EFFECTS OF LEARNING ON THE ECONOMIC ORDERING POLICIESFOR DEFECTIVE ITEMS UNDER FUZZY ENVIRONMENT WITH PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT
The phenomenon of learning canand has been implemented in several distinct disciplines such as environmental protection techniques, manufacturing processes as well as business transactions. As a matter of fact, learning effect is a phenomenon which occurs almost every where and enables the employees to perform new tasks with better performance after fluent repititons over a course of time. The learning effect acts as a considerable function for cost reduction. It consists of perfect and imperfect quality item's severance from the lot and instantly selling them at different prices. This paper is an adjunct of the commendable work by Jaggi et al.(2013) which considers learning in holding costs and percentage of defective items, follows the learning curve effects underthe fuzzy environment and in addition focuses on proving the concreteness of the fact, that the percentage of defective items together with the holding costs follow the learning effect. In obedience with the economic order quantity (EOQ) model, the ordered lot has 100% perfect items, but this is only suppositional. On contemplation and practical analysis, we have arrived to learn that some lots possess defective items because of process retrogression and other such factors.This research paper refines the economic order quantity model in the context that there are defective items present in each ordered lot confirmed after initial inspection and shortages are allowed under trade credit financingin accordance with the learning effect. The defective items are separated from the considered lot through vigilant inspection and are sold again at a fixed decisive cost price. Conclusively, we de-fuzzify the total profit functions using the triangular method, and for verification of the same, numerical examples and sensitive analysis have been presented in this paper for a clear understanding.

KEYWORDS: Inventory, Learning effect,Imperfect items, Shortages, Trade credit financing, Fuzzy system

MSC: 90B05

RESUMEN
El fenómeno del aprendizaje puede y se ha implementado en varias disciplinas distintas, como las técnicas de protección ambiental, los procesos de fabricación y las transacciones comerciales. De hecho, el efecto de aprendizaje es un fenómeno que se produce en casi todas partes y permite a los empleados realizar nuevas tareas con un mejor rendimiento después de repeticiones fluidas a lo largo del tiempo. El efecto de aprendizaje actúa como una función considerable para la reducción de costos. Consiste en la separación perfecta e imperfecta de los artículos de calidad del lote y que se venden al instante a diferentes precios. Este documento es un complemento del trabajo encomiable de Jaggi et al. (2013) que considera que el aprendizaje, en costos de tenencia y el porcentaje de elementos defectuosos, sigue los efectos de la curva de aprendizaje en el entorno difuso y, además, se centra en demostrar la concreción del hecho de que el porcentaje de artículos defectuosos, junto con los costos de mantenimiento, siguen el efecto de aprendizaje. De acuerdo con el modelo de la cantidad de orden económica (EOQ), el lote ordenado tiene artículos 100% perfectos, pero esto es solo supuesto. En cuanto a la contemplación y el análisis práctico, hemos llegado a saber que algunos lotes poseen artículos defectuosos debido al proceso de retroalimentación y otros factores similares. Este trabajo de investigación refina el modelo de cantidad de orden económica en el contexto de que hay artículos defectuosos presentes en cada lote ordenado confirmado después. La inspección inicial y la escasez están permitidas bajo la financiación de un crédito comercial, de acuerdo con el efecto de aprendizaje. Los artículos defectuosos se separan del lote considerado a través de una inspección minuciosa y se venden nuevamente a un precio fijo y decisivo. Para concluir, desfuzicicamos las funciones de ganancia total utilizando el método triangular, y para la verificación de las mismas, se han presentado ejemplos numéricos y análisis sensibles en este documento para una comprensión clara..

PALABRAS CLAVE: Inventario, artículos imperfectos, escasez, financiamiento de crédito comercial, sistema difuso y efecto de aprendizaje.

1. INTRODUCTION

EOQ model provides a procedure to determine the optimal order size by assuming the procurement costs, inventory holding,backorders and trade credit financing underthe learning effect with fuzzy system. In the contemporary world driven by technology, despite of manufacturing systems' efficient planning, advanced manufacturing mechanisms' and control systems' development and implementation, the articles manufactured may have afraction of defects. By considering this fact, researchers have put invigorousend eavours to improve the EPQ/EOQ models that contemplate upon the imperfect quality items.Salameh and Jaber (2000) improved the conventional model for economic production quantity/economic order quantity for defective quality articles.

Table 1. Different author's contribution

Author(s)	Fuzzy Environment	Learning Effects	Inspection	Trade credit financing
Write (1936)		✓		
Hammer (1957)		✓		
Baloff (1966)		✓		
Cunnigham (1980)		✓		
Dutton (1984)		✓		
Argote and Epple (1990)		✓		
Salameh et al. (1993)		✓	✓	
Jaber et al. (1996)		✓	✓	
Jaggi and Aggarwal(1996)			✓	✓
Jaber et al. (2000)			✓	
Jaber et al. (2008)		✓	✓	
Khan et al. (2010)		✓	✓	
Anazanello and Foglitti (2011)		✓		
Jaggi et al. (2013)			✓	✓
Sairetal.(2014)		✓		✓
Sangal et al. (2016)	✓	✓		
Sangal et al. (2017)		✓		
This Paper	✓	✓	✓	✓

Chang (2004) discussed a real-life implementation of fuzzy set concept for the mathematical model's formulation to improve the economic model for defective characteristic items and further explained about the perfect and imperfect items on the basis of features. LC (learning curve) developed by Wright is a mathematical tool formulated in 1936. In his first attempt, he derived the mathematical formula which established a relationship between learning variables in quantitative shape and got the result in the proposition of the LC (learning curve). Again, different to the excess of review on LC, there is a scarcity of review on forgetting curves. This scarcity of study has been credited almost certainly to the sensible difficulties occupied in getting information regarding the period of forgetting which is function of time developed by Globerson et al. (1989). Jaber et al.(1997) discussed a comparative study of learning and forgetting theory and focused on the comparison of different type of models such as VRVF, VRIF and LFCM. Jaber et al.(1995) discussed about the optimal lot sizing with shortages and backordering under learning. Jaber et al. (2008) took the EOQ model into consideration for imperfect quality items where defective percentage per batch decreases according to the LC (learning Curve). Jaber and Bonney (2003) considered the lot shape with respect to the theory of learning as well as forgetting the in set-up and in manufactured goods excellently and focused on minimizing production time, reducing rework process and optimizing production quantity. Jaber et al. (2004) presented the learning curve for process generating defects which required reworks and generated rate defects as stable and modified by Wright on learning curve.

Khan et al. (2010) considered an EOQ formulation for articles with defective features using learning for screening and maximizing production and minimizing the cost of production. Jaber et al. (2010) discussed on how to develop a merger of average dispensation time processes to give way with respect to the number of lots and plan the consequences in the learning curve parameters manufactured and revised for developed models. Konstantaras et al.(2011) developed a model to maximize production under the condition of shortages for imperfect items. Jaggiet al. (2013) discussed over the production inventory model with financing policies of imperfect items under acceptable backlogging cases. Jaber et al.(2013) considered a manufactured stock model with LC and FC "learning and forgetting" theory in manufacturing and also discussed by how much the number of order (shipments) of a batch should be minimized from manufacturing to the subsequent cycle. Teng et al.(2014) discussed and contemplated over lot size policies in EPQ models under the learning curve production costs with trade credit. Sangal et al.(2016) proposeda model for product archives or inventory with fuzzy environment with partial backlogging under the learning effect. Aggarwal et

al.(2017) improved the non-instantaneous model's optimal policy for deteriorating items with partial shortages and learning effects.
The work of Jaggi et al. (2013) asserts that each lot contains defective and non-defective items and it isalso assumed that a percentage of defective items is present in each lot, distributed uniformly as well as the unit selling price of the defective and non-defective is fixed. This paper extends the model of Jaggi et al.(2013)by considering the percentage of defective items, the holding cost that follows the learning effect and the unit-selling price of defective and non-defective items in fuzzy environment due to them being imprecision in nature. As per considerations, we have taken all the input parameters followed by the mathematical model by Jaggi et al. (2013) after using strategies like learning and fuzzy theory in this model and by incorporating sensitive analysis. We obtain maximum profit owing to the learning effect in holding cost as well as the percentage of defective items and a suitable range of unit selling price of perfect and imperfect items due to the fuzzy properties.

2. DEFINITIONS AND ASSUMPTIONS

2.1 Definitions

- **Defective items and Inspection**

In the classical inventory models, the common unrealistic assumption is that all the items produced are of good quality in nature. However, in reality there may be some defective items in an ordered lot. Thus, inspection of lot becomes indispensable in most of the organization.These items are usually picked up during the inspection/screening process and are sold as a single lot at the end of screening process.

- **Trade Credit**

In today's business transactions, as most of the suppliers allows a certain fixed period to encourage its retailers to order large quantity and charges no interest for this period but beyond this period interest is charged accordingly to the terms and conditions agreed upon.

