AN APPROACH TO DETECTION OF PERIODIC SIGNALS IN WHITE NOISE BASED ON DERIVATIVE OF POWER SPECTRUM AND CONVOLUTION

Fidel Hernandez*[1] and Miguel Iglesias**.
*Technological University of Havana.
**University of Pinar del Rio.

ABSTRACT
This research is based on the Derivative/Integration of the Power Spectrum for the reduction of noise in a periodic signal. The performed procedure, joined to a convolution process, results in the detection of the original values of amplitude, phase and frequency of the spectral components of the periodic signal. In this paper, second-order statistics foundations are presented and the validation of the proposed algorithm is exposed in both theoretical and practical sense.

KEYWORDS: Second-Order Statistics, Noise Cancellation, Power Spectrum, Convolution.

MSC: 60G35, 62M20, 93E10, 93E11

RESUMEN
El trabajo que se propone se basa en la aplicación de un proceso de Derivación/Integración a la Densidad Espectral de Potencia, con el objetivo de reducir el ruido de una señal periódica. El procedimiento realizado, en conjunto con una operación de convolución, permite la detección de los valores originales de amplitud, frecuencia y fase de las componentes espectrales de una señal periódica contaminada por ruido. En este artículo se exponen los fundamentos estadísticos de segundo orden utilizados, así como la validación del algoritmo propuesto, tanto desde el punto de vista teórico, como práctico.

PALABRAS CLAVE: Estadística de Segundo Orden, Cancelación de ruido, Densidad Espectral de Potencia, Convolución**.**

1. INTRODUCTION

Detection of periodic signals lying in noise with impulsive autocorrelation function (NIAC) is a significant problem that arises in many applications such as, radar, communication, biomedicine, fault diagnosis and others (Randall, Sawalhi and Coats (2011), Martin and Mailhes (2010)). Several methods have been developed for the estimation of the main parameters (amplitude, frequency and phase) of the spectral components of a periodic signal corrupted by Gaussian noise, a type of NIAC. These methods are based upon the analysis of the *k*th-order spectrum (Bartelt, Lohmann and Wirnitzer (1984), Geng, Liang and Wang (2011), Holambe, Ray and Basu (1996), Le, Clediere, Serviere and Lacoume (2007), Matsuoka and Ulrych (1984), Pan and Nikias (1987), Petropulu and Nikias (1992), Swami and Mendel (1991), Zhang and Wang (1998)).

Other methods addressing Polyspectrum Slice computations (Kachenoura, Albera, Bellanger and Senhadji (2008)) have also been reported. These are based on the use of some part of the polyspectrum information such as one or two fixed one-dimensional (1-D) polyspectrum slices (Dianat and Raghuveer (1990), Pozidis and Petropulu (1998), Petropulu and Pozidis (1998)). However, the estimated higher-order spectral parameters are result of multidimensional function calculations that make them no suitable for practical implementation and cannot be directly or simply applied on the problem of detection of periodic signals corrupted by Gaussian noise (Iglesias and Hernández (2013)).

The main limitations of the previous algorithms are related to the fact that they do not take into consideration the estimation of amplitude, frequency and phase (all at the same time) of the periodic signal spectral components. As some of them are focused on recovering only the phase information, the rest of them are used just for amplitude and frequency estimation. The computational cost of these applied amplitude, frequency or

[1] fhernandez@tele.cujae.edu.cu.

phase estimation procedures is very high due to the computation of higher-order statistics. In order to overcome the limitations of such algorithms, a signal detection method, based on fourth-order statistics (fourth-order cumulants calculation), is proposed by Iglesias and Hernández (2013). However, this method does not reach to cancel NIAC in general, it is only valid for Gaussian noise reduction.

Several works based on the principles of Welch-based denoising technique and its limitations have also been reported by Santoso (2008). Another method, proposed by Rainer (2001), uses the derivative of power spectral density (PSD) for cancellation of noise in speech signal based on an optimal signal PSD smoothing method and minimum statistics. In the work proposed by Othman and Qian (2006), a noise reduction algorithm based on the spectral derivative domain is introduced and applied on the problem of denoising hyperspectral imagery using 2D-Wavelet Transform. Although these algorithms implement a complete noise reduction procedure, its computational complexity is still high.

