OPTIMAL DECISION SUPPORT MIXTURE MODEL WITH WEIBULL DEMAND AND DETERIORATION

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ABSTRACT
In this paper, the performance of an inventory model is explored with deteriorating items under imprecision environment where the demand follows a three-parameter Weibull distribution. Deterioration and holding cost is considered as a linear function of time. Fuzziness has been allowed to deal with imprecision. Mathematical observations of both crisp and fuzzy models have been illustrated to determine the optimal cycle time and optimal inventory cost. The demand distribution, deterioration rate and all costs of models are expressed as triangular, trapezoidal and pentagonal fuzzy numbers. Graded mean integration method is used for defuzzification. Numerical illustrations are provided to validate the applications of the model. Sensitivity analysis with useful graphs and tables are performed to analyze the variability in the optimal solution with respect to change in various system parameters.

KEYWORDS: Weibull Demand, Triangular Fuzzy Number, Trapezoidal Fuzzy Number, Pentagonal Fuzzy Number, Graded Mean Integration

MSC: 90B05

RESUMEN
En este documento, se analiza el rendimiento de un modelo de inventario con elementos deteriorados en un entorno de imprecisión donde la demanda sigue una distribución de Weibull de tres parámetros. El deterioro y el costo de mantenimiento se consideran una función lineal del tiempo. Se ha permitido a la borrosidad lidiar con la imprecisión. Las observaciones matemáticas de los modelos nítidos y difusos se han ilustrado para determinar el tiempo de ciclo óptimo y el costo de inventario óptimo. La distribución de la demanda, la tasa de deterioro y todos los costos de los modelos se expresan como números borrosos triangulares, trapezoidales y pentagonales. Se utiliza el método de integración de medios graduados para la defuzzificación. Se proporcionan ilustraciones numéricas para validar las aplicaciones del modelo. Se realizan análisis de sensibilidad con gráficos y tablas útiles para analizar la variabilidad en la solución óptima con respecto al cambio en varios parámetros del sistema.

1. INTRODUCTION

Most of the existing inventory models based on assumptions that the items can be stored indefinitely to face the future demands. Deteriorating items are common in our daily life. If the rate of deterioration is high, its impact on modeling of such an inventory system cannot be neglected. Deteriorating items refer to the items that become decayed, damaged, evaporative, expired, invalid, devaluation in course of time. But certain types of items either deteriorate or become obsolete with respect to time. The commonly used goods like fruits, vegetables, meat, foodstuffs, fashionable items, alcohol, gasoline, medicines, radioactive substances, photographic films, electronic devices, etc., where deterioration is commonly observed during their normal storage period.
Inventory model with Weibull demand was considered earlier by Tadikamalla [16]. Ghosh *et. al.* [7] developed an inventory model with Weibull demand rate and production rate is assumed as finite. Tripathy and Pradhan [17] suggested inventory model having Weibull demand and variable deterioration rate. Covert and Philip [3], Giri *et. al.* [8], Ghosh and Choudhury [6] developed model with Weibull distribution deterioration with various pattern of demand. One of the weaknesses of the current model which is mostly used in business world is the unrealistic assumption of the different parameters. Fuzzy inventory models are more realistic than the traditional inventory models. The uncertainties are due to fuzziness and such cases explained in the fuzzy set theory which was demonstrated by Zadeh [21], Kaufman and Gupta [10]. Syed and Aziz [15] discussed a fuzzy inventory model using signed distance method. Chang *et. al.* [2], De and Rawat [4] and Jaggi *et. al* [9]. developed fuzzy models for deteriorating items and demand using triangular fuzzy

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number. Yao and Lee [20] discussed fuzzy inventory with or without backorder for fuzzy order quantity with trapezoidal fuzzy number. Since then many related research ware found in Dutta and Kumar [5], Mohanty and Tripathy [11], Nagar and Surana [12], Sahoo *et. al.* [14], Behera and Tripathy [1], Sahoo and Tripathy [13], Tripathy and Sahoo [18], Tripathy and Sukla [19] developed improved inventory model in fuzzy sense using triangular, trapezoidal and pentagonal fuzzy numbers separately.
This paper is developed for deteriorating items under imprecision condition where the demand function follows three-parameter Weibull distribution. Deterioration rate and holding cost are considered as time varying linear function. Mathematical models are developed for both crisp and fuzzy sense. The models have been illustrated to determine the optimal cycle time and inventory cost. The demand distribution, deterioration rate and all costs are measured in triangular, trapezoidal and pentagonal fuzzy numbers. The fuzzy model is defuzzified by using graded mean integration method. Empirical investigations are provided to validate the applications of model. Sensitivity analyses have been carried out to study the variability in the optimal solution with respect to change in various system parameters.

2. NOTATIONS AND ASSUMPTIONS

i. $d(t)$ is the demand rate per unit time.
ii. O_c is the ordering cost per order.
iii. ϕ is the deterioration rate per unit time.
iv. u_c is the unit cost per unit time.
v. h_c is the inventory holding cost per unit per unit time.
vi. t_2 is the length of the cycle.
vii. Q is the ordering quantity per unit.
viii. s_c is the shortage cost per unit time.
ix. $C(t_1,t_2)$ is the total inventory cost per unit time.
x. $C^{*}{}_{GM}(t_1,t_2)$ is the defuzzified value of $\tilde{C}(t_1,t_2)$ by applying graded mean integration method using parameters are triangular fuzzy number.
xi. $C^{**}{}_{GM}(t_1,t_2)$ is the defuzzified value of $\tilde{C}(t_1,t_2)$ by applying graded mean integration method using parameters are trapezoidal fuzzy number.
xii. $C^{***}{}_{GM}(t_1,t_2)$ is the defuzzified value of $\tilde{C}(t_1,t_2)$ by applying graded mean integration method using parameters are pentagonal fuzzy number.
xiii. Demand $d(t) = \alpha\beta(t-\gamma)^{\beta-1}$ is three parameter Weibull distribution function where $\alpha > 0$ is scale parameter, $\beta > 0$ is shape parameter and $0 < \gamma < 1$ is location parameter.
xiv. $\phi(t) = a + bt$ is the time varying deterioration rate.
xv. $h_c = p + qt$ is holding cost per unit time.
xvi. The replenishment rate is instantaneous and lead-time is zero.
xvii. Shortages are allowed and fully backlogged.

3. MATHEMATICAL MODEL

Let $I(t)$ be the on-hand inventory at any time t with initial inventory Q. During the period $[0, t_1]$ the on-hand inventory is depleted due to demand and deterioration and also completely exhausted at time t_1. The period $[t_1, t_2]$ is the period of shortages, which are fully backlogged. At any instant of time the inventory level $I(t)$ is expressed by the differential equation.

3.1. Crisp Model

$$\frac{dI(t)}{dt} + \phi(t)I(t) = -d(t), 0 \le t \le t_1 \tag{1}$$

with $I(0) = Q$ and $I(t_1) = 0$.

