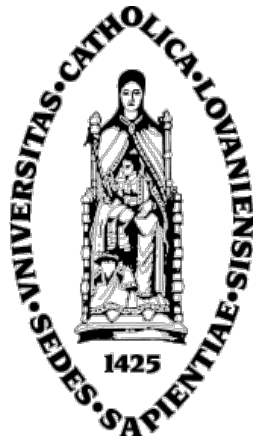
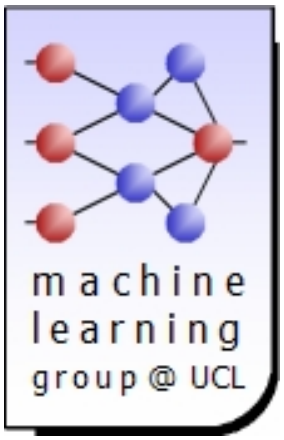
Curve Alignment: Theory and Applications

Benoît Frénay

UCL/ICTEAM - MLG

June 5, 2012





Curves: an Ubiquitous Representation in Sciences

Curves are everywhere in data analysis and sciences:

- time series
- handwriting recognition
- shapes of objects

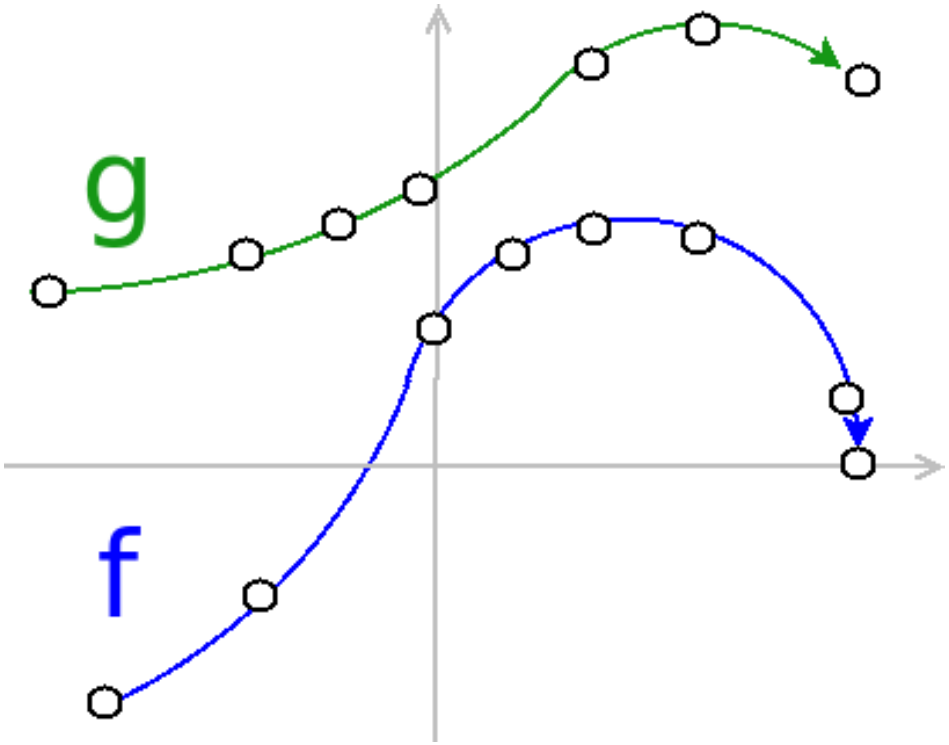
Common problems:

- compare curves
- cluster curves
- summarise curves

For the above problems, curve alignment is often essential !

What is Curve Alignment and Why do we Need it ?

Let us suppose we want to **compare two 2D curves** f and g:

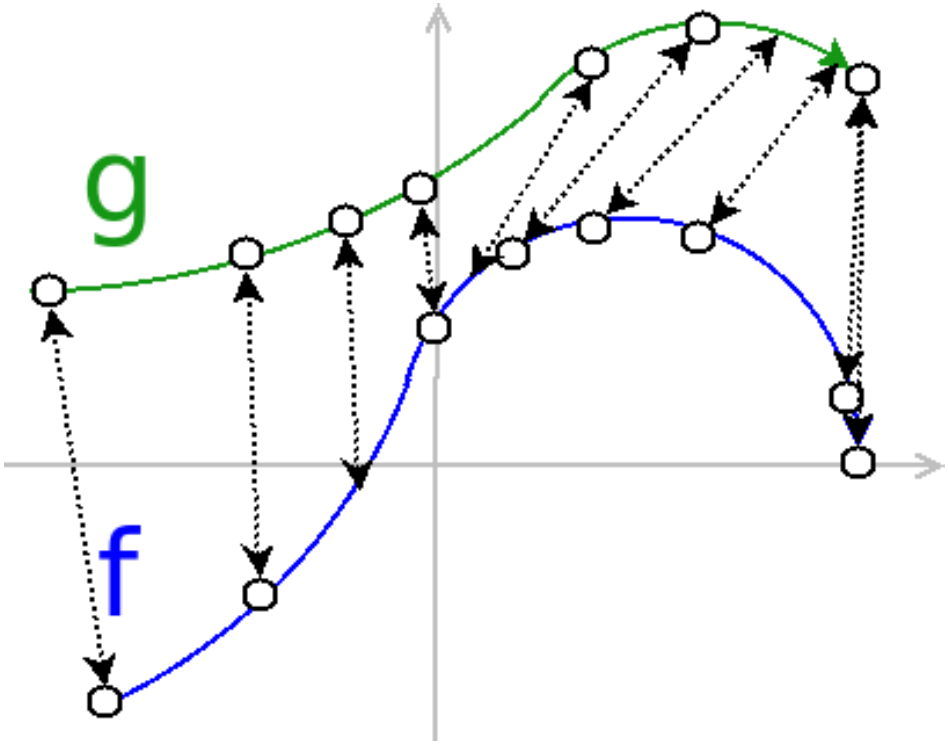


Difficult problem:

- each curve can be described by a **different number** of points
- the **x-components** are not necessarily the **same** / **coherent**

What is Curve Alignment and Why do we Need it ?

