dependent	Exponential (Ramp type)	2	Present
Muniappan et al.(2015)	Constant	Constant	4	Present
P.K.Tripathy&A.Bag(2018)	Time and credit period dependent	Quadratic	1	Present
Tripathi et al.(2018)	Stock dependent	Linear	4	Present
Present paper	Time and selling price dependent	Quadratic	4	Present

2. NOTATIONS AND ASSUMPTIONS

The notations that are used in this paper are as follows

I (t) : Inventory level at time t

p : Purchase cost per unit

s : Selling price per unit

$D(t)$: Demand rate at any instant of time

θ : Rate of deterioration, $0 < \theta < 1$

h (t) : Holding cost per item per unit time

A : Ordering cost per order

r : Rate of cash discount, $0 < r < 1$

I_c, I_d :The interest charged and earned / dollar / year respectively

M_1, M_2: Cash discount and permissible delay periods, respectively, $M_2 > M_1$

Q : Order quantity

T :Length of the cycle

$T_1^*, T_2^*, T_3^*, T_4^*$: The optimal cycle time for cases I, II, III, IV respectively.

$\emptyset_1, \emptyset_2, \emptyset_3, \emptyset_4$: The total relevant cost per year for cases I, II, III, IV respectively.

$\emptyset_1^*, \emptyset_2^*, \emptyset_3^*, \emptyset_4^*$: Optimal total relevant cost for cases I, II, III, IV respectively.

$Q_1^*, Q_2^*, Q_3^*, Q_4^*$: The optimal order quantity for cases I, II, III, IV respectively.

The assumptions used in this paper are as follow:

Demand rate of items are price and time induced: $D\{I(t)\} = -as^{-\eta}(1 + bt - ct^2)$

1. The time horizon is infinite and lead time is negligible.
2. The deterioration rate θ is time varying and $0 < \theta < 1$.
3. The period of cash discount is less than the permissible delay period.

3. MATHEMATICAL FORMULATION AND OPTIMAL SOLUTION

Since the level of inventory is depleted by the combined effect of demand and deterioration, the inventory level at any instance of time t is governed by the following differential equation.

$$\frac{dI(t)}{dt} + \theta I(t) = -as^{-\eta}(1+bt-ct^2), \quad 0 \le t \le T, \tag{1}$$

The solution of the above differential equation under the condition I (T) = 0 is

$$I(t) = \frac{-as^{-\eta}}{\theta^3}\begin{bmatrix}\theta^2(1+bt-ct^2) - \theta(b-2ct) - 2c + \\ \{\theta^2(cT^2 - bT - 1) - 2\theta(-\tfrac{b}{2} + cT) + 2c\}e^{\theta(T-t)}\end{bmatrix} \tag{2}$$

At the beginning, the supplier has 'Q' units in the inventory system.

$$Q = I(0) = \frac{-as^{-\eta}}{\theta^3}\left[\theta^2 - \theta b - 2c + \{\theta^2(cT^2 - bT - 1) - 2\theta(-\tfrac{b}{2} + cT) + 2c\}e^{\theta T}\right] \tag{3}$$

Total relevant cost/year consists of the following elements.

(i) Cost of placing order $= \frac{A}{T}$ (4)

(ii) Cost of purchasing units $= \frac{pQ}{T}$ (5)

(ii) Holding cost $= \frac{h}{T}\int_0^T I(t)t\,dt$

$$= \frac{h}{T}\left(\frac{-as^{-\eta}}{\theta^3}\right)\begin{bmatrix}\theta^2\left(\frac{T^2}{2} + b\frac{T^3}{3} - c\frac{T^4}{4}\right) - \theta\left(\frac{bT^2}{2} - 2\frac{cT^3}{3}\right) - cT^2 \\ +\left(\theta^2(cT^2 - bT - 1) + 2\theta\left(\frac{b}{2} - cT\right) + 2c\right) \\ \left(\frac{T^2}{2} + \frac{\theta T^3}{6} + \frac{\theta^2 T^4}{24}\right)\end{bmatrix} \tag{6}$$