And

$$\frac{dI(t)}{dt} = -d(t), t_1 \le t \le t_2 \tag{2}$$

with $I(t_1) = 0$

Solutions of equation (1) and (2) are

$$I(t) = \begin{cases} \alpha\left(\begin{array}{l} (t_1-\gamma)^{\beta}(1 - at - \frac{bt^2}{2} + at_1 + \frac{bt_1^{\,2}}{2}) - \frac{(t_1-\gamma)^{\beta+1}}{(\beta+1)}(a+bt_1) \\ + \frac{b(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - (t-\gamma)^{\beta} + \frac{(t-\gamma)^{\beta+1}}{(\beta+1)}(a+bt) - \frac{b(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} \end{array}\right), & 0 \le t \le t_1 \\ \alpha((t_1-\gamma)^{\beta} - (t-\gamma)^{\beta}), & t_1 \le t \le t_2 \end{cases} \tag{3}$$

Inventory is available in the system during the time period $(0, t_1)$. Hence the cost for holding inventory in stock is computed for time period $(0, t_1)$ only.

Total no. of holding cost unit IHC during period $[0, t_2]$ is given by

$$\begin{aligned} IHC = \int_0^{t_1} h_c I(t)dt = p\alpha &\left(\begin{array}{l} (t_1-\gamma)^{\beta}(t_1 + \frac{at_1}{2}^2 + \frac{bt_1^3}{3}) - \frac{(t_1-\gamma)^{\beta+1}}{(\beta+1)}(1 + at_1 + bt_1{}^2) + \frac{(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}(a+2bt_1) \\ - \frac{2b(t_1-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} + \frac{(-\gamma)^{\beta+1}}{\beta+1} - \frac{a(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{2b(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} \end{array}\right) \\ + q\alpha &\left(\begin{array}{l} (t_1-\gamma)^{\beta}(\frac{t_1^{\,2}}{2} + \frac{at_1^{\,3}}{6} + \frac{bt_1^{\,4}}{8}) - \frac{(t_1-\gamma)^{\beta+1}}{(\beta+1)}(t_1 + \frac{at_1^{\,2}}{2} + \frac{bt_1^{\,3}}{2}) \\ + \frac{(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}(1 + at_1 + \frac{3bt_1^{\,2}}{2}) - \frac{(t_1-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}(a+3bt_1) \\ + \frac{3b(t_1-\gamma)^{\beta+4}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)} - \frac{(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{a(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3b(-\gamma)^{\beta+4}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)} \end{array}\right) \end{aligned} \tag{4}$$

Total no of deteriorated units I_D during period $[0, t_2]$ is given by

$$\begin{aligned} I_D &= u_c[Q - Total\,Demand] \\ &= u_c\alpha\left((t_1-\gamma)^{\beta}(at_1 + \frac{bt_1^{\,2}}{2}) - \frac{(t_1-\gamma)^{\beta+1}}{(\beta+1)}(a+bt_1) + \frac{b(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{a(-\gamma)^{\beta+1}}{(\beta+1)} - \frac{b(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right) \end{aligned} \tag{5}$$

Total no. of shortage units I_S during period $[0, t_2]$ is given by

$$I_S = s_c\int_{t_1}^{t_2} -I(t)dt = -s_c\alpha\left((t_1-\gamma)^{\beta}(t_2 - t_1) - \frac{(t_2-\gamma)^{\beta+1}}{\beta+1} + \frac{(t_1-\gamma)^{\beta+1}}{\beta+1}\right) \tag{6}$$

Total cost of the system per unit time is given by

$$\begin{aligned} C(t_1, t_2) &= \tfrac{1}{t_2}[O_c + IHC + I_D + I_S] \\ &= \tfrac{1}{t_2}\Bigg[O_c + p\alpha\left(\begin{array}{l} (t_1-\gamma)^{\beta}(t_1 + \frac{at_1}{2}^2 + \frac{bt_1^3}{3}) - \frac{(t_1-\gamma)^{\beta+1}}{(\beta+1)}(1 + at_1 + bt_1{}^2) + \frac{(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}(a+2bt_1) \\ - \frac{2b(t_1-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} + \frac{(-\gamma)^{\beta+1}}{\beta+1} - \frac{a(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{2b(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} \end{array}\right) \end{aligned}$$

$$+q\alpha\begin{pmatrix}(t_1-\gamma)^{\beta}(\frac{t_1^2}{2}+\frac{at_1^3}{6}+\frac{bt_1^4}{8})-\frac{(t_1-\gamma)^{\beta+1}}{(\beta+1)}(t_1+\frac{at_1^2}{2}+\frac{bt_1^3}{2})\\ +\frac{(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}(1+at_1+\frac{3bt_1^2}{2})-\frac{(t_1-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}(a+3bt_1)+\\ \frac{3b(t_1-\gamma)^{\beta+4}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}-\frac{(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{a(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}-\frac{3b(-\gamma)^{\beta+4}}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}\end{pmatrix}$$

$$\left.\begin{array}{l}+u_c\alpha\left((t_1-\gamma)^{\beta}(at_1+\frac{bt_1^2}{2})-\frac{(t_1-\gamma)^{\beta+1}}{(\beta+1)}(a+bt_1)+\frac{b(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{a(-\gamma)^{\beta+1}}{(\beta+1)}-\frac{b(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right)\\ -s_c\alpha\left((t_1-\gamma)^{\beta}(t_2-t_1)-\frac{(t_2-\gamma)^{\beta+1}}{\beta+1}+\frac{(t_1-\gamma)^{\beta+1}}{\beta+1}\right)\end{array}\right] \quad (7)$$

3.2. Fuzzy Model

It is not easy to determine all the system parameters due to uncertainty in the environment. Accordingly it is assumed that some of these parameters namely $\tilde{\alpha},\tilde{\beta},\tilde{\gamma},\tilde{a},\tilde{b},\tilde{u}_c,\tilde{s}_c,\tilde{p},\tilde{q}$ may change within some limits are triangular, trapezoidal and pentagonal fuzzy numbers.
Total cost of the system per unit time in fuzzy sense is governed by

$$\tilde{C}(t_1,t_2)=\frac{1}{t_2}\left[O_c+\tilde{p}\tilde{\alpha}\begin{pmatrix}(t_1-\tilde{\gamma})^{\tilde{\beta}}(t_1+\frac{\tilde{a}t_1^2}{2}+\frac{\tilde{b}t_1^3}{3})-\frac{(t_1-\tilde{\gamma})^{\tilde{\beta}+1}}{(\tilde{\beta}+1)}(1+\tilde{a}t_1+\tilde{b}t_1^2)+\frac{(t_1-\tilde{\gamma})^{\tilde{\beta}+2}}{(\tilde{\beta}+1)(\tilde{\beta}+2)}(\tilde{a}+2\tilde{b}t_1)\\ -\frac{2\tilde{b}(t_1-\tilde{\gamma})^{\tilde{\beta}+3}}{(\tilde{\beta}+1)(\tilde{\beta}+2)(\tilde{\beta}+3)}+\frac{(-\tilde{\gamma})^{\tilde{\beta}+1}}{\tilde{\beta}+1}-\frac{\tilde{a}(-\tilde{\gamma})^{\tilde{\beta}+2}}{(\tilde{\beta}+1)(\tilde{\beta}+2)}+\frac{2\tilde{b}(-\tilde{\gamma})^{\tilde{\beta}+3}}{(\tilde{\beta}+1)(\tilde{\beta}+2)(\tilde{\beta}+3)}\end{pmatrix}\right.$$