- **Fuzzy Environment**

When we considerthemodel for fuzzy environment, the following definitions are necessitated. A fuzzy set $\tilde{A}$ on the universal set, X is denoted and defined by $\tilde{A}=\left\{\left(x,\lambda_{\tilde{A}}\left(\tilde{x}\right)\right):\ x\in X\right\}$ where $\lambda_{\tilde{A}}:X\to[0,1]$ is known as themembership function. The triplet (x_1,x_2,x_3), is used to specify a triangular fuzzy number, where $x_1<x_2<x_3$ and is defined by the continuous membership function $\tilde{\lambda}:X\to[0,1]$ as follows -

$$\lambda_{\tilde{A}}=\begin{cases}\dfrac{x-x_1}{x_2-x_1},\ x_1\le x\le x_2; \\ \dfrac{x_3-x}{x_3-x_2},\ x_2\le x\le x_3 \\ 0, \qquad otherwise\end{cases}$$

If $\tilde{A}=(x_1,x_2,x_3)$ is a triangular fuzzy number, then the centroid method on $\tilde{A}$ is defined as

$$C\left(\tilde{A}\right)=\frac{x_1+x_2+x_3}{3}.$$

2.2 Assumptions

1. The demand rate iswell known pre-hand with reliability as well as uniformity.
2. The replenishment is always assumed to be instantaneous.
3. Shortages are assumed to bepermitted and are completely backlogged.
4. The lead-time ispresumed to be zero.
5. A credit period which is fixed is provided by the supplier for settling the retailers' accounts.
6. Screening and demanding proceed concurrently, and screening rate is presumed and the demand rate is less than the screening rate as suggested by Jaggi, Goyal and Mittal (2013)
7. Time horizon is finite.
8. It is considered that holding cost is constant partially and decreases partially in each individual cycle due to employees' learning effects as followed by Aggarwal, Sangal and Singh (2017)
9. It is considered that there is some imperfect items' percentage in each individual lot as proposed by Salameh and Jaber (2000)
10. Defective items' percentage in each individual shipment is governed by the learning curve as discussed by Jaber, Goyal and Imran (2008)
11. Imperfect items are then sold at a pre-decisive discounted price.
12. Unit selling price of non-defective item is imprecise in nature.
13. Unit selling price of defective items is imprecise in nature.

2.3. Notations

D	Demand (order require) rate in units per unit time
Q	Orderrange for each cycle (**decision variable**)
S	Optimal backorder stage permitted **(decision variable**)
K	Preset cost of placing an order
c	Unit cost
p	Unit selling price of non-defective items
c_s	Unit selling price of defective items, $c_s < p$
β	Unit's screening cost
$h(n)$	Holding cost for each order is partially persistentconstant) and partially reduces in each individual shipment (n)owing to employees' learning effects
$p(n)$	Defective items' percentage in each individual shipment is governed bythelearningcurve
λ	Rate of screeningis in units/unit time, $\lambda > D$
t_1	Time taken for building up the backorder level of S units
t_2	Time taken to abolish level of backorder of S units
t_3	Time taken to screen Qunits that areordered per individualcycle
T_1	Time taken when the stock stage will eventually become zero
T	Length of the cycle
C_2	Cost of shortage in unit/unit time
I_e	Earned interest in unit/unit time
I_p	Paid interest in unit/unit time
L_1	Level of stock/ inventory at time t_3
L_2	Level of stock/ inventory at time t_2
$(1-p(n))\lambda$	Perfectquality items' rateat t_2

$(1-p(n))\lambda - D$ $(1-p(n))\lambda - D > 0$	Good quality items' rate in order to get the backorder abolished,
$\tilde{p}$	Fuzzy unit perfect item's selling price
$\tilde{c}_s$	Fuzzy unit defective items' selling price, $\tilde{p} > \tilde{c}_s$
$\Psi_i^*(Q,S)$	Retailer's total profit, for the different cases, $i = 1, 2, 3, 4, 5,\ viz$
$\Psi_i(Q,S)$	Retailer's total profit in fuzzy system for different cases, $i = 1, 2, 3, 4, 5,\ viz$

As per definition and assumption , each delivered lot contains percentage of defective items which follow learning curve and to stop defective items from being sold to customers, the buyer inspect the items at a fixed rate λ and inspection rate is assumed to be higher than the demand rate D. The sellor offers the buyer a fixed credit period M to stimulate sales.As we know that fuzzy theory involves the process to find out the suitable value of range for unfocused items in an imprecise environment, therefore the recent paper asserts the unit selling price of perfect and imperfect items in fuzzy environment as per the assumption that selling price of an item is a major parameter for a buyer and for smooth coordination between the buyer and the seller.The strategy of this paper has been discuseed in the next section which is given below.

2.3 Objective of this paper

The objective of this paper is to maximize the ordered quantity and backorders together with the corresponding profit for the retailer with trade-credit financing under the learning effect on holding cost and defective percentage for imperfect quality items in fuzzy environment. This paper is an adjuncted and an extensively developed form of the model proposed byJaggi et al.(2013). They suggested and deviced an implementation as to how to maximize the ordered quantity as well as the back orders and their corresponding profits under the trade credit financing for defective quality items without learning effects on holding cost and defective percentage. The present mathematical model differs from that of Jaggi et al.(2013) in the following ways;

- It is considered that the holding cost is constant initially and decreases partially in each individual cycle due to the employees' learning effects and can be represented in mathematical form as follows
  $$h(n) = h_0 + \frac{h_1}{n^\gamma},$$
  here n ,is the number of shipment and γ is learning factor.
- Percentage of imperfect items follows the S shaped learning curve which is mathematically represented below
  $$P(n) = \frac{a}{g + e^{bn}},$$
  here $a,\ g > 0$, n is the number of shipment and $b > 0$ is learning factor.
- Unit selling price of non-defective items is imprecise in nature.
- Unit selling price of defective items is imprecise in nature.
- We de-fuzzify the total profit functions using the triangular method which is mathematically presented in section-2.

Finally, our ambition is to illustrate the impacts of these parameters on optimal order quantity, backorders and corresponding profit under the fuzzy environment.

3. MATHEMATICAL MODEL

A mathematical model has been developed inthis paper under allowable setback in cash under the learning effect in a fuzzy systemwhich is fully backlogged. Shortage is authorized in addition to being completely backlogged which is removed during the procedure of screeningas $\lambda > D$. The inventory model's behavior is depicted in Figure 1. It is supposed that a lot comprising of Q unitscontains $p(n)$ defective items and enters the inventory procedure at time $t = 0$, which follows the learning curve. The wholelot which is received λ units per unit time at a fixed ratein order to detect perfect and imperfect articles is screened.Further, an assumption is madeto screen the imperfect items at the specified rate of $(1 - p(n))\lambda$, during time t_2 and perfect quality items' fraction satisfies the demand/ order at a rate D and the rest are used to abolish the backorders with the rate, $(1 - p(n))\lambda - D$. After the process of screening ends at time t_3, the defected articles are sold at once as a single lot and at a discounted price c_s after which the inventory/ stock stage decreases slowly owing to the demandand eventually reaches zero at time T_1.

The length of cycle T for the proposed inventory prototype is denoted by–

$$T = \frac{(1 - p(n))Q}{D} \tag{1}$$

The time to make up a backorder of S units is

$$t_1 = \frac{S}{D} \tag{2}$$

And the time taken to remove S units is

$$t_2 = \frac{S}{(1 - p(n))\lambda - D} \tag{3}$$

$$t_2 = \frac{Q - L_2}{(1 - p(n))\lambda} \tag{4}$$

From equations (3) and (4), we get the values of L_2 as follows

$$L_2 = Q - \frac{(1 - p(n))S}{(1 - p(n)) - \frac{D}{\lambda}} \tag{5}$$

The screening time, t_3 is found out to be as follows

$$t_3 = \frac{Q}{\lambda} \quad . \tag{6}$$

Using Figure 1, we can write

$$t_3 - t_2 = \frac{(L_2 - L_1 - p(n)Q)}{D} \tag{7}$$

$$L_1 = \left((1 - p(n)) - \frac{D}{\lambda} \right) Q - S \tag{8}$$

$$T_1 = \frac{\left((1 - p(n)) - \frac{D}{\lambda} \right) Q - S}{D} + \frac{Q}{\lambda} \tag{9}$$

The circumstances of allowable setback in payments have been considered to develop the present model. Therefore taking the credit period into consideration leads us to five different conditions for the retailer's entire gain $\Psi_i^*(B,Q),\ i=1,\ 2,3,4, and\ 5$

$$
\begin{array}{ll}
(i) & T_1 \geq t_3 \geq t_2 \geq M \\
(ii) & t_2 \leq M \leq t_3 \leq T_1 \\
(iii) & t_3 \leq t_3 \leq M \leq T_1 \\
(iv) & T \geq M \geq T_1 \quad and \\
(v) & M \geq T
\end{array}
\tag{10}
$$

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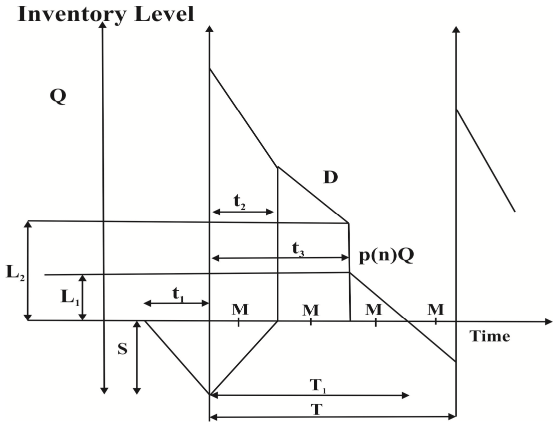

Figure1. Structure of inventory for the case with inspection $M \leq t_2 \leq t_3 \leq T_1, t_2 \leq M \leq t_3 \leq T_1,$ $t_2 \leq t_3 \leq M \leq T_1, T_1 \leq M \leq T$ and $T \leq M$

Since the total profit of the retailers, consists of the following mentioned components–