In order to obtain a method for the estimation of a signal corrupted by NIAC, exhibiting a computational complexity lower than that achieved in previous methods, a new algorithm based on the derivative of power spectrum and a convolution process is proposed in this work.

2. REMOVING NOISE WITH IMPULSIVE AUTOCORRELATION FUNCTION FROM A PERIODIC SIGNAL FROM SECOND-ORDER STATISTICS

2.1 Second-Order Statistics Calculation

Let's denote $y(t)$ a real value periodic signal corrupted by NIC as follows:

$$y(t) = \sum_{k=1}^{N} A_k \cos(w_k t + \phi_k) + n(t) = x(t) + n(t) \tag{1}$$

where $x(t)$ is the desired periodic signal (signal to be detected) and $n(t)$ is additive zero mean stationary noise with impulsive autocorrelation function. Furthermore, A_k, f_k and ϕ_k are the amplitude, frequency and phase of the signal spectral components, respectively, and N is the number of harmonics of the periodic signal.

It's not hard to find out that the autocovariance function of $y(t)$, $c_2^y(\tau)$ yields:

$$c_2^y(\tau) = \sum_{k=1}^{N} \frac{{A_k}^2}{2} \cos(w_k \tau) + \sigma_n^2 \delta(\tau) \tag{2}$$

where σ_n^2 is equal to the noise power spectral density.

In order to remove the impulsive term in Equation (2), the derivative and subsequent integration of the spectrum of $c_2^y(\tau)$ is proposed. Then, the noise can be completely removed due to its flat nature in the spectral domain.

If the power spectral density of the signal given by Equation (1) is as follows:

$$C_2^y(f) = \sum_{k=1}^{N} \frac{A_k^2}{4} \delta(f - f_k) + \sum_{k=1}^{N} \frac{A_k^2}{4} \delta(f + f_k) + \sigma_n^2 \tag{3}$$

then the derivative/integration of Equation (3) results in the following expression:

$$C_2^y(f)_{d/i} = \sum_{k=1}^{N} \frac{A_k^2}{4} \delta(f - f_k) + \sum_{k=1}^{N} \frac{A_k^2}{4} \delta(f + f_k) \tag{4}$$

The inverse Fourier transform of Equation (4) yields only, and conveniently, the desired periodic signal, however, the phase of the spectral components of the achieved signal are all zero:

$$c_2^y(\tau)_{d/i} = \sum_{k=1}^{N} \frac{{A_k}^2}{2} \cos(w_k \tau) \tag{5}$$

2.2 Phase Recovery

In order to recover the phase information of the spectral components in the original signal, a convolution procedure, similar to that proposed by Iglesias and Hernández (2013), is proposed. This convolution procedure involves both the original signal corrupted by noise, given by Equation (1), and the signal resulting from de noise cancellation procedure, given by Equation (5). Since Equations (1) and (5) describe functions in time domain and the current procedure is carried out after obtaining $c_2^y(\tau)_{d/i}$, the variable τ in Equation (5) is set to be equal to *t*. The convolution procedure performs as follows:

$$y(t) * c_2^y(t)_{d/i} = \left[\sum_{k=1}^{N} A_k \cos(w_k t + \phi_k) + n(t)\right] * \left[\sum_{k=1}^{N} \frac{A_k^2}{2}\cos(w_k t)\right]$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=1}^{N} \left[A_k \cos(w_k t' + \phi_k) + n(t')\right] \cdot \left[\frac{A_k^2}{2}\cos(w_k t - w_k t')\right] dt'$$

$$= \lim_{T\to\infty} \left[\frac{1}{T} \int_{-T/2}^{T/2} \sum_{k=1}^{N} \frac{A_k^3}{2} \cos(w_k t' + \phi_k)\cos(w_k t - w_k t')dt' + \int_{-T/2}^{T/2} \sum_{k=1}^{N} \frac{A_k{}^2}{2} \cos(w_k t - w_k t') n(t') dt'\right]$$

$$= \sum_{k=1}^{N} \frac{A_k^3}{4} \cos(w_k t + \phi_k) = \sum_{k=1}^{N} A_{k_c} \cos(w_k t + \phi_k) \tag{6}$$

where $A_{k_c} = \dfrac{A_k^3}{4}$.