$$+\tilde{q}\tilde{\alpha}\begin{pmatrix}(t_1-\tilde{\gamma})^{\tilde{\beta}}(\frac{t_1^2}{2}+\frac{\tilde{a}t_1^3}{6}+\frac{\tilde{b}t_1^4}{8})-\frac{(t_1-\tilde{\gamma})^{\tilde{\beta}+1}}{(\tilde{\beta}+1)}(t_1+\frac{\tilde{a}t_1^2}{2}+\frac{\tilde{b}t_1^3}{2})\\ +\frac{(t_1-\tilde{\gamma})^{\tilde{\beta}+2}}{(\tilde{\beta}+1)(\tilde{\beta}+2)}(1+\tilde{a}t_1+\frac{3\tilde{b}t_1^2}{2})-\frac{(t_1-\tilde{\gamma})^{\tilde{\beta}+3}}{(\tilde{\beta}+1)(\tilde{\beta}+2)(\tilde{\beta}+3)}(\tilde{a}+3\tilde{b}t_1)\\ +\frac{3\tilde{b}(t_1-\tilde{\gamma})^{\tilde{\beta}+4}}{(\tilde{\beta}+1)(\tilde{\beta}+2)(\tilde{\beta}+3)(\tilde{\beta}+4)}-\frac{(-\tilde{\gamma})^{\tilde{\beta}+2}}{(\tilde{\beta}+1)(\tilde{\beta}+2)}+\frac{\tilde{a}(-\tilde{\gamma})^{\tilde{\beta}+3}}{(\tilde{\beta}+1)(\tilde{\beta}+2)(\tilde{\beta}+3)}-\frac{3\tilde{b}(-\tilde{\gamma})^{\tilde{\beta}+4}}{(\tilde{\beta}+1)(\tilde{\beta}+2)(\tilde{\beta}+3)(\tilde{\beta}+4)}\end{pmatrix}$$

$$\left.\begin{array}{l}+\tilde{u}_c\tilde{\alpha}\left((t_1-\tilde{\gamma})^{\tilde{\beta}}(\tilde{a}t_1+\frac{\tilde{b}t_1^2}{2})-\frac{(t_1-\tilde{\gamma})^{\tilde{\beta}+1}}{(\tilde{\beta}+1)}(\tilde{a}+\tilde{b}t_1)+\frac{\tilde{b}(t_1-\tilde{\gamma})^{\tilde{\beta}+2}}{(\tilde{\beta}+1)(\tilde{\beta}+2)}+\frac{\tilde{a}(-\tilde{\gamma})^{\tilde{\beta}+1}}{(\tilde{\beta}+1)}-\frac{\tilde{b}(-\tilde{\gamma})^{\tilde{\beta}+2}}{(\tilde{\beta}+1)(\tilde{\beta}+2)}\right)\\ -\tilde{s}_c\tilde{\alpha}\left((t_1-\tilde{\gamma})^{\tilde{\beta}}(t_2-t_1)-\frac{(t_2-\tilde{\gamma})^{\tilde{\beta}+1}}{\tilde{\beta}+1}+\frac{(t_1-\tilde{\gamma})^{\tilde{\beta}+1}}{\tilde{\beta}+1}\right)\end{array}\right] \quad (8)$$

The total fuzzy cost $\tilde{C}(t_1,t_2)$ is defuzzified by graded mean integration taking system parameters as triangular, trapezoidal and pentagonal fuzzy numbers.

Case-1: (If Parameters are triangular fuzzy numbers)
Considering $\alpha=(\alpha_1,\alpha_2,\alpha_3)$, $\beta=(\beta_1,\beta_2,\beta_3)$, $\gamma=(\gamma_1,\gamma_2,\gamma_3)$, $a=(a_1,a_2,a_3)$, $b=(b_1,b_2,b_3)$, $u_c=(u_1,u_2,u_3)$, $s_c=(s_1,s_2,s_3)$, $p=(p_1,p_2,p_3)$ and $q=(q_1,q_2,q_3)$ are triangular fuzzy numbers, the total cost is calculated as

$$C^*_{GM}(t_1,t_2)=\frac{1}{6}\left(C^*_{GM_1}(t_1,t_2)+4C^*_{GM_2}(t_1,t_2)+C^*_{GM_3}(t_1,t_2)\right) \quad (9)$$

where

$$C^*_{GM_1}(t_1,t_2)=\frac{1}{t_2}\left[O_c+p_1\alpha_1\begin{pmatrix}(t_1-\gamma_1)^{\beta_1}(t_1+\frac{a_1t_1}{2}^2+\frac{b_1t_1^3}{3})-\frac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(1+a_1t_1+b_1t_1^2)\\ +\frac{(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}(a_1+2b_1t_1)-\frac{2b_1(t_1-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}\\ +\frac{(-\gamma_1)^{\beta_1+1}}{\beta_1+1}-\frac{a_1(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\frac{2b_1(-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}\end{pmatrix}\right.$$

$$+q_1\alpha_1\left(\begin{array}{l}(t_1-\gamma_1)^{\beta_1}(\frac{t_1^2}{2}+\frac{a_1t_1^3}{6}+\frac{b_1t_1^4}{8})-\frac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(t_1+\frac{a_1t_1^2}{2}+\frac{b_1t_1^3}{2})\\+\frac{(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}(1+a_1t_1+\frac{3b_1t_1^2}{2})-\frac{(t_1-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}(a_1+3b_1t_1)\\+\frac{3b_1(t_1-\gamma_1)^{\beta_1+4}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)(\beta_1+4)}-\frac{(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\frac{a_1(-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}-\frac{3b_1(-\gamma_1)^{\beta_1+4}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)(\beta_1+4)}\end{array}\right)$$
$$+u_1\alpha_1\left((t_1-\gamma_1)^{\beta_1}(a_1t_1+\frac{b_1t_1^2}{2})-\frac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(a_1+b_1t_1)+\frac{b_1(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\frac{a_1(-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}-\frac{b_1(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}\right)$$
$$\left.-s_1\alpha_1\left((t_1-\gamma_1)^{\beta_1}(t_2-t_1)-\frac{(t_2-\gamma_1)^{\beta_1+1}}{\beta_1+1}+\frac{(t_1-\gamma_1)^{\beta_1+1}}{\beta_1+1}\right)\right] \quad (10)$$

$C^*_{GM_2}(t_1,t_2)$ and $C^*_{GM_3}(t_1,t_2)$ can be written as similar as equation (10)