(iii) Deterioration cost $= C_3\theta\int_0^T I(t)t\,dt$ (7)

Four possible cases are discussed with respect to cash discount and interest charged and earned according to the length of credit periods M_1 or M_2 and the length of cycle time T. For case I, the payment is paid at the given credit period M_1 when M_1 lies within the cycle time T. For case II, the customer pays full payment at M_1, when the credit period M_1 exceeds the cycle time T.. In the same manner, for case III, the payment is paid at the permissible credit period M_2, when the credit period M_2 lies within the cycle time T. In case IV, the customer pays in full at M_2 but M_2 exceeds the cycle time T.

Case I: $(M_1 \le T)$

The discount saving per unit time by the customer $= \frac{rpQ}{T}$ (8)

According to assumptions the customer pays off all units ordered at time of cash discount to obtain profit. Consequently the items in stock have to be financed after the time period M_1.

Hence, the interest payable per year is

$$\frac{p(1-r)I_c}{T}\int_{M_1}^{T} I(t)dt \quad = \left(\frac{-as^{-\eta}}{\theta^3}\right) \tag{9}$$

During $[0,M_1]$,the customer sells products and deposits the revenue into an account that earns I_d per dollar per year.
Thus interest earned per unit time is

$$\frac{sI_d}{T}\int_0^{M_1} as^{-\eta}(1+bt-ct^2)tdt = as^{1-\eta}\frac{I_d}{T}M_1^2\left[\frac{1}{2}+\frac{bM_1}{3}-\frac{cM_1^2}{4}\right] \tag{10}$$

The total relevant cost per unit time ϕ_1 is given by

$\emptyset_1$= Cost of placing order + Cost of purchasing units after discount rate (r) + Holding cost+ Deterioration cost + Interest payable per unit time –Interest earned per unit time (11)

Case-II ($M_1 > T$)
In this case no interest is payable. Cash discount is same as that of case-I.
The interest earned per unit time is

$$\frac{sI_d}{T}\int_0^{T} as^{-\eta}\left(1+bt-ct^2\right)tdt + \left(M_1-T\right)\int_0^{T} as^{-\eta}\left(1+bt-ct^2\right)dt$$

$$= -as^{-\eta}T\left[sI_d\left(\frac{1}{2}+\frac{bT}{3}-\frac{cT^2}{4}\right)+\left(M_1-T\right)\left(1+\frac{bT}{2}-\frac{cT^2}{3}\right)\right] \tag{12}$$

Total relevant cost per year ϕ_2 is given by

$\emptyset_2$=Cost of placing order + Cost of purchasing units after discount rate (r)+Holding cost +Deterioration cost – Interest earned per unit time (13)

Case-III ($M_2 \le T$)
In this case, the payment is paid at time M_2 , there is no cash discount.

The interest payable per unit time is $\frac{pI_C}{T}\int_{M_2}^{T} I(t)dt$

$$= \frac{-as^{-\eta}}{\theta^3}\frac{pI_c}{T}\left[\begin{array}{l}\theta^2T+\frac{b}{2}\theta^2T^2-\frac{c}{3}\theta^2T^3+\theta-b-\frac{2c}{\theta}-\theta^2M_2-\frac{b}{2}\theta^2M_2^2+\frac{c}{3}\theta^2M_2^3+b\theta M_2-c\theta M_2^2+2cM_2-\\ \left(cT^2\theta-b\theta T-\theta+b-2cT+\frac{2c}{\theta}\right)\left(1+\theta T-\theta M_2+\theta^2(T^2-2TM_2+M_2^2)\right)\end{array}\right] \tag{14}$$

The interest earned per unit time is $\frac{sId}{T}\int_0^{M_2} as^{-\eta}\left(1+bt-ct^2\right)tdt$

$$= as^{1-\eta}\frac{I_d}{T}M_2^2\left[\frac{1}{2}+\frac{bM_2}{3}-\frac{cM_2^2}{4}\right] \tag{15}$$

