Correction Interrogation 1 2013

exo 1. ① $n=2k$, $k \geq 1$

1)

$Y \backslash X$	-1	0	1	marg. Y
-1	$2/9$	0	0	$2/9$
0	0	$1/3$	0	$1/3$
1	$1/9$	0	$1/3$	$4/9$
marg. X	$1/3$	$1/3$	$1/3$	

2)

$Y\|X=-1$	-1	0	1
$\mathbb{P}(Y\|X=-1)$	$2/3$	0	$1/3$

$Y\|X=0$	-1	0	1
$\mathbb{P}(Y\|X=0)$	0	1	0

$Y\|X=1$	-1	0	1
$\mathbb{P}(Y\|X=1)$	0	0	1

3)

$\mathbb{E}(Y|X=-1) = -1/3$, $\mathbb{E}(Y|X=0) = 0$, $\mathbb{E}(Y|X=1) = 1$

$\mathbb{E}(Y) = -1 \times 2/9 + 0 \times 1/3 + 1 \times 4/9 = 2/9$ méthode 1

$\mathbb{E}(Y) = -1/3 \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = 2/9$ méthode 2

4)

X^2	0	1
$\mathbb{P}_{X^2}$	$1/3$	$2/3$

$\mathbb{E}X = 0$ $\mathbb{E}X^2 = 2/3$ $\mathrm{Var}\,X = 2/3$

Y^2	0	1
$\mathbb{P}_{Y^2}$	$1/3$	$2/3$

$\mathbb{E}Y = 2/9$ $\mathbb{E}Y^2 = 2/3$ $\mathrm{Var}\,Y = \frac{50}{81}$

XY	-1	0	1
$\mathbb{P}_{XY}$	$1/9 + 0$	$1/3$	$2/9 + 1/3$

$\mathbb{E}XY = \frac{4}{9}$ $\mathrm{cov}(X,Y) = \frac{4}{9} - 0 \times \frac{2}{9} = \frac{4}{9}$

matrice var-cov : $\begin{pmatrix} 2/3 & 4/9 \\ 4/9 & 50/81 \end{pmatrix}$

② $n=2k-1 \quad k \geq 1$

1)

$Y \backslash X$	-1	0	1	marg. Y
-1	$1/9$	0	0	$1/9$
0	0	$1/3$	0	$1/3$
1	$2/9$	0	$1/3$	$5/9$
marg. X	$1/3$	$1/3$	$1/3$	

2)

$Y\|X=-1$	-1	0	1
$\mathbb{P}(Y\|X=-1)$	$1/3$	0	$2/3$

$Y\|X=0$	-1	0	1
$\mathbb{P}(Y\|X=0)$	0	1	0

$Y\|X=1$	-1	0	1
$\mathbb{P}(Y\|X=1)$	0	0	1

3) $\mathbb{E}(Y|X=-1) = 1/3 \qquad \mathbb{E}(Y|X=0) = 0 \qquad \mathbb{E}(Y|X=1) = 1$

$\mathbb{E}(Y) = -1 \times 1/9 + 0 \times 1/3 + 1 \times 5/9 = 4/9$ méthode 1

$\mathbb{E}(Y) = 1/3 \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = 4/9$ méthode 2

4)

X^2	0	1
$\mathbb{P}_{X^2}$	$1/3$	$2/3$

$\mathbb{E}X = 0 \qquad \mathbb{E}X^2 = 2/3 \qquad \mathrm{Var}\,X = 2/3$

Y^2	0	1
$\mathbb{P}_{Y^2}$	$1/3$	$2/3$

$\mathbb{E}Y = 4/9 \qquad \mathbb{E}Y^2 = 2/3 \qquad \mathrm{Var}\,Y = \frac{38}{81}$