$\Psi_i^*(B,Q) =$ Sales income - Setup cost - Screening cost-Holding cost +Interest gained –Interest paid. (11)

Consequently, the evaluation of these individual components is done as follows-

- Sales income is equal to the addition of generatedincome or revenueby an order demand met throughout the sum of range of time length (0, T) and the defective articles' sales which is

$$SR = p(1-p(n))Q + c_s p(n)Q \tag{12}$$

- Ordering Cost $= K$ (13)

- Purchase Cost $= cQ$ (14)

- Holding Cost during the time period 0 to T_1

$$
\begin{aligned}
IHC(h) &= h(n)\left[\frac{t_2(L_1+Q)}{2} + \frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\
&= \left(ho + \frac{h_1}{n^\gamma}\right)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right]
\end{aligned}
\tag{15}
$$

- Shortage Cost $=\dfrac{C_2(t_2+t_1)S}{2}$ (16)

- Screening Cost= βQ (17)

Now we can write retailer's entire profit;

$$\begin{aligned}\Psi^*{}_i(Q,B) = {} & p(1-p(n))Q + c_s\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} \\ & -\left(ho + \frac{h_1}{n^\gamma}\right)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\ & + IE - IC\end{aligned} \tag{18}$$

The earned interest and interest chargeisevaluated for the following five distinct situations:

Case 1: $M \le t_2 \le t_3 \le T_1$

The retailers earn an interest based uponprobable sales income that is produced for a time period 0 to M . At M the account is developed which is equal to $[(1-p(n))\lambda - D]I_e pM^2/2 + M^2 DpI_e/2$ and finances which are to be arranged to make the remaining stock's payment at some specified rate of interest for the period M to T_1 (Figure 2) which are equal to

$[(1-p(n))Q - DM - \{(1-p(n))\lambda + D\}M]cI_p(T_1-M)/2 + cpI_p\, p(n)Q(t_3-M)$

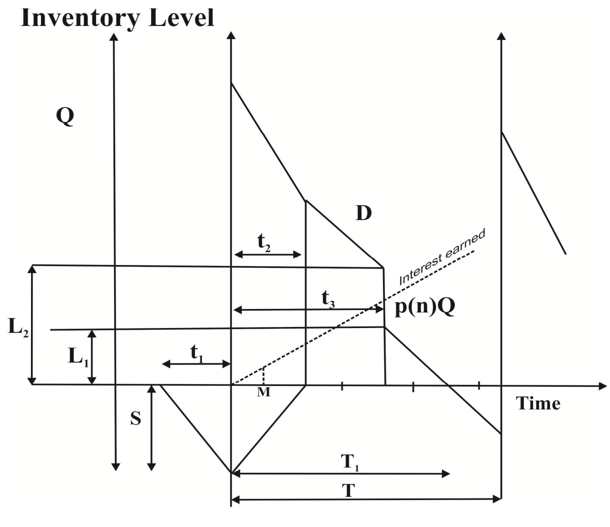

Figure 2. Structure of inventory for situation1 under inspection $M \le t_2 \le t_3 \le T_1$

From Equation (11), we derive retailers'profit in this case

$$
\begin{aligned}
\Psi^*{}_1(Q,B) = {} & p(1-p(n))Q + c_s\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} \\
& - c_s\, I_p\, p(n)Q(t_3 - M) + \left[\frac{\{(1-p(n))\lambda - D\}M^2 I_e\, p}{2} + \frac{DM^2 I_e\, p}{2}\right] \\
& - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\
& - \left[\frac{[(1-p(n)Q - DM - \{(1-p(n))\lambda - D\}M)]cI_p(T_1-M)}{2}\right]
\end{aligned}
\tag{21}
$$

Case 2: $T_1 \geq t_3 \geq M \geq t_2$

Under this situation, the retailer can earn an interest on the income produced from sales up to M . Although, the account at M, has to be settled, for which funds have to be arranged at some specified rate of interest in order to get his remaining stock financed for the period M *to* T_1. Due tothe shortage met for the considered time period $(M - t_2)$ (Figure 3), the retailer will gain an interest too. The total earning due to the interest on revenue and shortages met is equal to $[(1-p(n))\lambda - D]I_e t_2{}^2 p/2 + SpI_e(-t_2+M) + M^2 DI_e p/2$ and the interest charged is equal to $[(1-p(n))Q - DM - \{(1-p(n))\lambda + D\}t_2]cI_p(T_1 - M)/2 + cpI_p\, p(n)Q(t_3 - M)$. Now total profit in this case is

$$
\begin{aligned}
\Psi^*{}_2(Q,S) = {} & p(1-p(n))Q + c_s\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} - c\ I_p\, p(n)Q(t_3 - M) \\
& + \left[\frac{\{(1-p(n))\lambda - D\}t_2{}^2 I_e\, p}{2} + \frac{DM^2 I_e\, p}{2} + S\ p\ I_e(M - t_2)\right] \\
& - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\
& - \left[\frac{[(1-p(n)Q - MD - \{(1-p(n))\lambda - D\}t_2)]I_p c(T_1-M)}{2}\right]
\end{aligned}
\tag{24}
$$

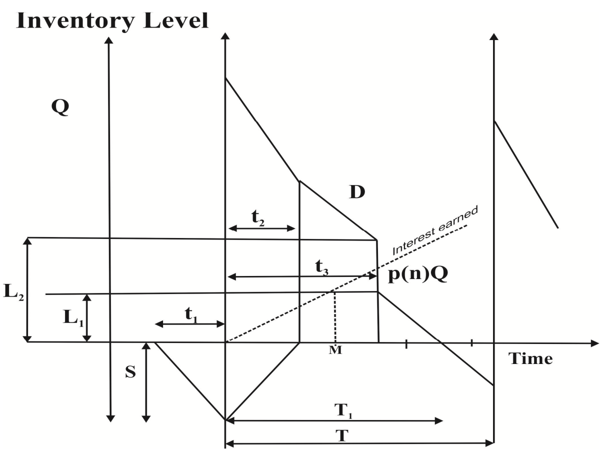

Figure 3. Structure of inventory for the cases under inspection process $t_2 \leq M \leq t_3 \leq T_1$

Case 3: $t_2 \leq t_3 \leq M \leq T_1$

Here, additional to the interest gained and the interest paid as it were in Case 1, the retailer does not only earns an interest due to the defective items' sales, $p(n)Q$, after the screening process for the considered time period $(M - t_3)$ but also earns an interest due to the scarcity which has been backlogged during $(M - t_2)$ (Figure4). Total earning in this case is equal to $[(1 - p(n))\lambda - D]t_2^{\,2} I_e p/2 + SpI_e(M - t_2) + DM^2 I_e p/2 + I_e c_s p(n)Q(M - t_3)$ and the total interest charges are equal to $[Q(1 - p(n)) - MD - \{(1 - p(n))\lambda + D\}t_2]I_p c(T_1 - M)/2$.

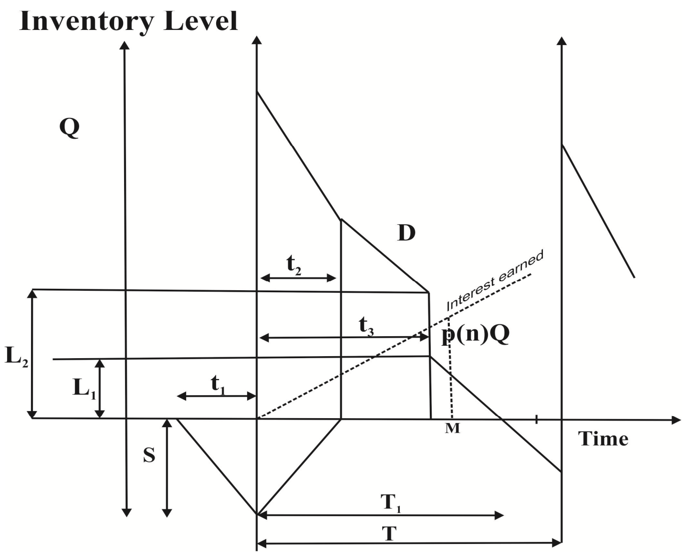

Figure 4. Structure of inventory system under the cases with inspection $t_2 \le t_3 \le M \le T_1$

Now, the total profit for the Case 3 becomes as follows

$$
\begin{aligned}
\Psi^*{}_3(Q,B) = {} & p(1 - p(n))Q + c_s\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1 + t_2)S}{2} + c_s\, I_e p(n)Q(M - t_3) \\
& + \left[\frac{\{(1 - p(n))\lambda - D\}t_2^{\,2} I_e\, p}{2} + \frac{DM^2 I_e\, p}{2} + S\, p\, I_e(M - t_2)\right] \\
& - h(n)\left[\frac{t_2(Q + L_1)}{2} + \frac{(t_2 - t_3)(L_1 + L_2 + p(n)Q)}{2} + \frac{(T_1 - t_3)L_2}{2}\right] \\
& - \left[\frac{[(1 - p(n)Q - DM - \{(1 - p(n))\lambda - D\}t_2)]cI_p(T_1 - M)}{2}\right]
\end{aligned} \tag{24}
$$

Case 4: $T_1 \le M \le T$

The condition when the inventory cycle is equal to or less than the permissible delay has been discussed under this case.Thus,the interest is not payable in this scenariofrom the retailer's end;the retailer can earn an interest on the produced income from the sales from 0 to M. Further, the retailer does not only earns an interest owing to the defective items' sales i.e. $p(n)Q$ for the time $(M - t_3)$ but also earns an interest from thescarcity that is backlogged for the time $(M - t_2)$ (Fig. 5). The total earned interest in this case is equal to $[(1 - p(n))\lambda - D]t_2^{\,2} I_e p/2 + SpI_e(M - t_2) + DM^2 I_e p/2 + I_e c_s p(n)Q(M - t_3) + DT_1 I_e p(M - t_3)$