Total relevant cost per unit time ϕ_3 is given by

ϕ_3 = Cost of placing order + Cost of purchasing units+ Holding cost +Deterioration cost+ Interest payable per unit time – Interest earned per unit time (16)

Case-IV ($M_2 > T$)
In this case no interest is payable.
The interest earned per unit time is

$$\frac{sI_d}{T}\int_0^{T} as^{-\eta}\left(1+bt-ct^2)tdt\right)+\left(M_2-T\right)\int_0^{T} as^{-\eta}\left(1+bt-ct^2\right)dt$$

$$= -as^{-\eta}T\left[sI_d\left(\frac{1}{2}+\frac{bT}{3}-\frac{cT^2}{4}\right)+\left(M_2-T\right)\left(1+\frac{bT}{2}-\frac{cT^2}{3}\right)\right] \tag{17}$$

Total relevant cost per unit time ϕ_4 is given by

ϕ_4 = Cost of placing order + Cost of purchasing units + Holding cost + Deterioration cost + Interest payable per unit time – Interest earned per unit time (18)

Differentiating the total cost functions for cases I, II, III, and IV partially with respect to T and equating them to zero, the optimal solutions for T are found out.

For convexity of the total cost function, it is verified that, the necessary and sufficient conditions $\frac{\partial^2 \phi_i}{\partial T^2} > 0$ are satisfied, for i = 1,2,3,4.

4. NUMERICAL ILLUSTRATIONS

The proposed model can be illustrated by the following numerical illustrations. Each illustration is demonstrated by four possible cases. Mathematica 5.1 software is used to obtain the relevant results.

Numerical illustration 1

Case:1 ($M_1 < T$)

To illustrate the effect of the model set a = 800,h = 15, $I_c = 0.08$,I_d = 0.05, p =15, s = 35, θ =0.1, r = 0.02,M = 1, A = 5, b = 0.2, c = 0.3, η = 2.5 ,$C_3 = 0.001$; Result: T= 5.4005, Total cost $\emptyset_1{}^*$= 239.142, Purchase Quantity $Q_1{}^*$= 67.9587

Case:2 ($M_1 \geq T$)

Set a = 900, h = 15, $I_c = 0.08$,I_d = 0.05, p =60, s = 95,θ =0.4, r = 0.02, M =2.3, A = 10, b = 0.2, c = 0.3, η = 1.2, $C_3 = 0.001$; Result: T = 1.68136, Total cost$\emptyset_2{}^*$=288.29, Purchase Quantity $Q_2{}^*$=27.0155

Case: 3 ($M_2 < T$)

Set a = 500, h = 10, $I_c = 0.08$,I_d = 0.05, p =20, s = 30,θ =0.1, r = 0.02, M = 1.5, A = 5, b = 0.2, c = 0.3, η = 2.5 ,$C_3 = 0.001$;Result: T = 6.09275, Total cost $\emptyset_3{}^*$= 250.201, Purchase Quantity $Q_3{}^*$= 62.5034

Case: 4 ($M_2 \geq T$)

Set a = 1000, h = 20, $I_c = 0.08$, I_d = 0.05, p =50, s = 95, θ =0.4, r = 0.02, M = 3, A = 10, b = 0.2, c = 0.3, η = 1.2 ,$C_3 = 0.001$; Result: T= 1.74667, Total cost $\emptyset_4{}^*$= 272.833, Purchase Quantity $Q_4{}^*$= 34.7845

Numerical illustration 2

Case: 1 ($M_1 < T$)

Set a = 900, h = 12, $I_c = 0.08$,I_d = 0.05, p =30, s = 60, θ =0.1, r = 0.2, M = 0.4, A = 5, b = 0.2, c = 0.3, η =2.5 ,$C_3 = 0.001$; Result: T = 6.2271, Total cost $\emptyset_1{}^*$= 95.6224, Purchase Quantity $Q_1{}^*$= 20.3219