XY	-1	0	1
$\mathbb{P}_{XY}$	$2/9 + 0$	$1/3$	$1/9 + 1/3$

$\mathbb{E}XY = \frac{2}{9} \qquad \mathrm{cov}(X,Y) = \frac{2}{9} - 0 \times \frac{4}{9} = \frac{2}{9}$

matrice var-cov : $\begin{pmatrix} 2/3 & 2/9 \\ 2/9 & 38/81 \end{pmatrix}$

③ $n=0$

1)

$Y \backslash X$	-1	0	1	marg. Y
-1	$2/9$	0	0	$2/9$
0	0	$2/9$	0	$2/9$
1	$1/9$	$1/9$	$1/3$	$5/9$
marg. X	$1/3$	$1/3$	$1/3$	

2)

$Y \mid X=-1$	-1	0	1
$\mathbb{P}(Y \mid X=-1)$	$2/3$	0	$1/3$

$Y \mid X=0$	-1	0	1
$\mathbb{P}(Y \mid X=0)$	0	$2/3$	$1/3$

$Y \mid X=1$	-1	0	1
$\mathbb{P}(Y \mid X=1)$	0	0	1

3) $\mathbb{E}(Y \mid X=-1) = -1/3 \qquad \mathbb{E}(Y \mid X=0) = 1/3 \qquad \mathbb{E}(Y \mid X=1) = 1$

$\mathbb{E}(Y) = -1 \times 2/9 + 0 \times 2/9 + 1 \times 5/9 = 1/3$ méthode 1

$\mathbb{E}(Y) = -\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + 1 \times \frac{1}{3} = 1/3$ méthode 2

4)

X^2	0	1
$\mathbb{P}_{X^2}$	$1/3$	$2/3$

$\mathbb{E}(X) = 0 \qquad \mathbb{E}X^2 = 2/3 \qquad \mathrm{Var}\, X = 2/3$

Y^2	0	1
$\mathbb{P}_{Y^2}$	$2/9$	$7/9$

$\mathbb{E}(Y) = 1/3 \qquad \mathbb{E}Y^2 = 7/9 \qquad \mathrm{Var}\, Y = 2/3$

XY	-1	0	1
$\mathbb{P}_{XY}$	$1/9+0$	$2/9+1/9$	$2/9+1/3$

$\mathbb{E}XY = \frac{4}{9} \qquad \mathrm{cov}(X,Y) = \frac{4}{9} - 0 \times 1/3 = \frac{4}{9}$

matrice Var–cov $\begin{pmatrix} 2/3 & 4/9 \\ 4/9 & 2/3 \end{pmatrix}$

exo 2.

1) $\mathbb{P}(X_{n+1}=x_{n+1} | X_n=x_n, X_{n-1}=x_{n-1}, \cdots, X_0=x_0) = \mathbb{P}(X_{n+1}=-x_n | X_n=x_n) = 1$

2) $E=\{X_0, -X_0\}$, $M=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3) tous les états sont récurrents

4) $\left.\begin{matrix} M\mu=\mu \\ u_1+u_2=1 \end{matrix}\right\} \Rightarrow u_1=u_2=\frac{1}{2}$

5) Non, la chaîne est périodique.

exo 3.

1) $\mathbb{P}(X_{n+1}=x_n | X_n=x_n) = \mathbb{P}(Y_n=0) = \frac{3}{4}$

$\mathbb{P}(X_{n+1}=-x_n | X_n=x_n) = \mathbb{P}(Y_n=1) = \frac{1}{4}$

$\mathbb{P}(X_{n+1}=x_{n+1} | X_n=x_n, \cdots, X_0=x_0) = \mathbb{P}(X_{n+1}=x_{n+1} | X_n=x_n)$

2) $E=\{-X_0, X_0\}$, $M=\begin{pmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{pmatrix}$

3) $\left.\begin{matrix} M\mu=\mu \\ u_1+u_2=1 \end{matrix}\right\} \Rightarrow u_1=u_2=\frac{1}{2}$

4) Oui, la chaîne est irreductible.