The optimal values of t_1 and t_2 can be obtained so as to minimize the total fuzzy cost $C^*_{GM}(t_1,t_2)$ by solving the following equations

$$\frac{\partial C^*_{GM}(t_1,t_2)}{\partial t_1}=0 \text{ and } \frac{\partial C^*_{GM}(t_1,t_2)}{\partial t_2}=0 \quad (11)$$

Further, for the convexity of total fuzzy cost function $C^*_{GM}(t_1,t_2)$, the following conditions must be satisfied

$$\frac{\partial^2 C^*_{GM}(t_1,t_2)}{\partial t_1^2}>0,\ \frac{\partial^2 C^*_{GM}(t_1,t_2)}{\partial t_2^2}>0 \quad (12)$$

and

$$\left(\frac{\partial^2 C^*_{GM}(t_1,t_2)}{\partial t_1^2}\right)\left(\frac{\partial^2 C^*_{GM}(t_1,t_2)}{\partial t_2^2}\right)-\left(\frac{\partial^2 C^*_{GM}(t_1,t_2)}{\partial t_1\partial t_2}\right)>0 \quad (13)$$

It is difficult to prove the convexity mathematically, since it is complicated to determine the second derivatives of the total fuzzy cost function $C^*_{GM}(t_1,t_2)$. Therefore it is constrained to show the convexity of total fuzzy cost in graph (Figure-5).

Case-2: (If Parameters are trapezoidal fuzzy numbers)

Assuming $\alpha=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$, $\beta=(\beta_1,\beta_2,\beta_3,\beta_4)$, $\gamma=(\gamma_1,\gamma_2,\gamma_3,\gamma_4)$, $a=(a_1,a_2,a_3,a_4)$, $b=(b_1,b_2,b_3,b_4)$, $u_c=(u_1,u_2,u_3,u_4)$, $s_c=(s_1,s_2,s_3,s_4)$, $p=(p_1,p_2,p_3,p_4)$ and $q=(q_1,q_2,q_3,q_4)$ are trapezoidal fuzzy numbers, the total cost is evaluated as

$$C^{**}_{GM}(t_1,t_2)=\tfrac{1}{6}\left(C^{**}_{GM_1}(t_1,t_2)+2C^{**}_{GM_2}(t_1,t_2)+2C^{**}_{GM_3}(t_1,t_2)+C^{**}_{GM_4}(t_1,t_2)\right) \quad (14)$$

where

$$C^{**}_{GM_1}(t_1,t_2)=\tfrac{1}{t_2}\left[O_c+p_1\alpha_1\left(\begin{array}{l}(t_1-\gamma_1)^{\beta_1}(t_1+\frac{a_1t_1}{2}^2+\frac{b_1t_1^3}{3})-\frac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(1+a_1t_1+b_1t_1^2)\\+\frac{(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}(a_1+2b_1t_1)-\frac{2b_1(t_1-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}\\+\frac{(-\gamma_1)^{\beta_1+1}}{\beta_1+1}-\frac{a_1(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\frac{2b_1(-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}\end{array}\right)\right.$$

$$+q_1\alpha_1\left(\begin{array}{l}(t_1-\gamma_1)^{\beta_1}(\frac{t_1^2}{2}+\frac{a_1t_1^3}{6}+\frac{b_1t_1^4}{8})-\frac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(t_1+\frac{a_1t_1^2}{2}+\frac{b_1t_1^3}{2})\\+\frac{(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}(1+a_1t_1+\frac{3b_1t_1^2}{2})-\frac{(t_1-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}(a_1+3b_1t_1)\\+\frac{3b_1(t_1-\gamma_1)^{\beta_1+4}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)(\beta_1+4)}-\frac{(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\frac{a_1(-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}-\frac{3b_1(-\gamma_1)^{\beta_1+4}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)(\beta_1+4)}\end{array}\right)$$

$$+u_1\alpha_1\left((t_1-\gamma_1)^{\beta_1}(a_1t_1+\tfrac{b_1t_1^2}{2})-\tfrac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(a_1+b_1t_1)+\tfrac{b_1(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\tfrac{a_1(-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}-\tfrac{b_1(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}\right)$$
$$\left.-s_1\alpha_1\left((t_1-\gamma_1)^{\beta_1}(t_2-t_1)-\tfrac{(t_2-\gamma_1)^{\beta_1+1}}{\beta_1+1}+\tfrac{(t_1-\gamma_1)^{\beta_1+1}}{\beta_1+1}\right)\right] \quad (15)$$

$C^{**}{}_{GM_2}(t_1,t_2)$, $C^{**}{}_{GM_3}(t_1,t_2)$ and $C^{**}{}_{GM_4}(t_1,t_2)$ can be written as the pattern of equation (15)

The optimal values of t_1 and t_2 can be obtained so as to minimize the total fuzzy cost $C^{**}{}_{GM}(t_1,t_2)$ by solving the following equations

$$\frac{\partial C^{**}{}_{GM}(t_1,t_2)}{\partial t_1}=0 \text{ and } \frac{\partial C^{**}{}_{GM}(t_1,t_2)}{\partial t_2}=0 \quad (16)$$

Further, for the convexity of total fuzzy cost function $C^{**}{}_{GM}(t_1,t_2)$, the following conditions must be satisfied

$$\frac{\partial^2 C^{**}{}_{GM}(t_1,t_2)}{\partial t_1^2}>0,\ \frac{\partial^2 C^{**}{}_{GM}(t_1,t_2)}{\partial t_2^2}>0 \quad (17)$$

and

$$\left(\frac{\partial^2 C^{**}{}_{GM}(t_1,t_2)}{\partial t_1^2}\right)\left(\frac{\partial^2 C^{**}{}_{GM}(t_1,t_2)}{\partial t_2^2}\right)-\left(\frac{\partial^2 C^{**}{}_{GM}(t_1,t_2)}{\partial t_1\partial t_2}\right)>0 \quad (18)$$

It is difficult to prove the convexity mathematically, since it is complicated to determine the second derivatives of the total fuzzy cost function $C^{**}{}_{GM}(t_1,t_2)$. Therefore it is constrained to show the convexity of total fuzzy cost in graph (Figure-6).