Case: 2 ($M_1 \geq T$)

Set a = 800, h = 15,$I_c = 0.08$,I_d = 0.05, p =50, s = 90,θ = 0.4, r = 0.02, M = 2.5, A = 10, b = 0.2, c = 0.3, η = 1.2, $C_3 = 0.001$; Result: T = 1.70332, Total cost $\emptyset_2{}^*$ = 229.738, Purchase Quantity $Q_2{}^*$= 29.7518

Case: 3 ($M_2 < T$)

Set a = 1000, h = 15, $I_c = 0.08$,I_d = 0.05, p =40, s = 70,θ = 0.1, r = 0.1, M = 0.5, A = 5, b = 0.2, c = 0.3, η = 2.5 , $C_3 = 0.001$; Result: T = 6.55039, Total cost $\emptyset_3{}^*$= 123.849, Purchase Quantity $Q_3{}^*$= 15.5197

Case: 4 ($M_2 \geq T$)

Set a = 1500, h = 20, $I_c = 0.08$,I_d = 0.05, p =35, s = 60, θ =0.3, r = 0.2, M = 3, A = 10, b = 0.2, c = 0.3, η = 1.2 ,$C_3 = 0.001$; Result: T= 1.91546, Total cost $\emptyset_4{}^*$= 480.413, Purchase Quantity $Q_4{}^*$= 233.28

Numerical illustration 3

Case: 1 ($M_1 < T$)

Set a = 1100, h = 18, $I_c = 0.06$,I_d = 0.03, p =25, s = 50, θ =0.2, r = 0.3, M = 0.5, A = 5, b = 0.3, c = 0.4, η =1.8 ,$C_3 = 0.001$; Result: T = 2.96168, Total cost $\emptyset_1{}^*$= 310.735, Purchase Quantity $Q_1{}^*$= 99.4366

Case: 2 ($M_1 \geq T$)

Set a = 900, h = 17, $I_c = 0.06$,I_d = 0.04, p =25, s = 50,θ = 0.4, r = 0.03, M = 2.3, A = 8, b = 0.2, c = 0.3, η = 1.4, C_3 =0.001 ;Result: T = 1.85869, Total cost $\emptyset_2{}^*$ = 135.024, Purchase Quantity $Q_2{}^*$= 30.9641

Case: 3 ($M_2 < T$)

Set a = 900, h = 15, $I_c = 0.06$, I_d = 0.04, p =30, s = 65,θ = 0.2, r = 0.3, M = 0.7, A = 8, b = 0.2, c = 0.3, η = 2.1 ,$C_3 = 0.001$; Result: T = 3.30733, Total cost $\emptyset_3{}^{*}$= 50.7166, Purchase Quantity $Q_3{}^{*}$= 11.1208

Case: 4 ($M_2 \geq T$)

Set a = 1400, h = 20, $I_c = 0.08$,I_d = 0.05, p =30, s = 60, θ =0.3, r = 0.2, M = 3.2, A = 10, b = 0.2, c = 0.3, η = 1.2 ,$C_3 = 0.002$; Result: T= 1.93856, Total cost $\emptyset_4{}^{*}$= 389.501, Purchase Quantity $Q_4{}^{*}$= 217.713

4.1. Sensitivity Analysis

The change in the values of parameters may happen due to uncertainties in any decision making situation. In order to examine the implications of these changes the sensitivity analysis will be of great help in decision making. Using the numerical examples of the proceeding section, the sensitivity analysis of the parameters has been carried out in the below table.