Equation (6) reveals that an equivalent of the original periodic signal, preserving the original phase of the spectral components, is achieved. However, the original magnitude of the spectral components is not achieved, but a cubic version of it. That is why a restoration to the original magnitude of the spectral components must be performed by operating on the resulting spectral component amplitudes as follows:

$$A(f) = \sqrt[3]{4 \cdot A(f)_c} \tag{7}$$

where $A(f)_c$ is the amplitude spectrum of signal resulting from the convolution procedure. The amplitude restoration is carried out along the whole frequency axis. This way, the amplitude of spectral components corresponding to the periodic signal, the frequency of which can be unknown, is restored to the original value. Furthermore, since this amplitude restoration is performed in the frequency domain, when the Inverse Fourier transform is applied in order to obtain the original waveform, the resulting signal exhibits an envelope distortion that must be corrected. Then, an envelope detector can be used in order to overcome such an envelope distortion (see the work by Iglesias and Hernández (2013) for further details). A diagram of the entire procedure is shown in Figure 1. This way, the noise with impulsive autocorrelation function can be in theory completely cancelled from a periodic signal. The noise cancellation is performed over the full effective frequency domain.

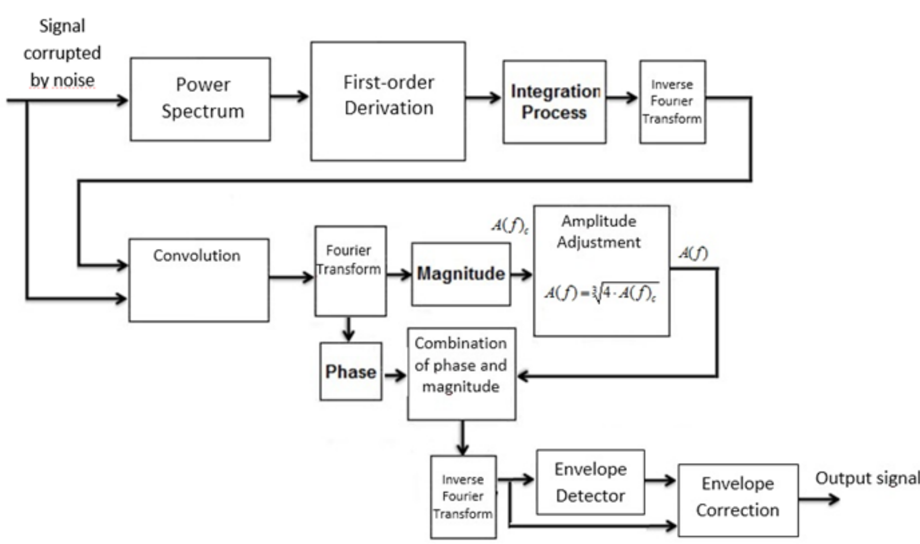

Figure 1. Block Diagram of the Proposed Algorithm

The objective comparison of the computational cost of the proposed algorithm with other algorithms is very difficult to be done. A proper comparison can be achieved if the other algorithms perform the cancellation process over the full effective frequency domain, provided only that the noise autocorrelation is impulsive. For example, filter-based noise reduction procedures do not reduce the noise along the full effective frequency domain, and the wavelet transform-based noise reduction algorithms, on the other hand, require information about the noise power. Comparing the proposed algorithm with such techniques does not provide useful information about the achieved performance, since indeed they have different application domains.