Case-3: (If Parameters are pentagonal fuzzy numbers)

Assuming $\alpha=(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5)$, $\beta=(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)$, $\gamma=(\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5)$, $a=(a_1,a_2,a_3,a_4,a_5)$, $b=(b_1,b_2,b_3,b_4,b_5)$, $u_c=(u_1,u_2,u_3,u_4,u_5)$, $s_c=(s_1,s_2,s_3,s_4,s_5)$, $p=(p_1,p_2,p_3,p_4,p_5)$ and $q=(q_1,q_2,q_3,q_4,q_5)$ are pentagonal fuzzy numbers, the total cost is evaluated as

$$C^{***}{}_{GM}(t_1,t_2)=\tfrac{1}{12}\begin{pmatrix}C^{***}{}_{GM_1}(t_1,t_2)+3C^{***}{}_{GM_2}(t_1,t_2)\\ +4C^{***}{}_{GM_3}(t_1,t_2)+3C^{***}{}_{GM_4}(t_1,t_2)+C^{***}{}_{GM_5}(t_1,t_2)\end{pmatrix} \quad (19)$$

where

$$C^{***}{}_{GM_1}(t_1,t_2)=\tfrac{1}{t_2}\left[O_c+p_1\alpha_1\begin{pmatrix}(t_1-\gamma_1)^{\beta_1}(t_1+\tfrac{a_1t_1}{2}^2+\tfrac{b_1t_1^3}{3})-\tfrac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(1+a_1t_1+b_1t_1^2)\\ +\tfrac{(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}(a_1+2b_1t_1)-\tfrac{2b_1(t_1-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}\\ +\tfrac{(-\gamma_1)^{\beta_1+1}}{\beta_1+1}-\tfrac{a_1(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\tfrac{2b_1(-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}\end{pmatrix}\right.$$

$$+q_1\alpha_1\begin{pmatrix}(t_1-\gamma_1)^{\beta_1}(\tfrac{t_1^2}{2}+\tfrac{a_1t_1^3}{6}+\tfrac{b_1t_1^4}{8})-\tfrac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(t_1+\tfrac{a_1t_1^2}{2}+\tfrac{b_1t_1^3}{2})\\ +\tfrac{(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}(1+a_1t_1+\tfrac{3b_1t_1^2}{2})-\tfrac{(t_1-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}(a_1+3b_1t_1)\\ +\tfrac{3b_1(t_1-\gamma_1)^{\beta_1+4}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)(\beta_1+4)}-\tfrac{(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\tfrac{a_1(-\gamma_1)^{\beta_1+3}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)}-\tfrac{3b_1(-\gamma_1)^{\beta_1+4}}{(\beta_1+1)(\beta_1+2)(\beta_1+3)(\beta_1+4)}\end{pmatrix}$$
$$+u_1\alpha_1\left((t_1-\gamma_1)^{\beta_1}(a_1t_1+\tfrac{b_1t_1^2}{2})-\tfrac{(t_1-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}(a_1+b_1t_1)+\tfrac{b_1(t_1-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}+\tfrac{a_1(-\gamma_1)^{\beta_1+1}}{(\beta_1+1)}-\tfrac{b_1(-\gamma_1)^{\beta_1+2}}{(\beta_1+1)(\beta_1+2)}\right)$$
$$\left.-s_1\alpha_1\left((t_1-\gamma_1)^{\beta_1}(t_2-t_1)-\tfrac{(t_2-\gamma_1)^{\beta_1+1}}{\beta_1+1}+\tfrac{(t_1-\gamma_1)^{\beta_1+1}}{\beta_1+1}\right)\right] \quad (20)$$

$C^{***}{}_{GM_2}(t_1,t_2), C^{***}{}_{GM_3}(t_1,t_2), C^{***}{}_{GM_4}(t_1,t_2)$ and $C^{***}{}_{GM_5}(t_1,t_2)$ can be written as the format of equation (20).

The optimal values of t_1 and t_2 can be obtained so as to minimize the total fuzzy cost $C^{***}{}_{GM}(t_1,t_2)$ by solving the following equations

$$\frac{\partial C^{***}{}_{GM}(t_1,t_2)}{\partial t_1} = 0 \text{ and } \frac{\partial C^{***}{}_{GM}(t_1,t_2)}{\partial t_2} = 0 \tag{21}$$

Further, for the convexity of total fuzzy cost function $C^{***}{}_{GM}(t_1,t_2)$, the following conditions must be satisfied

$$\frac{\partial^2 C^{***}{}_{GM}(t_1,t_2)}{\partial {t_1}^2} > 0,\ \frac{\partial^2 C^{***}{}_{GM}(t_1,t_2)}{\partial {t_2}^2} > 0 \tag{22}$$

and

$$\left(\frac{\partial^2 C^{***}{}_{GM}(t_1,t_2)}{\partial {t_1}^2}\right)\left(\frac{\partial^2 C^{***}{}_{GM}(t_1,t_2)}{\partial {t_2}^2}\right) - \left(\frac{\partial^2 C^{***}{}_{GM}(t_1,t_2)}{\partial t_1 \partial t_2}\right) > 0 \tag{23}$$

It is difficult to prove the convexity mathematically, since it is complicated to determine the second derivatives of the total fuzzy cost function $C^{***}{}_{GM}(t_1,t_2)$. Therefore it is constrained to show the convexity of total fuzzy cost in graph (Figure-7).

4. EMPIRICAL INVESTIGATIONS

Crisp Model

Numerical Illustration-1: To understand the effect of solution process, assuming $O_c = Rs\,200/year$, $\alpha = 100$, $\beta = 4$, $\gamma = 0.01$, $a = 3$, $b = 0.5$, $p = 5$, $q = 0.3$, $u_c = Rs\,12/unit$ and $s_c = Rs\,6/unit$ and using Mathematica-9, optimal cycle time is evaluated as $t_1 = 0.105568\,year, t_2 = 0.846922\,year$ and optimum total cost is calculated as $C(t_1,t_2) = Rs\,294.319/year$.

Fuzzy Model (Graded Mean Integration Method)

Case-1 (Using triangular fuzzy number)

Numerical Illustration-2: Based on the computational process, considering $O_c = Rs\,200/year$, $\tilde{\alpha} = (80,100,120)$, $\tilde{\beta} = (2,4,6)$, $\tilde{\gamma} = (0.007,0.01,0.013)$, $\tilde{a} = (2,3,4)$, $\tilde{b} = (0.3,0.5,0.7)$, $\tilde{p} = (3,5,7)$, $\tilde{q} = (0.1,0.3,0.5)$, $\tilde{u}_c = (9,12,15)$ & $\tilde{s}_c = (4,6,8)$ and using Mathematica-9, the optimal cycle time is found as $t_1 = 0.0070004\,year, t_2 = 0.851986\,year$ and optimal fuzzy cost is obtained as $C^{*}{}_{GM}(t_1,t_2) = Rs\,294.921/year.$

Case-2 (Using trapezoidal fuzzy number)

Numerical Illustration-3: To illustrate the solution process, setting $O_c = Rs\,200/year$, $\tilde{\alpha} = (70,90,110,130)$, $\tilde{\beta} = (1,3,5,7)$, $\tilde{\gamma} = (0.007,0.009,0.011,0.013)$, $\tilde{a} = (1.5,2.5,3.5,4.5)$, $\tilde{b} = (0.2,0.4,0.6,0.8)$, $\tilde{p} = (2,4,6,8)$, $\tilde{q} = (0.15,0.25,0.35,0.45)$, $\tilde{u}_c = (9,11,13,15)$ & $\tilde{s}_c = (3,5,7,9)$ and using Mathematica-9, the optimum time is determined as $t_1 = 0.1327\,year$, $t_2 = 0.861536\,year$ and optimum fuzzy cost is evaluated as $C^{**}{}_{GM}(t_1,t_2) = Rs\,294.17/year$.