4.1.1. Sensitivity analysis with variation in single parameter value

Illustration: 1

Case: 1 Variations of T, Q and ∅ with s, I_c and r

Parameters	% Change	Value of parameter	T_1	Q_1	$\emptyset_1$
s	-10	31.50	5.39973	88.2505	311.006
	-5	33.25	5.40009	77.1716	271.771
	5	36.75	5.40094	60.2265	211.758
	10	38.5	5.40142	53.6825	188.581
I_c	-10	0.072	5.29405	67.9576	219.709
	-5	0.076	5.34837	67.9582	229.471
	5	0.084	*	*	*
	10	0.088	*	*	*
r	-10	0.018	5.40251	67.9588	239.533
	-5	0.019	5.40151	67.9588	239.337
	5	0.021	5.3995	67.9587	238.946
	10	0.022	5.39851	67.9587	238.749

Case: 2 Variations of T, Q and ∅ with s, and r

Parameters	% Change	Value of parameter	T_2	Q_2	$\emptyset_2$
s	-10	85.50	1.68601	30.6102	327.627
	-5	90.25	1.68371	28.7088	306.816
	5	99.75	1.67895	25.4987	271.702
	10	104.50	1.67649	24.1329	256.772
r	-10	0.018	1.68108	27.0155	288.853
	-5	0.019	1.68122	27.0155	288.571
	5	0.021	1.6815	27.0155	288.008
	10	0.022	1.68164	27.0155	287.726

Case: 3 Variations of T, Q and ∅ with s, I_c and r

Parameters	% Change	Value of parameter	T_3	Q_3	$\emptyset_3$
s	-10	27	6.09201	81.1495	325.424
	-5	28.50	6.09237	70.9696	284.353
	5	31.50	6.09316	55.3995	221.538
	10	33.0	6.09361	49.3865	197.279
I_c	-10	0.072	*	*	*
	-5	0.076	*	*	*
	5	0.084	6.151	62.5053	260.348
	10	0.088	6.20716	62.5062	270.417
h	-10	9.0	6.21103	62.5063	245.244

	-5	9.5	6.15005	62.5053	243.515
	5	10.5	*	*	*
	10	11	*	*	*

Case: 4 Variations of T, Q and ∅ with s, and b

Parameters	% Change	Value of parameter	T_4	Q_4	$\emptyset_4$
s	-10	85.50	1.75016	39.4205	310.31
	-5	90.25	1.74844	36.9683	290.482
	5	99.75	1.74485	32.8284	257.031
	10	104.50	1.743	31.067	242.808
b	-10	0.18	1.69498	34.2532	267.232
	-5	0.19	1.72089	34.5189	269.987
	5	0.21	1.77232	35.0501	275.77
	10	0.22	1.79786	35.3156	278.8

- The increased value of 's' sensitizes the value of T, Q and ∅ to decrease.
- The forwarding value of 'b' sensitizes the value of T, Q and ∅ to move forward.
- The corresponding values of T, Q and ∅ ascend with a forward movement in the values ofI_c .
- The corresponding values of T, Q and ∅ descend with a forward movement in the values of h.
- The corresponding values of T, Q and ∅ rise with a forward movement in the values of b.
- There is very little impact of change in the values of T, Q and ∅ with a forward movement in the values of r.

Illustration: 2

Case: 1 Variations of T, Q and ∅ with s, I_c and r

Parameters	% Change	Value of parameter	T_1	Q_1	$\emptyset_1$
s	-10	54	6.22531	26.2547	124.262
	-5	57	6.22616	23.0155	108.626
	5	63	6.22809	18.0612	84.7095
	10	66	6.22917	16.1478	75.4731
I_c	-10	0.072	5.29405	67.9576	84.4462
	-5	0.076	5.34837	67.9582	91.7283
	5	0.084	*	*	99.4852
	10	0.088	*	*	*
r	-10	0.18	6.25517	20.3224	103.214
	-5	0.19	*	*	*
	5	0.21	*	*	*
	10	0.22	6.19844	20.3214	99.144

Case: 2 Variations of T, Q and ∅ with s, and r

Parameters	% Change	Value of parameter	T_2	Q_2	$\emptyset_2$
s	-10	81	1.70848	33.709	261.07
	-5	85.5	1.70593	31.6159	244.494
	5	94.5	1.70063	28.0821	216.528
	10	99	1.69786	26.5786	204.638
r	-10	0.018	1.70301	29.7518	230.183
	-5	0.019	1.70317	29.7518	229.96
	5	0.021	1.70347	29.7518	229.516
	10	0.022	1.70363	29.7518	229.294