Since the comparison must be performed between techniques with the same application requirements, one could compare the proposed algorithm with those based on higher-order statistics, for example, those mentioned in Section 1. The main problem in this case is that these techniques do not achieve the detection of every parameters of the periodic signal spectral components. Only the technique proposed by Iglesias and Hernández (2013) performs the detection of the periodic signal spectral components with the original amplitude, phase and frequency, however, this technique requires that corrupting noise be Gaussian. This technique is based on a higher-order statistical signal processing thus the incurred computational cost is higher than that achieved by the algorithm proposed in this paper.

2. EXPERIMENTAL RESULTS

In order to verify the effectiveness of the algorithm proposed in this paper, an experiment using a multitone signal was performed. In this case, the sum of nine tones with different amplitudes (0.1, 0.2, 0.3, 0.5, 1.0, 0.8, 0.6, 0.4, 0.9), frequencies (50 Hz, 200 Hz, 300 Hz, 100 Hz, 500 Hz, 150 Hz, 600 Hz, 350 Hz and 250 Hz) and phases (π/4 rad, π/3 rad, π/6 rad, π/8 rad, π rad, π/2 rad, π/5 rad, 3π/4 rad and 2π/3 rad), respectively, corrupted by zero-mean white noise with a Gaussian distribution, was generated (500 noise realizations were used in order to validate the algorithm). The work was performed using Matlab. The signal-to-noise rate (SNR) of the signal corrupted by noise was – 17.74 dB and the correlation between the corrupted signal and the desired signal was 0.37. The sampling frequency was 10 kHz.

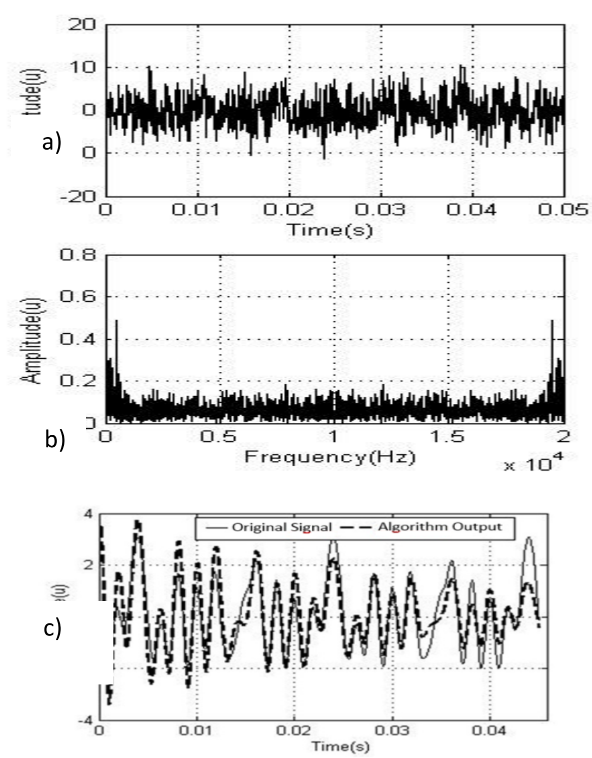

Figure 2. Sketch of a) the original signal (multitone signal) corrupted by noise (time domain), b) its spectrum and c) both the original signal (without noise) and the signal resulting from the noise reduction procedure.

In this work, the Welch's method was applied in order to estimate the power spectrum and the derivation and integration procedures were implemented by using the functions *diff* and *cumtrapz*.

The correlation index between the desired periodic signal and the signal resulting from de noise reduction procedure was used in order to measure the effectiveness of the proposed algorithm. The SNR of these two signals was also calculated and used in order to validate the proposed algorithm.

Figure 2a and Figure 2b show a sample of the corrupted signal and its spectrum, respectively. A comparison between the signal resulting from the noise reduction procedure and the original signal (without noise) is shown in Figure 2c.

The mean and statistical deviation of the SNR estimated at the proposed algorithm output was 31.12 dB and 0.02 dB, respectively. The mean and statistical deviation of the correlation between multitone signal without noise and the signal at the noise canceller output was 0.83 and 0.072, respectively.