Now substituting the values from Equation (28) and (29), in Equation (18), the total profit for Case 4 becomes

$$\Psi^*{}_4(Q,B) = p(1-p(n))Q + c_s Qp(n) - K - Qc - Q\beta - \frac{C_2(t_1+t_2)S}{2} + c_s I_e p(n)Q(M-t_3)$$
$$+\left[\frac{\{(1-p(n))\lambda - D\}t_2^{\,2} I_e\, p}{2} + \frac{DT_1^{\,2} I_e\, p}{2} + S\; p\, I_e(M-t_2) + DT_1 I_e\; p(M-T_1)\right] \qquad (30)$$
$$h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right]$$

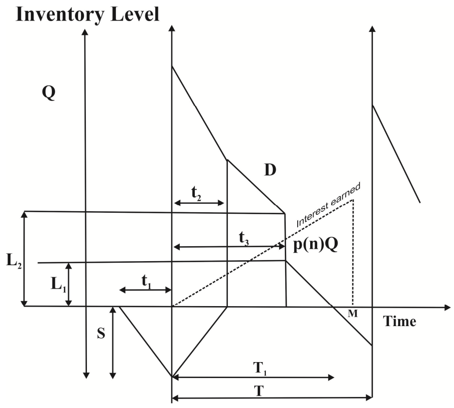

Figure 5. Structure of inventory under the cases with inspection $T_1 \le M \le T$

Case 5: $M \ge T$

The expressionisal together for the earned interest as well as for the paid interest coinciding in this case with those of in Case 4. Here, effectivelyfour various cases for the whole gain per cycle of the retailer, have been expressed as follows –

$$\Psi^*(S,Q) = \begin{cases} \Psi^*{}_1(S,Q), & T_1 \ge t_3 \ge t_2 \ge M \quad case \quad 1, \\ \Psi^*{}_2(S,Q), & T_1 \ge t_3 \ge M \ge t_2 \quad case \quad 2, \\ \Psi^*{}_3(S,Q), & T_1 \le M \le t_3 \le t_2 \quad case \quad 3 \\ \Psi^*{}_4(S,Q), & T_1 \le M \qquad\qquad case \quad 4 \end{cases} \qquad (31)$$

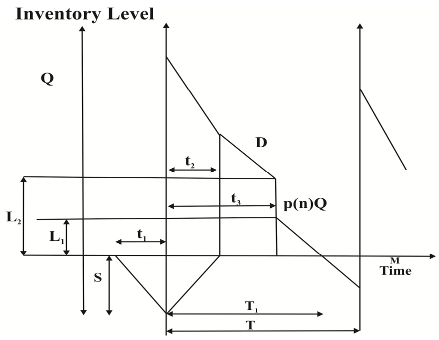

Figure 6 Structure of inventory system under the cases with inspection $T_1 \le M$

4. FUZZY MODEL

Let us assume that due to uncertainty existing in parameters, the inventory model is in fuzzy environment. Also, we have assumed that the parameters, $\tilde{p} = (p_{1,}\ p_2,\ p_3)$, $\tilde{c}_s = (c_1, c_2, c_3)$ are triangular fuzzy numbers, then the entire gain per unit time in fuzzy environment. We have found out four individual cases for retailer's entire gain per cycle in fuzzy environment $\Psi_i(S, Q),\ i = 1, 2, 3, 4\, viz$.

4.1. The total profit of retailers for case1 in fuzzy environment $M \le t_2 \ \le t_3 \le T_1$

$$\begin{aligned}\Psi_1(Q,S) = {} & \tilde{p}(1-p(n))Q + \tilde{c}_s\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} - \tilde{c}_s\, I_p\, p(n)Q(t_3 - M) \\ & + \left[\frac{\{(1-p(n))\lambda - D\}M^2 I_e\, \tilde{p}}{2} + \frac{DM^2 I_e\, \tilde{p}}{2}\right] \\ & - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\ & - \left[\frac{[(1-p(n)Q - DM - \{(1-p(n))\lambda - D\}M)]cI_p(T_1-M)}{2}\right]\end{aligned} \tag{32}$$

Now we defuzzify the entire profit per unit time by Centroid Method

$$\Psi_1(Q,S) = \frac{[\Psi_{11}(Q,S) + \Psi_{12}(Q,S) + \Psi_{13}(Q,S)]}{3T} \tag{33}$$

$$\begin{aligned}\Psi_{11}(Q,S) = {} & p_1(1-p(n))Q + c_1\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} - c_1\, I_p\, p(n)Q(t_3 - M) \\ & + \left[\frac{\{(1-p(n))\lambda - D\}M^2 I_e\, p_1}{2} + \frac{DM^2 .I_e\, p_1}{2}\right] \\ & - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\ & - \left[\frac{[(1-p(n)Q - DM - \{(1-p(n))\lambda - D\}M)]cI_p(T_1-M)}{2}\right]\end{aligned} \tag{34}$$

$$\Psi_{12}(Q,S)= p_2(1-p(n))Q+c_2\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}-c_2\,I_p\,p(n)Q(t_3-M)$$
$$+\left[\frac{\{(1-p(n))\lambda-D\}M^2 I_e\,p_2}{2}+\frac{DM^2.I_e\,p_2}{2}\right]$$
$$-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right] \quad (35)$$
$$-\left[\frac{[(1-p(n)Q-DM-\{(1-p(n))\lambda-D\}M)]cI_p(T_1-M)}{2}\right]$$

$$\Psi_{13}(Q,S)= p_3(1-p(n))Q+c_3\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}-c_3\,I_p\,p(n)Q(t_3-M)$$
$$+\left[\frac{\{(1-p(n))\lambda-D\}M^2 I_e\,p_3}{2}+\frac{DM^2.I_e\,p_3}{2}\right]$$
$$-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right] \quad (36)$$
$$-\left[\frac{[(1-p(n)Q-DM-\{(1-p(n))\lambda-D\}M)]cI_p(T_1-M)}{2}\right]$$

The values of $\Psi_{11}(Q,S),\Psi_{12}(Q,S)$ *and* $\Psi_{13}(Q,S)$ from Equations (34) ,(35) and (36)are substituted in Equation (33), we get

$$\Psi_{11}(Q,S)=\frac{\begin{array}{l}(p_1+p_2+p_3)\left[(1-p(n))Q+\frac{\{(1-p(n))\lambda-D\}M^2 I_e}{2}+\frac{DM^2 I_e}{2}\right]\\ +(c_1+c_2+c_3)\big[p(n)Q-I_p\,p(n)Q(t_3-M)\big]-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}\\ -h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\\ -\left[\frac{[(1-p(n))Q-DM-\{(1-p(n))\lambda-D\}M]\,cI_p(T_1-M)}{2}\right]\end{array}}{3T} \quad (37)$$

4.2. The total profit of retailers for case 2 in fuzzy environment $t_2\le M\le t_3\le T_1$

$$\Psi_2(Q,S)=\tilde{p}(1-p(n))Q+\tilde{c}_s\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}-\tilde{c}_s\,I_p\,p(n)Q(t_3-M)$$
$$+\left[\frac{\{(1-p(n))\lambda-D\}t_2^2 I_e\,\tilde{p}}{2}+\frac{DM^2.I_e\,\tilde{p}}{2}+S\,\tilde{p}\,I_e(M-t_2)\right]$$
$$-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right] \quad (38)$$
$$-\left[\frac{[(1-p(n)Q-DM-\{(1-p(n))\lambda-D\}t_2)]cI_p(T_1-M)}{2}\right]$$

Now we defuzzify the total profit per unit time by Centroid Method

$$\Psi_2(Q,S)=\frac{[\Psi_{21}(Q,S)+\Psi_{22}(Q,S)+\Psi_{23}(Q,S)]}{3T} \tag{39}$$

$$\begin{aligned}\Psi_{21}(Q,S)=\ & p_1(1-p(n))Q+c_1\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}-c_1 I_p\,p(n)Q(t_3-M)\\ &+\left[\frac{\{(1-p(n))\lambda-D\}t_2{}^2 I_e\,p_1}{2}+\frac{DM^2.I_e\,p_1}{2}+S\,p_1 I_e(M-t_2)\right]\\ &-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\\ &-\left[\frac{[(1-p(n)Q-DM-\{(1-p(n))\lambda-D\}t_2)]cI_p(T_1-M)}{2}\right]\end{aligned} \tag{40}$$

$$\begin{aligned}\Psi_{22}(Q,S)=\ & p_2(1-p(n))Q+c_2\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}-c_2 I_p\,p(n)Q(t_3-M)\\ &+\left[\frac{\{(1-p(n))\lambda-D\}t_2{}^2 I_e\,p_2}{2}+\frac{DM^2.I_e\,p_2}{2}+S\,p_2 I_e(M-t_2)\right]\\ &-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\\ &-\left[\frac{[(1-p(n)Q-DM-\{(1-p(n))\lambda-D\}t_2)]cI_p(T_1-M)}{2}\right]\end{aligned} \tag{41}$$

$$\begin{aligned}\Psi_{23}(Q,S)=\ & p_3(1-p(n))Q+c_3\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}-c_3 I_p\,p(n)Q(t_3-M)\\ &+\left[\frac{\{(1-p(n))\lambda-D\}t_2{}^2 I_e\,p_3}{2}+\frac{DM^2.I_e\,p_3}{2}+S\,p_3 I_e(M-t_2)\right]\\ &-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\\ &-\left[\frac{[(1-p(n)Q-DM-\{(1-p(n))\lambda-D\}t_2)]cI_p(T_1-M)}{2}\right]\end{aligned} \tag{42}$$