Case-3 (Using pentagonal fuzzy number)

Numerical Illustration-4: To demonstrate the effect of solution process, assuming $O_c = Rs\,200/\,year$, $\widetilde{\alpha} = (60, 80, 100, 120, 140)$, $\widetilde{\beta} = (2,3,4,5,6)$, $\widetilde{\gamma} = (0.006, 0.008, 0.01, 0.012, 0.014)$, $\widetilde{a} = (1.8,\ 2.4,\ 3,\ 3.6,\ 4.2)$, $\widetilde{b} = (0.3, 0.4, 0.5, 0.6, 0.7)$, $\widetilde{p} = (3,4,5,6,7)$, $\widetilde{q} = 0.18, 0.24, 0.3, 0.36, 0.42)$, $\widetilde{u}_c = (8,10,12,14,16)$ & $\widetilde{s}_c = (2,4,6,8,10)$ and using Mathematica-9, the optimum time is determined as $t_1 = 0.0954874\,year$, $t_2 = 0.848497\,year$ and optimum fuzzy cost is found as $C^{***}{}_{GM}(t_1, t_2) = Rs\,291.012/\,year.$

Taking various parameters as triangular, trapezoidal and pentagonal fuzzy numbers, the performance can be compared from the values of t_1, t_2 and optimal cost as given in Table-1, Table-2 and Table-3 respectively:

Table-1: The optimal time and optimal cost using triangular fuzzy numbers

Parameters are triangular fuzzy number	t_1	t_2	**Optimal Fuzzy cost** $C^{*}{}_{GM}(t_1,t_2)$
$\widetilde{\alpha},\widetilde{\beta},\widetilde{\gamma},\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.0070004	0.851986	294.921
$\widetilde{\beta},\widetilde{\gamma},\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.00700032	0.853139	296.758
$\widetilde{\gamma},\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.104025	0.847168	294.206
$\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.103374	0.84692	294.319
$\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.105544	0.846922	294.319
$\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.105568	0.846922	294.319
$\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.105568	0.846922	294.319
$\widetilde{u}_c,\widetilde{s}_c$	0.105568	0.846922	294.319
$\widetilde{s}_c$	0.105568	0.846922	294.319

Table-2: The optimal time and optimal cost using trapezoidal fuzzy numbers

Parameters are trapezoidal fuzzy number	t_1	t_2	**Optimal Fuzzy cost** $C^{**}{}_{GM}(t_1,t_2)$
$\widetilde{\alpha},\widetilde{\beta},\widetilde{\gamma},\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.1327	0.861536	294.17
$\widetilde{\beta},\widetilde{\gamma},\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.134339	0.86711	298.659
$\widetilde{\gamma},\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.0104267	0.847349	294.118
$\widetilde{a},\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.101553	0.846918	294.32
$\widetilde{b},\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.105523	0.846922	294.319
$\widetilde{p},\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.00999998	0.846893	294.328
$\widetilde{q},\widetilde{u}_c,\widetilde{s}_c$	0.00999998	0.846893	294.328
$\widetilde{u}_c,\widetilde{s}_c$	0.00999998	0.846893	294.328
$\widetilde{s}_c$	0.00999998	0.846893	294.328

Table-3: The optimal time and optimal cost using pentagonal fuzzy number

Parameters are pentagonal fuzzy number	t_1	t_2	Optimal Fuzzy cost $C^{***}_{GM}(t_1,t_2)$
$\tilde{\alpha},\tilde{\beta},\tilde{\gamma},\tilde{a},\tilde{b},\tilde{p},\tilde{q},\tilde{u}_c,\tilde{s}_c$	0.0954874	0.848497	291.012
$\tilde{\beta},\tilde{\gamma},\tilde{a},\tilde{b},\tilde{p},\tilde{q},\tilde{u}_c,\tilde{s}_c$	0.0947362	0.856389	291.176
$\tilde{\gamma},\tilde{a},\tilde{b},\tilde{p},\tilde{q},\tilde{u}_c,\tilde{s}_c$	0.102631	0.847499	294.053
$\tilde{a},\tilde{b},\tilde{p},\tilde{q},\tilde{u}_c,\tilde{s}_c$	0.102551	0.846919	294.32
$\tilde{b},\tilde{p},\tilde{q},\tilde{u}_c,\tilde{s}_c$	0.10554	0.846922	294.319
$\tilde{p},\tilde{q},\tilde{u}_c,\tilde{s}_c$	0.105568	0.846922	294.319
$\tilde{q},\tilde{u}_c,\tilde{s}_c$	0.105568	0.846922	294.319
$\tilde{u}_c,\tilde{s}_c$	0.105568	0.846922	294.319
$\tilde{s}_c$	0.105568	0.846922	294.319

5. SENSITIVITY ANALYSIS

The effects of changes in the system parameters $\tilde{\alpha},\ \tilde{\gamma},\ \tilde{a},\ \tilde{b},\ \tilde{p},\ \ \tilde{q},\ \tilde{O}_c, \tilde{u}_c\ and\ \tilde{s}_c$ as triangular, trapezoidal & pentagonal fuzzy numbers on the fuzzy cost derived by the proposed methods are now studied. The sensitivity analysis is performed by keeping all but one system parameter fixed at a time and study the change in the identified variable by fluctuating it from 25 % to 50 %.

Table-4: Sensitivity analysis on parameters α and γ as triangular, trapezoidal and pentagonal fuzzy numbers

Parameters	% change in parameters	$C^{*}_{GM}(t_1,t_2)$	$C^{**}_{GM}(t_1,t_2)$	$C^{***}_{GM}(t_1,t_2)$
α	+50	320.641	321.198	314.496
	+25	308.431	308.562	303.668
	-25	278.227	277.169	275.605
	-50	256.475	255.771	255.497
γ	+50	293.169	292.506	289.177
	+25	294.042	293.335	290.092
	-25	295.804	295.01	291.938
	-50	296.693	295.855	292.869

The above table indicates that as the value of α increases, fuzzy costs $C^{*}_{GM}(t_1,t_2)$, $C^{**}_{GM}(t_1,t_2)$ and $C^{***}_{GM}(t_1,t_2)$ increase regularly. But fuzzy costs $C^{*}_{GM}(t_1,t_2)$, $C^{**}_{GM}(t_1,t_2)$ and $C^{***}_{GM}(t_1,t_2)$ decrease slightly, while the value of γ increases.