Case: 3 Variations of T,Q and ∅ with s, I_c and r

Parameters	% Change	Value of parameter	T_3	Q_3	$\emptyset_3$
s	-10	63	6.5489	20.0036	161.004
	-5	66.5	6.54961	17.5555	140.718
	5	73.5	6.55123	13.8111	109.69
	10	77	6.55215	12.3651	97.7072
I_c	-10	0.072	*	*	*
	-5	0.076	6.48424	15.5184	118.769
	5	0.084	6.61411	15.521	128.888
	10	0.088	6.6756	15.5223	133.89
h	-10	13.5	6.6784	15.5224	121.421
	-5	14.25	6.61239	15.521	122.66
	5	15.75	6.49198	15.5185	124.991
	10	16.5	*	*	*

Case: 4 Variations of T,Q and Ø with s, and b

Parameters	% Change	Value of parameter	T_4	Q_4	$Ø_4$
s	-10	54	1.91577	264.591	546.968
	-5	57	1.91562	248.002	511.758
	5	63	1.9153	219.994	452.347
	10	66	1.91513	208.078	427.084
b	-10	0.18	1.86494	230.774	469.658
	-5	0.19	1.89024	232.001	474.948
	5	0.21	1.94062	234.455	486.055
	10	0.22	1.96571	235.681	491.877

- The increased value of 's' sensitizes the values of Q and Ø to decrease but has a little impact on the values of T.
- The increased value of 'b' sensitizes the values of T, Q and Ø to accelerate.
- The corresponding values of T, Q and Ø ascend with a forward movement in the values ofI_c .
- The corresponding values of T and Q descend with a forward movement in the values of h but the total cost descends.
- The corresponding values of T, Q and Ø ascend with a forward movement in the values of b.
- There is very little change in the values of T, Q and Ø with a forward movement in the values of r.

Illustration: 3

Case: 1 Variations of T, Q and Ø with s, I_c and r

Parameters	% Change	Value of parameter	T_1	Q_1	$Ø_1$
s	-10	45	2.96122	120.031	375.367
	-5	47.50	2.96144	108.976	340.673
	5	52.5	2.96193	91.1449	284.713
	10	55	2.96219	83.8891	261.942
I_c	-10	0.054	2.90443	99.4354	287.723
	-5	0.057	2.93363	99.436	299.287
	5	0.063	2.98867	99.4373	322.077
	10	0.066	3.0147	99.4379	333.316
r	-10	0.27	2.98153	99.4371	353.684
	-5	0.285	2.97169	99.4369	347.928
	5	0.315	2.95148	99.4364	336.347
	10	0.33	2.94108	99.4362	330.522

Case: 2 Variations of T, Q and Ø with s, and r

Parameters	% Change	Value of parameter	T_2	Q_2	$Ø_2$
s	-10	45	1.86195	35.8251	156.363
	-5	47.50	1.86035	33.2411	145.016
	5	52.5	1.85697	28.9448	126.167
	10	55	1.8552	27.1437	118.272
r	-10	0.027	1.85814	27.1436	135.362
	-5	0.0285	1.85841	27.1436	135.193
	5	0.0315	1.85896	27.1436	134.854
	10	0.033	1.85923	27.1435	134.685

Case: 3 Variations of T, Q and Ø with s, I_c and r

Parameters	% Change	Value of parameter	T_3	Q_3	$Ø_3$
s	-10	58.50	3.30189	13.7269	62.8411
	-5	61.75	3.30453	12.3176	56.285
	5	68.25	3.31027	10.0959	45.9485
	10	71.5	3.31336	9.21191	41.8353
I_c	-10	0.054	3.24245	11.1192	47.2338
	-5	0.057	3.2755	11.12	48.9833
	5	0.063	*	*	*
	10	0.066	3.36773	11.1223	54.1381
h	-10	13.50	*	*	*
	-5	14.25	3.33339	11.1214	50.1833
	5	15.75	3.28264	11.1201	51.2313
	10	16.50	3.25922	11.1196	51.7284