The values of $\Psi_{21}(Q,S),\Psi_{22}(Q,S)$ *and* $\Psi_{23}(Q,S)$ from Equations (40),(41) and (42),are substituted in Equation (39), we get

$$\Psi_2(Q,S)=\frac{\begin{aligned}&(p_1+p_2+p_3)\left[(1-p(n))Q+\frac{\{(1-p(n))\lambda-D\}M^2 I_e}{2}+\frac{DM^2 I_e}{2}+SI_e(M-t_2)\right]\\ &+(c_1+c_2+c_3)[p(n)Q-I_p\,p(n)Q(t_3-M)]-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}\\ &-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\\ &-\left[\frac{[(1-p(n))Q-DM-\{(1-p(n))\lambda-D\}t_2]\ cI_p(T_1-M)}{2}\right]\end{aligned}}{3T} \tag{43}$$

4.3. The total profit of retailers for case 3 in fuzzy environment, $t_2 \le t_3 \le M \le T_1$

$$\begin{aligned}\Psi_3(Q,S) = {} & \tilde{p}(1-p(n))Q + \tilde{c}_s\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} + \tilde{c}_s\, I_e p(n)Q(M-t_3) \\ & + \left[\frac{\{(1-p(n))\lambda - D\}t_2^{\,2} I_e \tilde{p}}{2} + \frac{DM^{\,2}.I_e \tilde{p}}{2} + S\,\tilde{p}\, I_e(M-t_2)\right] \\ & - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\ & - \left[\frac{[(1-p(n)Q - DM - \{(1-p(n))\lambda - D\}t_2)]cI_p(T_1-M)}{2}\right]\end{aligned} \quad (44)$$

Now we defuzzify the total profit per unit time by centroid method;

$$\Psi_3(S,Q) = \frac{[\Psi_{31}(S,Q) + \Psi_{32}(S,Q) + \Psi_{33}(S,Q)]}{3T} \quad (45)$$

$$\begin{aligned}\Psi_{31}(Q,S) = {} & p_1(1-p(n))Q + c_1\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} + c_1\, I_e p(n)Q(M-t_3) \\ & + \left[\frac{\{(1-p(n))\lambda - D\}t_2^{\,2} I_e\, p_1}{2} + \frac{DM^{\,2} I_e\, p_1}{2} + S\, p_1\, I_e(M-t_2)\right] \\ & - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\ & - \left[\frac{[(1-p(n)Q - DM - \{(1-p(n))\lambda - D\}t_2)]cI_p(T_1-M)}{2}\right]\end{aligned} \quad (46)$$

$$\begin{aligned}\Psi_{32}(Q,S) = {} & p_2(1-p(n))Q + c_2\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} + c_2\, I_e p(n)Q(M-t_3) \\ & + \left[\frac{\{(1-p(n))\lambda - D\}t_2^{\,2} I_e\, p_2}{2} + \frac{DM^{\,2} I_e\, p_2}{2} + S\, p_2\, I_e(M-t_2)\right] \\ & - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right] \\ & - \left[\frac{[(1-p(n)Q - DM - \{(1-p(n))\lambda - D\}t_2)]cI_p(T_1-M)}{2}\right]\end{aligned} \quad (47)$$

$$\begin{aligned}\Psi_{33}(Q,S) = {} & p_3(1-p(n))Q + c_3\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} + c_3\, I_e p(n)Q(M-t_3) \\ & + \left[\frac{\{(1-p(n))\lambda - D\}t_2^{\,2} I_e\, p_3}{2} + \frac{DM^{\,2} I_e\, p_3}{2} + S\, p_3\, I_e(M-t_2)\right] \\ & - h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3).L_2}{2}\right] \\ & - \left[\frac{[(1-p(n)Q - DM - \{(1-p(n))\lambda - D\}t_2)]cI_p(T_1-M)}{2}\right]\end{aligned} \quad (48)$$

The values of $\Psi_{31}(Q,S), \Psi_{32}(Q,S)$ *and* $\Psi_{33}(Q,S)$ from Equations (46) ,(47) and (48)are substituted in Equation (45), we get

$$\Psi_3(Q,S)=\frac{\begin{array}{l}(p_1+p_2+p_3)\left[(1-p(n))Q+\frac{\{(1-p(n))\lambda-D\}M^2I_e}{2}+\frac{DM^2I_e}{2}+SI_e(M-t_2)\right]\\+(c_1+c_2+c_3)[p(n)Q+I_p p(n)Q(t_3-M)]-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}\\-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\\-\left[\frac{[(1-p(n))Q-DM-\{(1-p(n))\lambda-D\}t_2]\ cI_p(T_1-M)}{2}\right]\end{array}}{3T} \tag{49}$$

4.4. The total profit of retailers for case 4 in fuzzy environment $t_2 \le t_3 \le T_1 \le M$

$$\begin{aligned}\Psi_4(Q,S)=&\tilde{p}(1-p(n))Q+\tilde{c}_s\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}+\tilde{c}_s\,I_e\,p(n)Q(M-t_3)\\&+\left[\frac{\{(1-p(n))\lambda-D\}t_2{}^2I_e\,\tilde{p}}{2}+\frac{DT_1{}^2I_e\,\tilde{p}}{2}+S\,\tilde{p}\,I_e(M-t_2)+DT_1I_e\,\tilde{p}\,(M-T_1)\right]\\&-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\end{aligned} \tag{50}$$

Now we de-fuzzify the total profit per unit time by Centroid Method

$$\Psi_4(Q,S)=\frac{[\Psi_{41}(Q,S)+\Psi_{42}(Q,S)+\Psi_{43}(Q,S)]}{3T} \tag{51}$$

$$\begin{aligned}\Psi_{41}(Q,S)=&\tilde{p}_1(1-p(n))Q+c_1\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}+c_1\,I_e\,p(n)Q(M-t_3)\\&+\left[\frac{\{(1-p(n))\lambda-D\}t_2{}^2I_e\,p_1}{2}+\frac{DT_1{}^2I_e\,p_2}{2}+S\,p_1\,I_e(M-t_2)+DT_1I_e\,\tilde{p}_1\,(\tilde{M}-T_1)\right]\\&-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\end{aligned} \tag{52}$$

$$\begin{aligned}\Psi_{42}(Q,S)=&p_2(1-p(n))Q+c_2\,p(n)Q-K-cQ-\beta Q-\frac{C_2(t_1+t_2)S}{2}+c_2\,I_e\,p(n)Q(M-t_3)\\&+\left[\frac{\{(1-p(n))\lambda-D\}t_2{}^2I_e\,p_2}{2}+\frac{DT_1{}^2I_e\,p_2}{2}+S\,p_2\,I_e(M-t_2)+DT_1I_e\,p_2(M-T_1)\right]\\&-h(n)\left[\frac{t_2(Q+L_1)}{2}+\frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2}+\frac{(T_1-t_3)L_2}{2}\right]\end{aligned} \tag{53}$$

$$\Psi_{43}(S,Q) = p_3(1-p(n))Q + c_3\, p(n)Q - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2} + c_3\, I_e\, p(n)Q(M-t_3)$$
$$+\left[\frac{\{(1-p(n))\lambda - D\}t_2^{\,2} I_e\, p_3}{2} + \frac{DT_1^{\,2} I_e\, p_3}{2} + S\, p_3\, I_e(M-t_2) + DT_1 I_e\, p_3(M-T_1)\right] \tag{54}$$
$$-h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_2-t_3)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right]$$

The values of $\Psi_{41}(S,Q), \Psi_{42}(S,Q)$ *and* $\Psi_{43}(S,Q)$ from Equations (52) ,(53)and (54)are substituted in Equation (51), we get

$$\Psi_4(S,Q) = \frac{\begin{array}{l}(p_1+p_2+p_3)\left[\begin{array}{l}(1-p(n))Q + \frac{\{(1-p(n))\lambda - D\}I_e M^2}{2} + \frac{DM^2 I_e}{2} + SI_e(M-t_2)\\ + DT_1\, I_e.(M-t_3)\end{array}\right]\\ +(c_1+c_2+c_3)\left[p(n)Q + I_p\, p(n)Q(t_3-M)\right] - K - cQ - \beta Q - \frac{C_2(t_1+t_2)S}{2}\\ -h(n)\left[\frac{t_2(Q+L_1)}{2} + \frac{(t_3-t_2)(L_1+L_2+p(n)Q)}{2} + \frac{(T_1-t_3)L_2}{2}\right]\end{array}}{3T} \tag{55}$$

4.5. The total profit of retailers for all the casesin fuzzy environment

This case consists of four discrete cases for the entire fuzzy profit percycle of the retailer, which can be depicted as follows –

$$\Psi(S,Q) = \begin{cases}\Psi_1(S,Q), & T_1 \ge t_3 \ge t_2 \ge M \quad case \quad 1\\ \Psi_2(S,Q), & T_1 \ge t_3 \ge M \ge t_2 \quad case \quad 2\\ \Psi_3(S,Q), & T_1 \ge M \ge t_3 \ge t_2 \quad case \quad 3\\ \Psi_4(S,Q), & T_1 \le M \quad case \quad 4\end{cases} \tag{56}$$