3.1. Working with True Communication Signals

This noise reduction procedure was applied on real communication signals, in this case, a QPSK modulation signal (symbol frequency equal to 250 Hz), down converted to a carrier frequency of 880 Hz and sampled at 44100 kHz. The only consideration to be taken into account for the noise cancellation procedure is that the noise can be assumed as white and the desired signal is periodic in consecutive times of duration 1/250 s; there is not information about the carrier frequency, nor the modulation bandwidth, nor the noise power, etc.

It is obvious that in order to apply the algorithm proposed in this paper, an accurate synchronization with the symbol period must be attained. Then, the noise reduction procedure is set to run over each single symbol signal in order to deliver a version of the original desired signal.

Here, the SNR of the signal at the algorithm input, and the correlation between the QPSK signal without noise and the signal at the algorithm input are unknown. Thus, these parameters will not be used in this experiment.

The signal achieved at the algorithm output, compared with the QPSK modulation signal given at the algorithm input, is shown in Figure 3. A comparison between the spectrum of the signal at the noise reduction algorithm output and the spectrum of the signal at the algorithm input is shown in Figure 4. The noise has been clearly reduced.

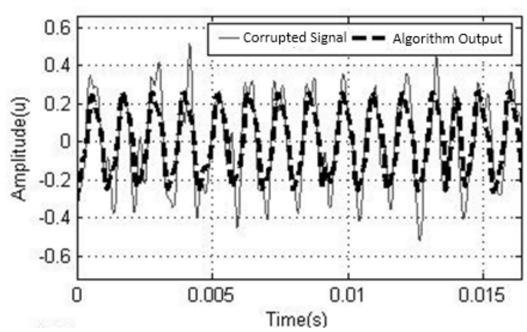

Figure 3. Sketch of the comparison between corrupted QPSK signal and signal at the algorithm output.

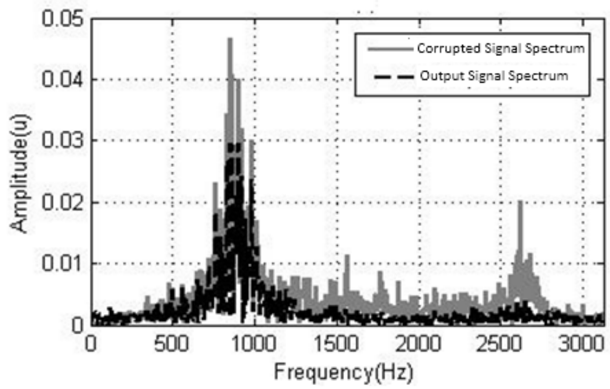

Figure 4. Sketch of the spectrum of the signal at the algorithm output, compared with the QPSK signal with noise.

4. CONCLUSIONS

This research confirmed the convenience of the application of the power spectrum combined with a convolution process and spectral amplitude estimation for detection of periodic signals in noise with impulsive autocorrelation function.

In this work, the use of the derivative of the power spectrum for reducing the noise component was proposed and argued. Proposed algorithm, based on second-order statistics, represents a computational cost lower than those incurred by techniques based on higher-order statistics.

Experimental results performed in Matlab were presented using a periodic signal corrupted by Gaussian noise and a real communication signal corrupted by white noise. Results revealed the effectiveness of the proposed algorithm application.

The application of the proposed algorithm on the reduction of noise in periodic signal does not require any specific information about the noise or the periodic signal. The only information required is that the noise autocorrelation function be impulsive.

RECEIVED: JUNE, 2018
REVISED: MARCH, 2019

REFERENCES

[1] BARTELT, H., LOHMANN, A. W. and WIRNITZER, B. (1984): Phase and amplitude recovery from bispectra, **Appl. Opt.**, 23, 3121–3129.

[2] DIANAT, S. A., and RAGHUVEER, M. R. (1990): Fast algorithms for bispectral reconstruction of two-dimensional signals, **Proc. IEEE Int. Conf. Acoust. Speech Signal Process**, 2377–2379.