Case: 4 Variations of T, Q and Ø with s, and b

Parameters	% Change	Value of parameter	T_4	Q_4	$Ø_4$
s	-10	54	1.93874	246.986	443.648

	-5	57	1.93865	231.502	415.002
	5	63	1.93846	205.362	366.669
	10	66	1.93835	194.25	346.119
b	-10	0.18	1.88867	192.197	380.233
	-5	0.19	1.91365	193.218	384.792
	5	0.21	1.96341	195.261	394.363
	10	0.22	1.98821	196.283	399.38

- The increased value of‘s’ sensitizes the values of Q and ∅ to decrease but it has a little effect on the values of T.
- The increased value of ‘b’ sensitizes the values of T, Q and ∅ to increase.
- The corresponding values of T, and ∅ascend with a forward movement in the values of I_c but have a little impact on the values of Q.
- The corresponding values of T and Q descend with a forward movement in the values of h, but the total cost descends.
- The corresponding values of T, Q and ∅ ascend with a forward movement in the values of b
- There is very little impact of change in the values of T, Q and ∅ with a forward movement in the values of r.

4.2. Sensitivity analysis with simultaneous variations in parameter values

Numerical illustration 1

Case: 1$(M_1 \leq T)$

Percentage change	S	I_c	r	T_1	Q_1	$\emptyset_1$
-10	31.50	0.072	0.018	5.29515	88.2493	286.183
-5	33.25	0.076	0.019	5.34895	77.171	260.987
5	36.75	0.084	0.021	*	*	*
10	38.5	0.088	0.022	*	*	*

Case: 2$(M_1 > T)$

Percentage change	S	r	T_1	Q_1	$\emptyset_1$
-10	85.50	0.018	1.68573	30.6102	328.266
-5	90.25	0.019	1.68357	28.7088	307.115
5	99.75	0.021	1.67909	25.4987	271.436
10	104.50	0.022	1.67676	24.1329	256.269

Case: 3$(M_2 \leq T)$

Percentage change	S	I_c	h	T_3	Q_3	$\emptyset_3$
-10	27	0.072	9	6.0836	81.1493	292.715
-5	28.50	0.076	9.5	6.08839	70.9695	270.067
5	31.50	0.084	10.5	6.09672	55.3996	232.664
10	33	0.088	11	6.10037	49.3866	217.089

Case: 4$(M_2 > T)$

Percentage change	S	b	T_4	Q_4	$\emptyset_4$
-10	85.50	0.18	1.69898	38.8178	303.87
-5	90.25	0.19	1.72278	36.6859	287.437
5	99.75	0.21	1.77063	33.0789	259.784
10	104.50	0.22	1.79466	31.5409	248.065

Numerical illustration 2:

Case: 1$(M_1 \leq T)$

Percentage change	S	I_c	r	T_1	Q_1	$\emptyset_1$
-10	54	0.072	0.18	6.1264	26.253	116.361
-5	57	0.076	0.19	6.17826	23.0147	105.241
5	63	0.084	0.21	6.27311	18.062	87.2434
10	66	0.088	0.22	6.31654	16.1494	79.896

Case: 2$(M_1 > T)$

Percentage change	S	r	T_2	Q_2	$\emptyset_2$
-10	81	0.018	1.70817	33.709	261.574
-5	85.5	0.019	1.70578	31.6159	244.73
5	94.5	0.021	1.70079	28.0821	216.318
10	99	0.022	1.69818	26.5786	204.242