5. SOLUTION PROCEDURE

Our primary target is to identify the maximum and the most favourable values of $Q$and S which optimize the whole profit /gain function $\Psi^*(Q,S)$ and the total fuzzy profit $\Psi(Q,S)$, therefore the necessary conditions so that $\Psi^*(Q,S)$ and $\Psi(Q,S)$are to be optimal are

$$\frac{\partial \Psi^*(S,Q)}{\partial Q} = 0,\ \frac{\partial \Psi^*(S,Q)}{\partial S} = 0 \quad and \quad \frac{\partial \Psi(S,Q)}{\partial Q} = 0,\ \frac{\partial \Psi(S,Q)}{\partial S} = 0.$$

The following sufficient conditions must be satisfied, for the total profit function and total fuzzy profit function to concave -

$$\left(\frac{\partial^2 \Psi^*(Q,S)}{\partial Q \partial S}\right)^2 - \left(\frac{\partial^2 \Psi^*(Q,S)}{\partial Q^2}\right) X \left(\frac{\partial^2 \Psi^*(Q,S)}{\partial S^2}\right) \le 0,\ and\ \left(\frac{\partial^2 \Psi^*(Q,S)}{\partial Q^2}\right) \le 0,\ \left(\frac{\partial^2 \Psi^*(Q,S)}{\partial S^2}\right) \le 0$$

And,

$$\left(\frac{\partial^2\Psi\ (Q,S)}{\partial Q\partial S}\right)^2-\left(\frac{\partial^2\Psi\ (Q,S)}{\partial Q^2}\right)X\left(\frac{\partial^2\Psi\ (Q,S)}{\partial S^2}\right)\le 0,\ and\ \left(\frac{\partial^2\Psi\ (Q,S)}{\partial Q^2}\right)\le 0,\ \left(\frac{\partial^2\Psi\ (Q,S)}{\partial S^2}\right)\le 0$$

Second order derivatives are very complicated when they are calculated. Therefore, it is a hassle some process to prove the concavity by a mathematical approach. Hence, the total profit functions' concavity and total fuzzy profit function's concavity has been graphicallydepicted and graph for one of the cases has been shown(Fig.7-Fig.14). Now,for finding the maximum values of S^* and Q^* which maximizes the entire fuzzy gain. Here, we adopt the following algorithm for the solution

Stage 1: Enter the values of $[D,K,ho,h_1,\lambda,c,p,c_s,\beta,C_2,p(n),n,M,I_e,I_p]$ and $[D,K,ho,h_1,\lambda,c,p_1,p_2,p_3\ c_{1s}\ c_3,c_3\ \beta,C_2,p(n),n,M,I_e,I_p]$ into Equations (31) and (40)
Stage 2: Find out $Q^*=Q_1(say)\ and\ S^*=S_1(say)$ from Equations (21) and (33). Now, using the values of Q_1 and S_1, compute the numerical values of t_2, t_3 and T_1 from Equations (3),(6) and (9). If $M\le t_2\le t_3\le T_1$, thenthetotal profit's and total fuzzy profit's optimized values are obtained from Equations (21) and (33).
Stage 3: Repeat Step 2 for the case 2, $Q^*=Q_2(assume)\ and\ S^*=S_2(assume)$ from Equations (24) and (35) . Now, utilizing the values of Q_2 and S_2 , compute the values of t_2,t_3 and T_1 from equations (3),(6) and (9). If $T_1\ge t_3\ge M\ge t_2$, after that the optimized values of total gain and total fuzzy profit are obtained from Equations (24) and (35)
Stage 4: Repeat Step 2 for case 3, from Equations (27) and (37) . Now, utilizing the values of Q_3 and S_3 , compute the numerical values of t_2,t_3 and T_1 from Equations (3),(6) and(9). If $T_1\ge M\ge t_3\ge t_2$, after that the optimized values of total profit and total fuzzy profit are obtained from Equations (27) and (37).
Stage5: Repeat Stage 2 for the case 4, $Q^*=Q_4(say)\ and\ S^*=S_4(say)$ from Equations(30) and (39) . Now, utilizing the values of Q_4 and S_4 , compute the numerical values of t_2,t_3 and T_1 from equations (3), (6) and (9). If we take $T_1\le M$, then the optimized values of total gain and total fuzzy profit are obtained from Equations (30) and (39).

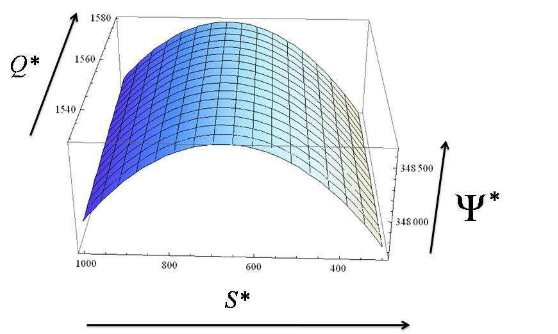

Fig. 7 Concavity of expected total profit function (Case 3)

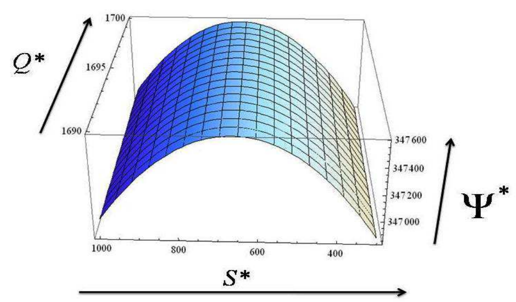

Fig. 8 Concavity of expected total profit function (Case 3)

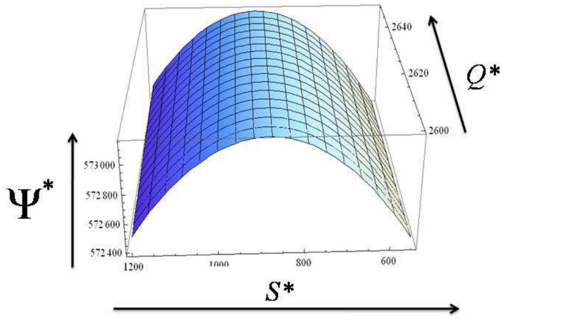

Fig. 9 Concavity of expected total profit function (Case 3)

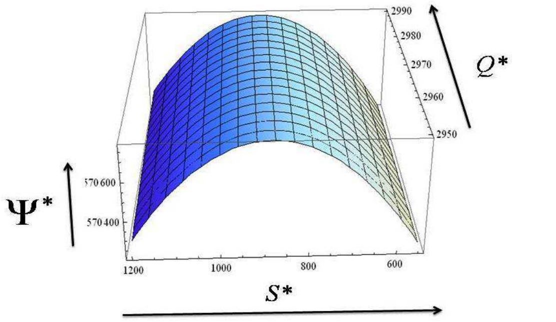

Fig. 10 Concavity of expected total profit function (Case 3)

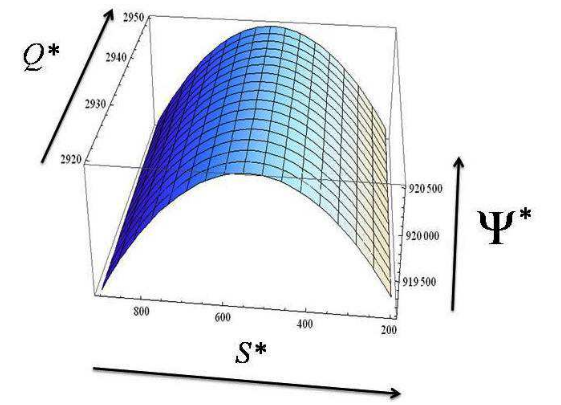

Fig. 11 Concavity of expected total profit function (Case 3)

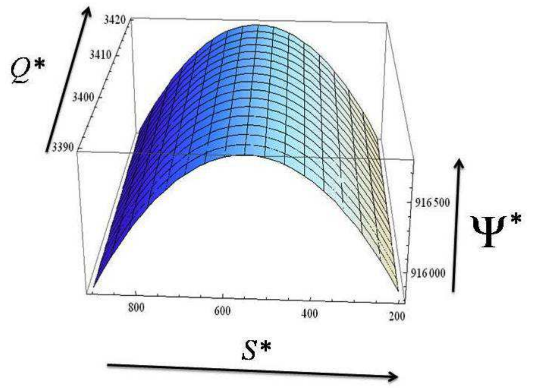

Fig. 12 Concavity of expected total profit function (Case 3)

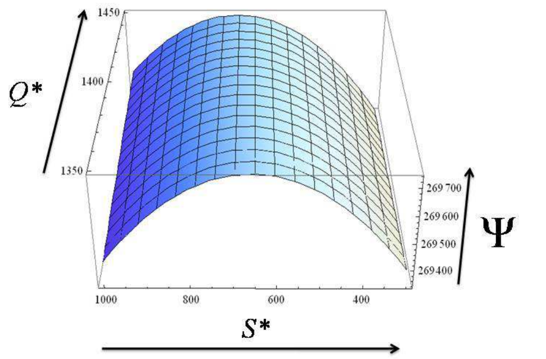

Fig. 13 Concavity of expected total profit function (Case 3)