[3] GENG, M., LIANG, H. and WANG J. (2011): Research on Methods of Higher-order Statistics for Phase Difference Detection and Frequency Estimation, **4th International Congress on Image and Signal Processing**, 4, 2189-2193.

[4] HOLAMBE, R. S., RAY, A. K. and BASU, T. K. (1996): Signal phase recovery using the bispectrum, **Signal Processing**, 55, 321-337.

[5] IGLESIAS M. E. and HERNÁNDEZ, F. E. (2013): Detection of Periodic Signals in Noise Based on Higher-Order Statistics Joined to Convolution Process and Spectral Analysis, **Lecture Notes in Computer Science**, 8258, 488-495.

[6] KACHENOURA, A., ALBERA, L., BELLANGER, J. J. and SENHADJI, L. (2008): Non-Minimum Phase Identification Based on Higher Order Spectrum Slices. **IEEE Transactions on Signal Processing**, 56, 1821-1829.

[7] LE, T. H., CLEDIERE, J., SERVIERE, C. and LACOUME, J. L. (2007): Noise Reduction in Side Channel Attack Using Fourth-Order Cumulant. **IEEE Transactions on Information Forensics and Security**, 2, 710-720.

[8] MARTIN, N. and MAILHES, C. (2010): About periodicity and signal to noise ratio - The strength of the autocorrelation function. **Seventh International Conference on Condition Monitoring and Machinery Failure Prevention Technologies**, 1-13.

[9] MATSUOKA, T. and ULRYCH, T. J. (1984): Phase estimation using the bispectrum, **Proc. IEEE**, 72, 1403–1411.
[10] OTHMAN, H. and QIAN, S. E. (2006): Noise Reduction of Hyperspectral Imagery Using Hybrid Spatial-Spectral Derivative-Domain Wavelet Shrinkage, **IEEE Transactions on Geoscience and Remote Sensing**, 44, 397-408.
[11] PAN, R. and NIKIAS, C. L. (1987): Phase Reconstruction in the Trispectrum Domain, **IEEE Transaction On Signal Processing**, 35, 895-897.
[12] PETROPULU, A. P., and NIKIAS, C. L. (1992): Signal reconstruction from the phase of bispectrum, **IEEE Trans. Signal Process**, 40, 601–610.
[13] PETROPULU A. P. and POZIDIS, H. (1998): Phase reconstruction from bispectrum slices, **IEEE Trans. Signal Process**, 46, 527–530.
[14] POZIDIS, H., PETROPULU, A. P. (1998): System reconstruction based on selected regions of discretized higher order spectra, **IEEE Trans. Signal Process**, 46, 3360–3377.
[15] RAINER, M. (2001): Noise Power Spectral Density Estimation Based on Optimal Smoothing and Minimum Statistics, **IEEE Transactions on Speech and Audio Processing**, 9, 504-512.
[16] RANDALL, R.B., SAWALHI, N and COATS, M. (2011): A comparison of methods for separation of deterministic and random signals. **The International Journal of Condition Monitoring**, 1, 11-19.
[17] SANTOSO F. (2008): Welch Based Denoising Technique for a Set of Chirp Signals Corrupted by Gaussian Noises, **ITB Journal**, 2, 115-129.
[18] SWAMI, A. and MENDEL, J. M. (1988): Cumulant-Based Approach to the Harmonic Retrieval Problem. **International Conference on Acoustics, Speech, and Signal Processing ICASSP-88**,4, 2264-2267.
[19] SWAMI, A. and MENDEL, J. M. (1991): Cumulant-Based Approach to Harmonic Retrieval and Related Problems. **IEEE Transactions on Signal Processing**, 39, 1099-1109.
[20] ZHANG, Y. and WANG S. (1998): A Hybrid Approach to Harmonic Retrieval in Non-Gaussian Noise Using Fourth-Order Moment and Autocorrelation.**Fourth International Conference on Signal Processing ICSP '98**, 1, 411- 414.