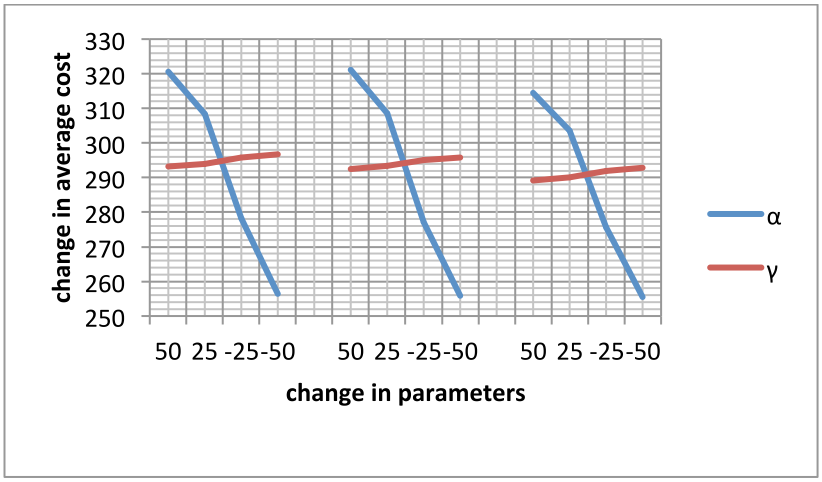

Figure-1 (Behavior of parameters α and γ)

Table-5: Sensitivity analysis on parameters *a* and *b* as triangular, trapezoidal and pentagonal fuzzy numbers

Parameters	% change in parameters	$C^{*}{}_{GM}(t_1,t_2)$	$C^{**}{}_{GM}(t_1,t_2)$	$C^{***}{}_{GM}(t_1,t_2)$
a	+50	294.782	294.83	291.034
	+25	294.731	294.552	291.026
	-25	294.504	293.607	290.981
	-50	294.205	292.687	290.902
b	+50	294.921	294.176	291.012
	+25	294.921	294.173	291.012
	-25	294.648	294.167	291.012
	-50	294.648	294.164	291.011

The above table shows that fuzzy cost $C^{*}{}_{GM}(t_1,t_2)$ decreases slowly and moderately with the increase with respect to the parameters a and b respectively. The fuzzy cost $C^{**}{}_{GM}(t_1,t_2)$ increases gradually and moderately with the increase with respect to the parameters a and b respectively and the fuzzy cost $C^{***}{}_{GM}(t_1,t_2)$ decreases moderately with the increase with respect to the parameter a, but fuzzy cost $C^{***}{}_{GM}(t_1,t_2)$ insensitively with parameter b.

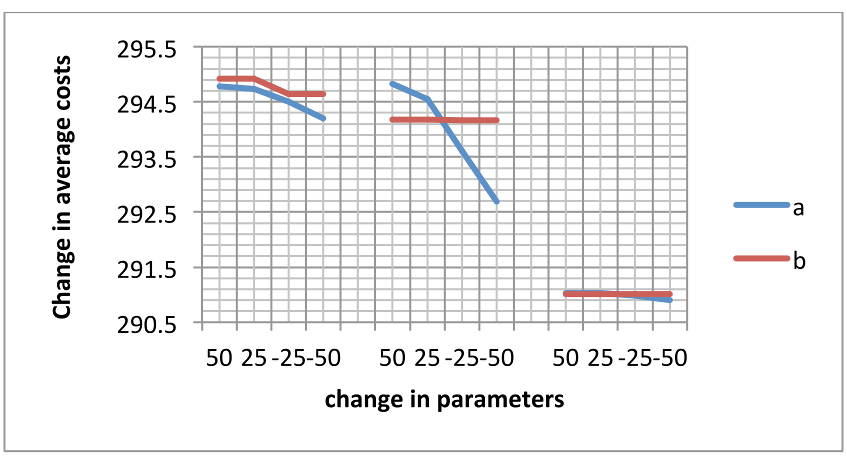

Figure-2 (Behavior of parameters *a* and *b*)

Table-6: Sensitivity analysis on parameters p and q as triangular, trapezoidal and pentagonal fuzzy numbers

Parameters	% change in parameters	$C^{*}{}_{GM}(t_1,t_2)$	$C^{**}{}_{GM}(t_1,t_2)$	$C^{***}{}_{GM}(t_1,t_2)$
p	+50	294.921	294.305	291.019
	+25	294.667	294.239	291.015
	-25	294.629	294.096	291.007
	-50	294.608	294.171	291.002
q	+50	294.921	294.171	291.011
	+25	294.921	294.17	291.011
	-25	294.649	294.17	291.011
	-50	294.921	294.17	291.011

According to above results, while the values of p and q increase, the fuzzy cost $C^{*}{}_{GM}(t_1,t_2)$ decreases moderately. The value of p increases, the fuzzy cost $C^{**}{}_{GM}(t_1,t_2)$ increases slowly. But fuzzy cost $C^{**}{}_{GM}(t_1,t_2)$ insensitive with respect to the parameter q. The fuzzy cost $C^{***}{}_{GM}(t_1,t_2)$ is insensitive with respect to the parameters p and q.

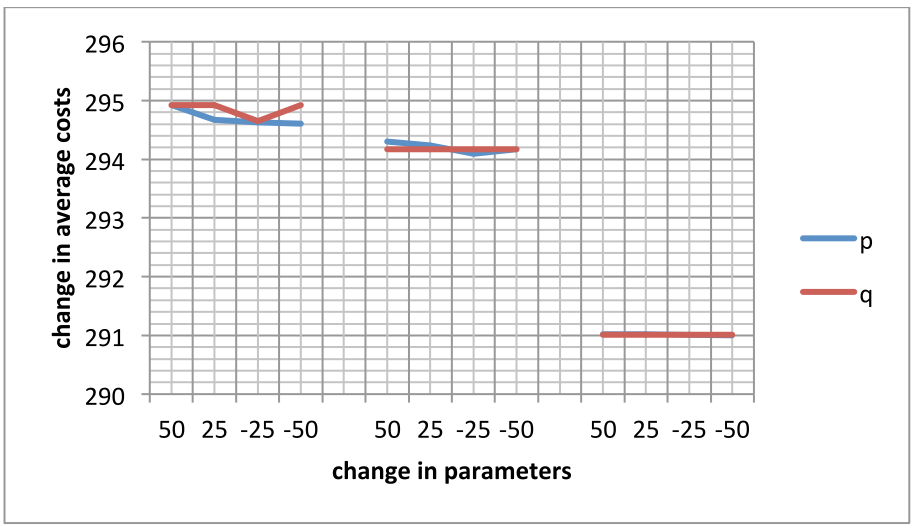

Figure-3 (Behavior of parameters p and q)

Table-7: Sensitivity analysis on parameters O_c, u_c and s_c as triangular, trapezoidal and pentagonal fuzzy numbers

Parameters	% change in parameters	$C^{*}{}_{GM}(t_1,t_2)$	$C^{**}{}_{GM}(t_1,t_2)$	$C^{***}{}_{GM}(t_1,t_2)$
O_c	+50	407.602	405.996	404.366
	+25	352.334	351.052	348.72
	-25	234.425	234.705	230.587
	-50	170.106	171.469	166.265
u_c	+50	294.781	294.829	291.034
	+25	294.921	294.551	291.026
	-25	294.506	293.612	290.981
	-50	294.218	292.708	290.904

s_c	+50	319.94	319.822	314.766
	+25	308.28	307.952	303.636
	-25	278.093	277.597	275.622
	-50	256.607	256.406	255.52

The above table indicates that the fuzzy cost $C^{*}{}_{GM}(t_1,t_2)$ is highly sensitive with respect to the parameters O_c and s_c, while fuzzy $C^{*}{}_{GM}(t_1,t_2)$ is low sensitive with parameter u_c. The fuzzy cost $C^{**}{}_{GM}(t_1,t_2)$ and $C^{***}{}_{GM}(t_1,t_2)$ are highly sensitive with respect to the parameters O_c and s_c, while fuzzy $C^{**}{}_{GM}(t_1,t_2)$ and $C^{***}{}_{GM}(t_1,t_2)$ are moderately sensitive with parameter u_c.