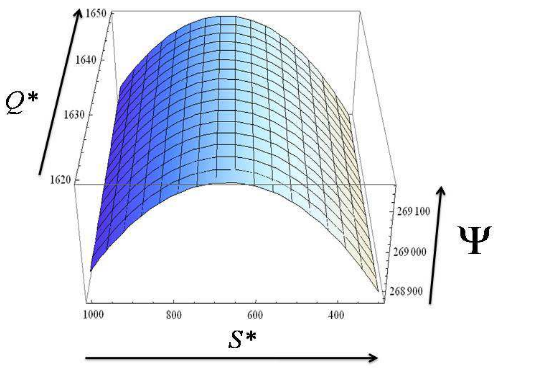

Fig. 14 Concavity of expected total profit function (Case 3)

6. NUMERICAL EXAMPLES

6.1. Example-1

A suitable example has been devised to demonstrate the allowable late in payments' effects on the retailer's ordering strategy for the urbanized model with the following below data.

$$D = 15000\, units\ per\ year, K = 400\$\, per\ cycle,\ ho = 4\$\ per\ unit\ per\ year,\ h1 = 1\$\, per\ unit\ per\ year,$$
$$\lambda = 60000 unit\ per\ year,\ c = 35\$\, per\ unit,\ p = 60\$\ per\ unit,\ c_s = 25\$\ per\ unit,\ \beta = 1.0\$\ per\ unit, b = 1,$$
$$C_2 = 6\$\, per\ year,\ n = 1,\ p(n) = 0.0399333,\ \gamma = 0.20,\ M = 18/360\, year.$$

Two cases have been assumed

(a) Suppose $I_e = 8\%\ per\ year\ and\ \ I_p = 10\%\ per\ year,\ \left(cI_p = 3.5 < 4.80 = pIe\right)$

Outcome have been got by use of the planned algorithm as $Q^* = 1554\, units,\ S^* = 680\, units,$ replacing the optimal values of Q^* in equations (1) and (6), we obtain

$$T^* = 0.099\ year,\ t_3^* = 0.025\, year\ and\ \ \Psi^*(Q,S) = 348660\$.$$

The total expected profit corresponding tothis ordered quantity and back order is graphically presented in Figure -7 using the Case-3(best case).

(b) $I_e = 4\%\ and\ \ I_p = 7\%,\ \left(cI_p = 2.45\ >\ 2.4 = pIe\right)$

Results have been computed using the planned algorithm as replacing the maximum values of $Q^* = 1698\ units, S^* = 666\ units,$ in equations (1) and (6), we obtain

$$T^* = 0.108\ year,\ t_3^* = 0.028\, year, and\ \ \Psi^*(Q,S) = 347588\$.$$

The total expected profit corresponding to this ordered quantity and backorder is graphically presented in Figure 8 using the Case-3(best case).

Now the output of the base model with same parameters without learning effects is that $Q^* = 1642\, units, S^* = 674\, units,$

$$T^* = 0.104\ year,\ t_3^* = 0.0274\, year, and\ \ \Psi^*(Q,S) = 347086\$.$$

In the present model,the ordered quantity is less as compared to that of the base model owing to the separation of defective and non-defective quality items from the lot and the backorder is geater due to unsatisfied demand and therefore,the retailer will earn a profit, due learning in cost reduction.

(b) $I_e = 4\%\ and\ \ I_p = 7\%,\ \left(cI_p = 2.45\ >\ 2.4 = pIe\right)$

$$Q^* = 1804\, units, S^* = 653\, units,$$

$$T^* = 0.114\ year,\ t_3^* = 0.0301\, year, and\ \ \Psi^*(Q,S) = 346095\$.$$

On comparison, it is deduced that more profit is obtained as compared to that in the base model (Jaggi et al.(2013)) owingto the learning effects. It is easily evident in each of the numerical examples which are explained below.

6.2. Example-2

A suitable example has been devised to demonstrate the permittedsetback in payments' effects on the retailer's ordering strategy for the urbanized model using the following data.

$$D = 25000\, units\ per\ year, K = 1000\$\ per\ cycle,\ ho = 3\$\ per\ unit\ per\ year, h1 =, 2\$\ per\ unit\ per\ year,$$
$$\lambda = 60000 unit\ per\ year, c = 45\$\ per\ unit, p = 70\$\ per\ unit, c_s = 30\$\ per\ unit, \beta = 1.0\$\ per\ unit, b = 1,$$
$$C_2 = 7\$\ per\ year, n = 1, p(n) = 0.0399333,\ \gamma = 0.20,\ M = 20/360\, year.$$

Two cases have been taken into assumption,

(a) Suppose $I_e = 10\%\ per\ year\ and\ \ I_p = 12\%\ per\ year,\ \left(cI_p = 5.4 < 7 = pIe\right)$

Results have been got using the planned algorithm as $Q^* = 2637\ units, S^* = 892\ units,$ replacing the maximum values of Q^* in equations (1) and (6), we obtain

$T^* = 0.101\ year,\ t_3^* = 0.043\ year, and\ \ \Psi^*(Q,S) = 573144\$.$

The total expected profit corresponding to this ordered quantity and backorder is graphically presented in Figure -9 using the case-3 (best case).

(b) $I_e = 5\%\ \ and\ \ I_p = 8\%, \left(cI_p = 3.6 > 3.5 = pI_e\right)$

The results have been derived by applying the planned algorithm as $Q^* = 2976\ units, S^* = 889 units,$ replacing the maximum values of Q^* in equations (1) and(6), we obtain

$T^* = 0.114\ year,\ t_3^* = 0.049 year, and\ \ \Psi^*(Q,S) = 570777\$.$

The total expected profit corresponding to this ordered quantity and backorder is graphically presented in Figure -10 using the case-3 (best case).

Now the output of the base model with same parameters without learning effects is as follows

$(a)\ I_e = 10\%\ per\ year\ and\ \ I_p = 12\%\ per\ year, \left(cI_p = 5.4 < 7 = pIe\right)$

$Q^* = 2671 units, S^* = 886\ units,$

$T^* = 0.101/\ year,\ t^*{}_3 = 0.0445 year\ and\ \ \Psi^*(Q,S) = 568588\$..$

(b) $I_e = 5\%\ \ and\ \ I_p = 8\%, \left(cI_p = 3.6 > 3.5 = pI_e\right)$

$Q^* = 3004\ units, S^* = 881\ units,$.

6.3. Example-3

A suitable example has been devised to demonstrate the permitted setback in cash effect on the retailer's demanding strategy for the urbanized model by use of the subsequent data.

$$D = 40000\ units\ per\ year, K = 1000\$\ per\ cycle,\ ho = 3\$\ per\ unit\ per\ year, h1 =, 2\$\ per\ unit\ per\ year,$$
$$\lambda = 60000 unit\ per\ year, c = 45\$\ per\ unit,\ p = 70\$\ per\ unit, c_s = 30\$\ per\ unit, \beta = 1.0\$\ pertunit, b = 1,$$
$$C_2 = 7\$\ per\ year, n = 1,\ p(n) = 0.0399333;\ \gamma = 0.20,\ M = 20/360\ year.$$

Two cases have been considered,

(a) Suppose $I_e = 10\%\ per\ year\ \ and\ \ I_p = 12\%\ per\ year, \left(cI_p = 5.4 < 7 = pI_c\right)$

Results have been calculated and derived after adopting the planned algorithm as $Q^* = 2935\ units, S^* = 541\ units,$ by replacing the maximum values of Q^* in Equations (1) and (6), we obtain $T^* = 0.070\ \ year,\ t_3^* = 0.048 year,\ \ and\ \ \Psi^*(Q,S) = 920507\$.$

The total expected profit corresponding to this ordered quantity and backorderisgraphically presented in Figure -11 using the case-3 (best case)

(b) $I_e = 5\%\ \ and\ \ I_p = 8\%, \left(pI_e = 3.5 < 3.6 = cI_p\right)$

Results have been derived by the help of the planned algorithm and we got $Q^* = 3407\ units, S^* = 549 units,$ by replacing the maximum values of Q^* in Equations (1) and (6), we obtain $T^* = 0.081\ year,\ t_3^* = 0.056 year, and\ \ \Psi^*(Q,S) = 916769\$.$

The total expected profit corresponding to this ordered quantity and backorder is graphically presented in Figure 12 using the case-3 (best case).

Now the output of the base model with the same parameters without learning effects is as follows

(a) $I_e = 10\%\, per\ year\ \ and\ \ I_p = 12\%\ per\ year,\ \left(cI_p = 5.4 < 7 = pI_c\right)$

$Q^* = 2941\, units, S^* = 530\, units,$

$T^* = 0.070\ year,\ t^*{}_3 = 0.049 year,\ and\ \Psi^*(Q,S) = 914760\$..$

(b) $I_e = 5\%\ \ and\ \ I_p = 8\%,\ \left(pI_e = 3.5 <\ 3.6 = cI_p\right), Q^* = 3439 units, S^* = 535\, units,$

$T^* = 0.082\ year,\ t^*{}_3 = 0.0573 year,\ and\ \Psi^*(Q,S) = 910292\$..$

6.4. Fuzzy numerical example

As per considerations, the unit selling price of defective and non-defective items are imprecise in nature. For verification and establishing the accuracy of the model we have taken a suitable example which has been devised for illustrating the allowable set back in payments' effects on the retailer's demanding strategy for the urbanized Fuzzy model by use of subsequent data.