Case: 3$(M_2 \leq T)$

Percentage change	S	I_c	h	T_3	Q_3	$\emptyset_3$
-10	63	0.072	13.5	6.53818	20.0033	144.808
-5	66.5	0.076	14.25	6.54458	17.5554	133.643
5	73.5	0.084	15.75	6.55574	13.8112	115.2
10	77	0.088	16.5	6.56065	12.3653	107.518

Case: $4(M_2 > T)$

Percentage change	s	b	T_4	Q_4	$\emptyset_4$
-10	54	0.18	1.86545	261.808	534.653
-5	57	0.19	1.89044	246.697	505.921
5	63	0.21	1.9405	221.151	457.645
10	66	0.22	1.96556	210.266	437.221

Numerical illustration 3:

Case: $1(M_1 \leq T)$

Percentage change	s	I_c	r	T_1	Q_1	$\emptyset_1$
-10	45	0.054	0.27	2.92321	120.03	359.16
-5	47.50	0.057	0.285	*	*	*
5	52.5	0.063	0.315	2.97857	91.1453	290.134
10	55	0.066	0.33	2.99402	83.8899	271.482

Case: $2(M_1 > T)$

Percentage change	s	r	T_2	Q_2	$\emptyset_2$
-10	45	0.027	1.8614	35.8251	156.755
-5	47.50	0.0285	1.86007	33.2411	145.198
5	52.5	0.0315	1.85724	28.9448	126.009
10	55	0.033	1.85574	27.1437	117.976

Case: $3(M_2 \leq T)$

Percentage change	s	I_c	h	T_3	Q_3	$\emptyset_3$
-10	58.50	0.054	13.50	3.28831	13.7265	57.234
-5	61.75	0.057	14.25	3.2982	12.3175	53.7812
5	68.25	0.063	15.75	3.31578	10.0961	47.9795
10	71.5	0.066	16.50	3.32365	9.21217	45.5205

Case: $4(M_2 > T)$

Percentage change	s	b	T_4	Q_4	$\emptyset_4$
-10	54	0.18	1.88906	244.388	433.03
-5	57	0.19	1.91379	230.285	409.97
5	63	0.21	1.96336	206.442	371.233
10	66	0.22	1.98819	196.283	354.85

Sensitivity analysis for different numerical illustrations summarizes the followings.

- The simultaneous increment in the values of s, I_c and r sensitize the corresponding values of T to increase but $Q_1$and $\emptyset_1$ to decrease.
- The simultaneous increment in the values of s and r simultaneously sensitize the corresponding values of $T_{2,}Q_2$ and $\emptyset_2$ to decrease.
- The simultaneous increment in the values of s, I_c and h sensitize the corresponding values of T to increase but $Q_1$and $\emptyset_1$ to decrease.
- The simultaneous increment in the values of s and b sensitize the corresponding values of T to increase but $Q_1$and $\emptyset_1$ to decrease.

5. CONCLUSION

Allowing multiple conditional deliveries with different credit periods and discounting can justifiably be used in actual situations for an inventory system of decaying item to stimulate sale. However introducing credit period is increasingly recognized as an important strategy to increase retailer's profitability. In this paper the model provides a significant direction to obtain the optimal replenishment policy under the existing condition. Four special cases are presented for different credit periods and the effects of selling price, rate of discount, holding cost and interest payable on order quantity, cycle time and total cost are studied. Sensitivity analysis of the numerical examples reveal that higher values of holding cost, selling price and cash discount result in lower values of order quantity, cycle time and total cost. On the contrary the higher values of interest payable sensitize the order quantity, cycle time and total cost to increase. So a significant insight observed is that it is more beneficial for the inventory system to make holding cost, selling price and cash discount higher. The results in this chapter not only provide a valuable reference for decision makers in planning procurement but also in controlling the inventories.

The chapter has a limitation that a smaller batch of order size is desirable; because large size will welcome disordering cost. The model can be further extended to some more practical situations such as multiple items and inflation with discounting. Compensation mechanism should also be included in future.

RECEIVED: SEPTEMBER, 2018.
REVISED: DECEMBER, 2018

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