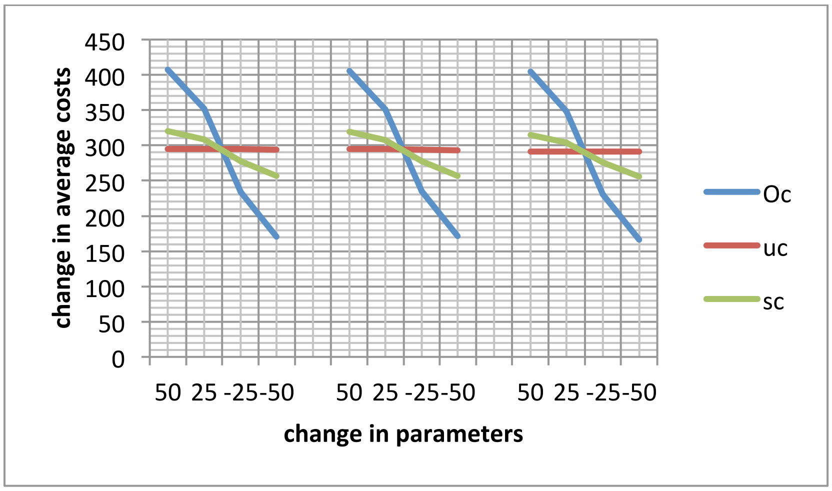

Figure-4 (Behavior of parameters Oc, u_c & s_c)

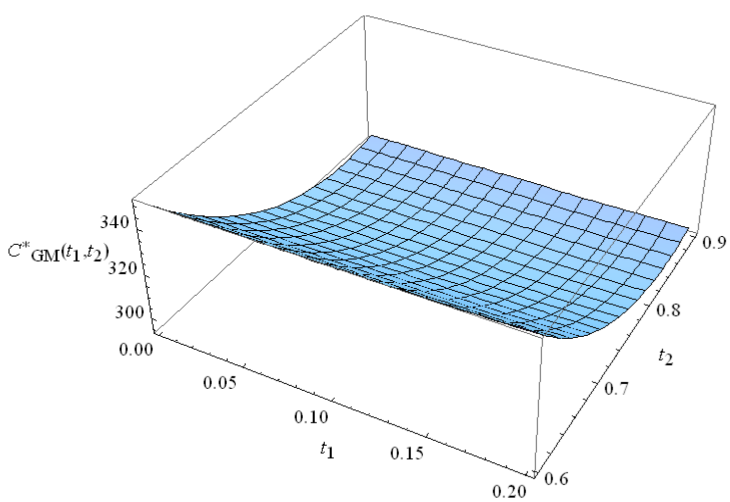

Figure.-5. Convexity graph of cost function $C^{*}{}_{GM}(t_1,t_2)$ with t_1 and t_2

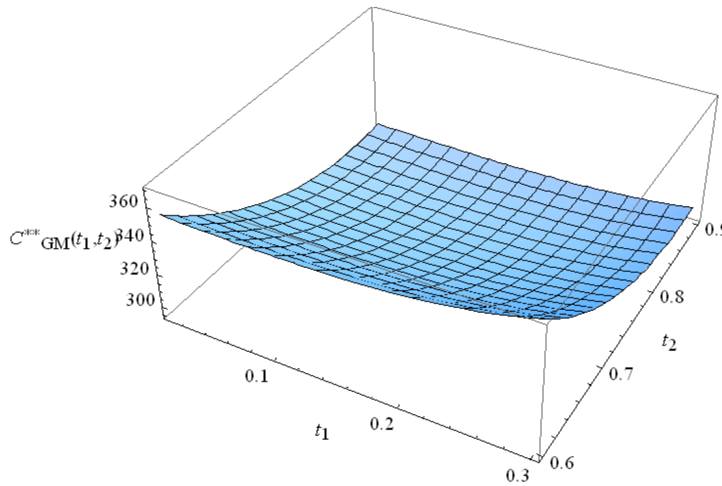

Figure.-6. Convexity graph of cost function $C^{**}{}_{GM}(t_1,t_2)$ with t_1 and t_2

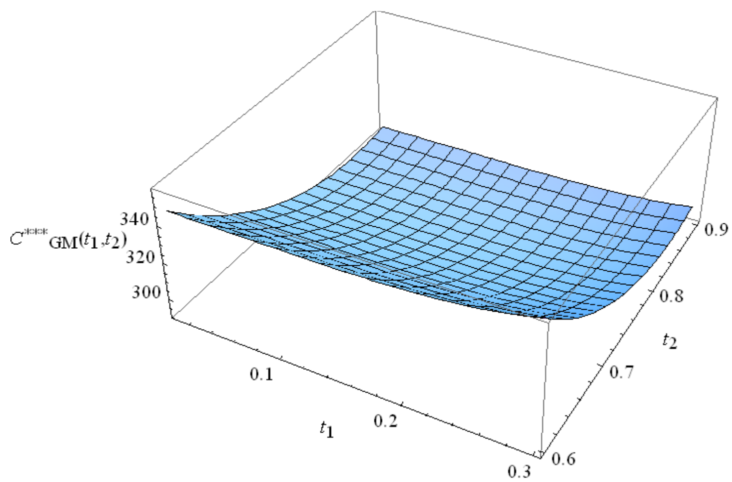

Figure-7. Convexity graph of cost function $C^{***}{}_{GM}(t_1,t_2)$ with t_1 and t_2

6. CONCLUSION

The paper deals with optimal decision model with three-parameter Weibull demand, linear deterioration rate and time dependent holding cost. The demand rate, deterioration rate and various costs are taken as triangular, trapezoidal and pentagonal fuzzy numbers. Graded mean integration method is introduced for defuzzification of total inventory cost under fuzzy sense. The above model with pentagonal fuzzy number deals with more realistic approach for storage of the deteriorating goods or commodities. Numerical illustrations are provided to validate the applications of model. Sensitivity analysis with useful graphs and tables are performed to analyze the variability in the optimal solution with respect to change in various system parameters.
After comparison, it is concluded that if system parameters are pentagonal fuzzy number then graded mean integration method provides optimum inventory cost. This model can be improved by introducing stochastic demand, freezing technology and dispatch policy.

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