$D = 15000\ units\ per\ year, K = 400\$\ per\ cycle,\ ho = 4\$\ per\ unit\ per\ year, h1 =,1\$\ per\ unit\ per\ year,$

$\lambda = 60000 unit\ per\ year, c = 35\$\ per\ unit,\ \tilde{p} = (20,\ 30,\ 40),\ \tilde{c}_s = (5,\ 15,\ 20),\ \beta = 1.0\$\ per\ unit, b = 1,$

$C_2 = 6\$\ per\ year, n = 1,\ p(n) = 0.0399333, \gamma = 0.20,\ \ M = 18/360\, year.$

Two cases have been assumed

(a) Suppose $I_e = 8\%\ per\ year,\ I_p = 10\%\ per\ year,\ \left(pI_e = 4.80 > 3.5 = cI_p\right)$

Results have been derived by following the planned algorithm as $Q^* = 1414\ units, S^* = 683\ units,$ by replacing the maximum values of Q^* in Equations (1) and (6), we obtain

$T^* = 0.090\ \ year,\ t^*{}_3 = 0.023\, year\ \ and\ \ \Psi(Q,S) = 269736.\$$

The total expected profit corresponding to this ordered quantity and backorder is graphically presented in Figure -13 using the case-3(best case)

(b) $I_e = 4\%\ and\ \ I_p = 7,\%\ \left(pI_e = 2.4\ <\ 2.45 = c.I_p\right)$

Results have been derived by following the planned algorithm as $Q^* = 1634\ units, S^* = 674\ units,$ by replacing the maximum values of Q^* in Equations (1) and (6), we obtain

$T^* = 0.104\ \ year,\ t^*{}_3 = 0.027\, year\ \ and\ \ \Psi\ (Q,S) = 269157\$.$

The total expected profit corresponding to this ordered quantity and backorder is graphically presented in Figure -14 using the case-3(best case)

The behaviour of the considered fuzzy model has been briefy explained with the help of the numerical example under the discussion heading. This numerical fuzzy example explained that if the value of unit selling of non-defective items was in the suitable range (20-40) and also if the value of unit selling price of the defective items was in the range (5-15), then such values were beneficial for the buyer and the seller during successful dealing transactions in business.

Taking Table 2 into consideration it can be analyzed and deduced that as the learning rate increases (1-1.5), the organization's profit increases. FromTable 3, it can be observed that if the number of shipment increases (1-5) then the ordered quantity and profit of the organization increase due to the learning effect. FromTable 4, it can be analyzed that if M increases, then the organization's profit increases.

Table -2 Learning's effects on profit

Number of Shipment (n)	Rate of Learning				
	$b=1$	$b=1.2$	$b=1.3$	$b=1.4$	$b=1.5$
1	348660	348664	348666	348669	348672
2	348730	348754	348771	348790	348814
3	348834	348939	349018	349121	349255
4	349063	349439	349737	350130	350626
5	349587	350635	351385	352226	353065

Table-3 Learning's effects on lot size, backorder, defective items' percentage, holding cost and profit

Number of Shipment	Lot Size	Backorder Quantities	Percentage of Defective items	Holding Cost	Total Profit
1	1554.05	680.043	0.0399333	5.00000	348660
2	1562.28	679.805	0.0397563	4.87055	348730
3	1566.37	679.891	0.0392914	4.80275	348834
4	1568.41	680.472	0.0381017	4.75786	349063
5	1568.45	682.079	0.0352518	4.72478	349587

Table-4Learning's effects on lot size, backorder quantities and profit

Number of Shipment (n)	Trade Credit Period								
	$M=10$			$M=20$			$M=30$		
	Q	S	$\Psi^*(Q,S)$	Q	S	$\Psi^*(Q,S)$	Q	S	$\Psi^*(Q,S)$
1	1589.83	673.294	347127	1541.72	680.354	349059	1458.02	673.935	351164
2	1598.27	672.860	347199	1549.88	680.166	349129	1465.68	672.680	351230
3	1602.47	672.843	347305	1553.94	680.277	349233	1469.49	673.235	351332
4	1604.56	673.361	347536	1555.95	680.873	349461	1471.37	673.897	351558
5	1605.00	674.931	348062	1556.00	682.485	349984	1471..29	675.518	352077

Observations

In this research paper we have discussed about the cases, and have tried to determine which case was better for this model after we procured the solution with the assistance of the concerned algorithm and compared it this paper to that of Jaggi et al.(2013) paper. All the numerical input parameters were taken from the previous model excluding the learning and fuzzy parameters using present mathematical model. Learning effects acted as cost reduction parameters when implemented by the buyer. For the motive of generating more profit, the fuzzyness technique was used by the seller to decide the unit selling price for defective and non-defective items that would be beneficial for the buyers. Conclusively, if there was no learning and no fuzzy concept present in this model then it would go back approximately to the base model, as per the mathematical aspects discussed in each numerical example mentioned above briefly. The ordered quantity was less as compared to that in the base model due to the separation of defective items from the lot and because of the fact that the backorder's quantity was more when compared to that in the base model owing to the demand of good items but profit was more as compared to that of the base model due to the learning effect. After getting all the values from the above four cases, we concluded that the maximum profit was given by case-4 $T_1 \leq M \leq T$.

But this was not always the situation that the credit period would lie beyond the total cycle length (T). Hence, it is not beneficial from the seller's perception. So after pondering upon the same, we considered that, Case-3 ($t_2 \le t_3 \le M \le T_1$) was perfect and apt to be implemented for any situation. This case gave us the approximate value for all the parameters.

7. CONCLUSIONS

The original EOQ model had not been apt for the conditions when lots had been ordered, as they had some imperfect quality articles. Consequently, new modified models were required for more pragmatic solutions in genuine daily-life scenarios. Eventually such a required EOQ formulation was framed when each lot, which was ordered,contained some imperfect quality articles and shortages were backlogged with financing under fuzzy environment with learning process. This model provided the foundation to theconclusion to consider the learning effect simultaneously while taking decisions which would as a result help them to generate greater gain for the system. Optimized batch size had been obtained by employing the calculus' method in order to optimize the entire gain function. The credibility and utility of the developed model were checked through numerical examples. Learning phenomenon is related to scheduling, uncontrolled inventory management, quality management, inspection, unbalance supply chain management. Finally, we were able to generatemore profit under such assumptions which have been stated under section-2.1 due to the impact of learning in holding cost and percentage of defective articles under the fuzzy environment. We have efficiently and mathematically compared the parameters and the associated observations from the base paper with learning effect as well as without learning effect and the same has been shown through the medium of numerical examples. This paper allows scope for extention for more realistic situations such as deteriorating items, stock dependencies and stochastic demands with partial-trade credit,etc.

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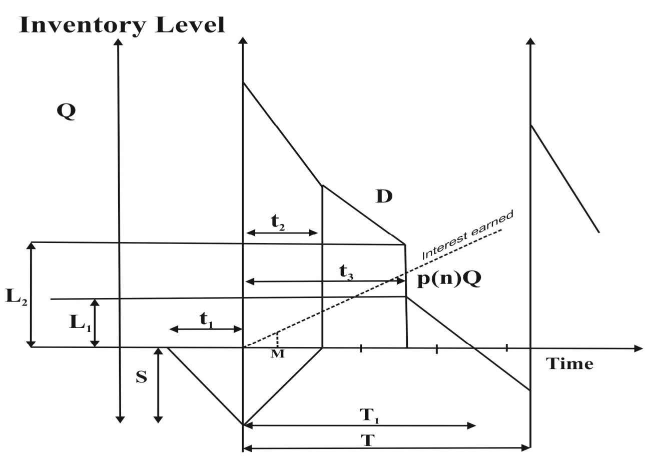
Appendix-1: Calculation of interest earn and interest charges for the different cases:



For case-1:

The interest earned for the cycle from the time period 0 to M

$$IE = pI_e\left(\left(1-p(n)\right)\lambda - D\right) + D\Big)\int_0^M t\,dt$$

Or $IE = \left[\left(1-p(n)\right)\lambda - D\right]I_e pM^2/2 + M^2 DpI_e/2$

And the interest paid per cycle for the inventory not sold after trade period

M to T_1

$$IC = \left[\left(1-p(n)\right)Q - DM - \{\left(1-p(n)\right)\lambda + D\}M\right]cI_p\left(T_1 - M\right)/2 + cpI_p p(n)Q\left(t_3 - M\right)$$

(A1)

For case-2

Similarly, can be calculated from the figure-3

$$IE = \left[\left(1-p(n)\right)\lambda - D\right]I_e t_2^{\,2} p/2 + SpI_e\left(-t_2 + M\right) + M^2 DI_e p/2$$

And

$$IC = \left[\left(1-p(n)\right)Q - DM - \{\left(1-p(n)\right)\lambda + D\}t_2\right]cI_p\left(T_1 - M\right)/2 + cpI_p p(n)Q\left(t_3 - M\right)$$

(A2)**For case-3**Similarly, can be calculated from the figure-3

$$IE = \left[\left(1-p(n)\right)\lambda - D\right]t_2^{\,2} I_e p/2 + SpI_e\left(M - t_2\right) + DM^2 I_e p/2 + I_e c_s p(n)Q\left(M - t_3\right) \text{ and}$$

$$IC = \left[Q\left(1-p(n)\right) - MD - \{\left(1-p(n)\right)\lambda + D\}t_2\right]I_p c\left(T_1 - M\right)/2. \qquad (A_3)$$

For case-4 Similarly, can be calculated from the figure-3

$$IE = \left[\left(1-p(n)\right)\lambda - D\right]t_2^{\,2} I_e p/2 + SpI_e\left(M - t_2\right) + DM^2 I_e p/2 + I_e c_s p(n)Q\left(M - t_3\right) + DT_1 I_e p\left(M - t_3\right)$$

$$IC = 0. \qquad (A_4)$$