Let us suppose we want to **compare two 2D curves** f and g:



Difficult problem:

- each curve can be described by a **different number** of points
- the **x-components** are not necessarily the **same** / **coherent**

Problem(s) Statement

Problem #1:

- how can we define a **distance measure** d between f and g?
- how can we **compute efficiently** $d(f, g)$?

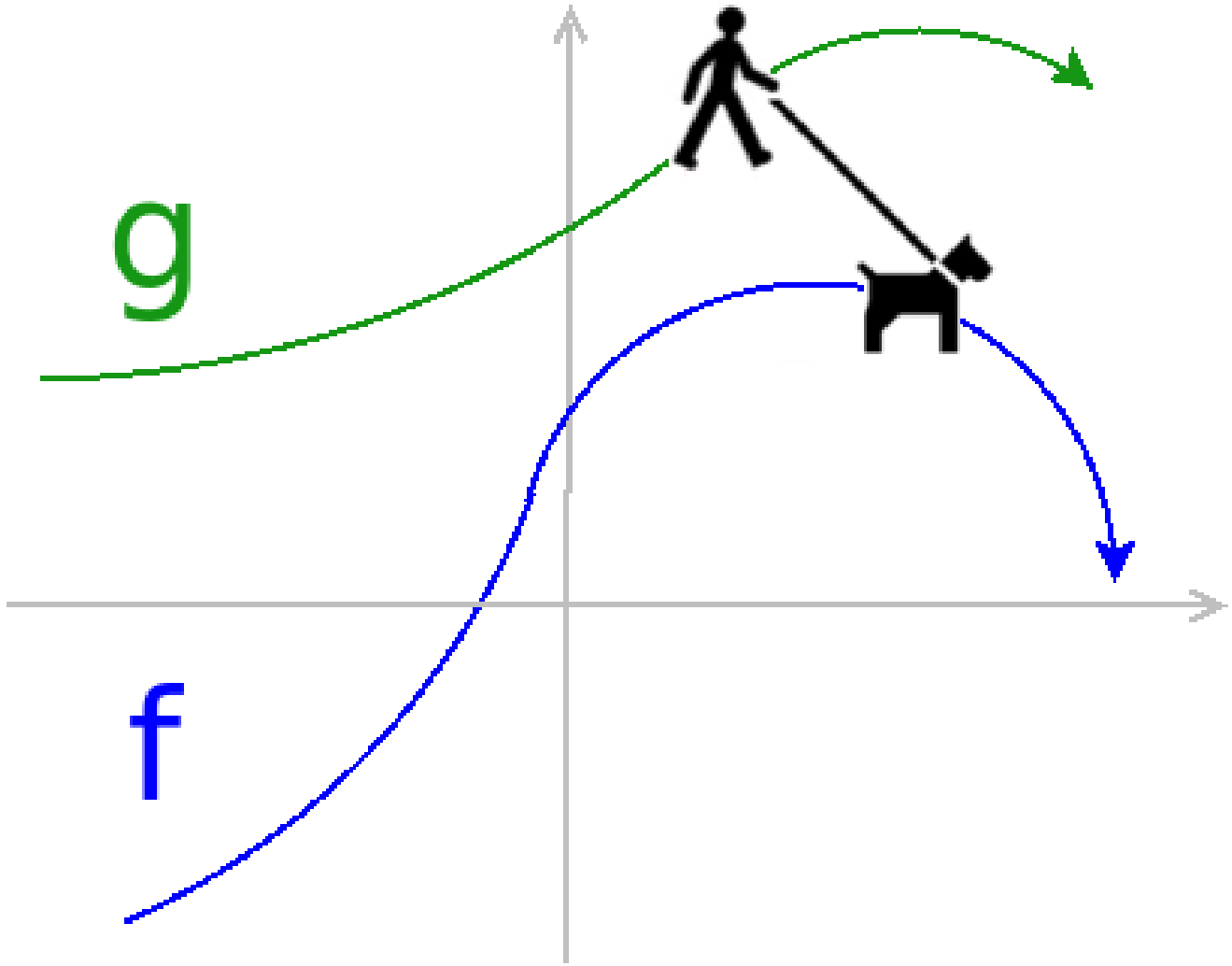
Problem #2:

- what is a **matching** ϕ between f and g?
- how can we **compute efficiently** ϕ?

Both problems can be addressed at the **same time**!

Curve Alignment with Dynamic Time Warping

The Dog-Man Analogy



Warping Functions

The curves f and g are **parametric functions**

- $f : [1, T_F] \rightarrow \Re^2$
- $g : [1, T_G] \rightarrow \Re^2$

We can define the **warping functions** ϕ_f and ϕ_g as **reparametrisations**

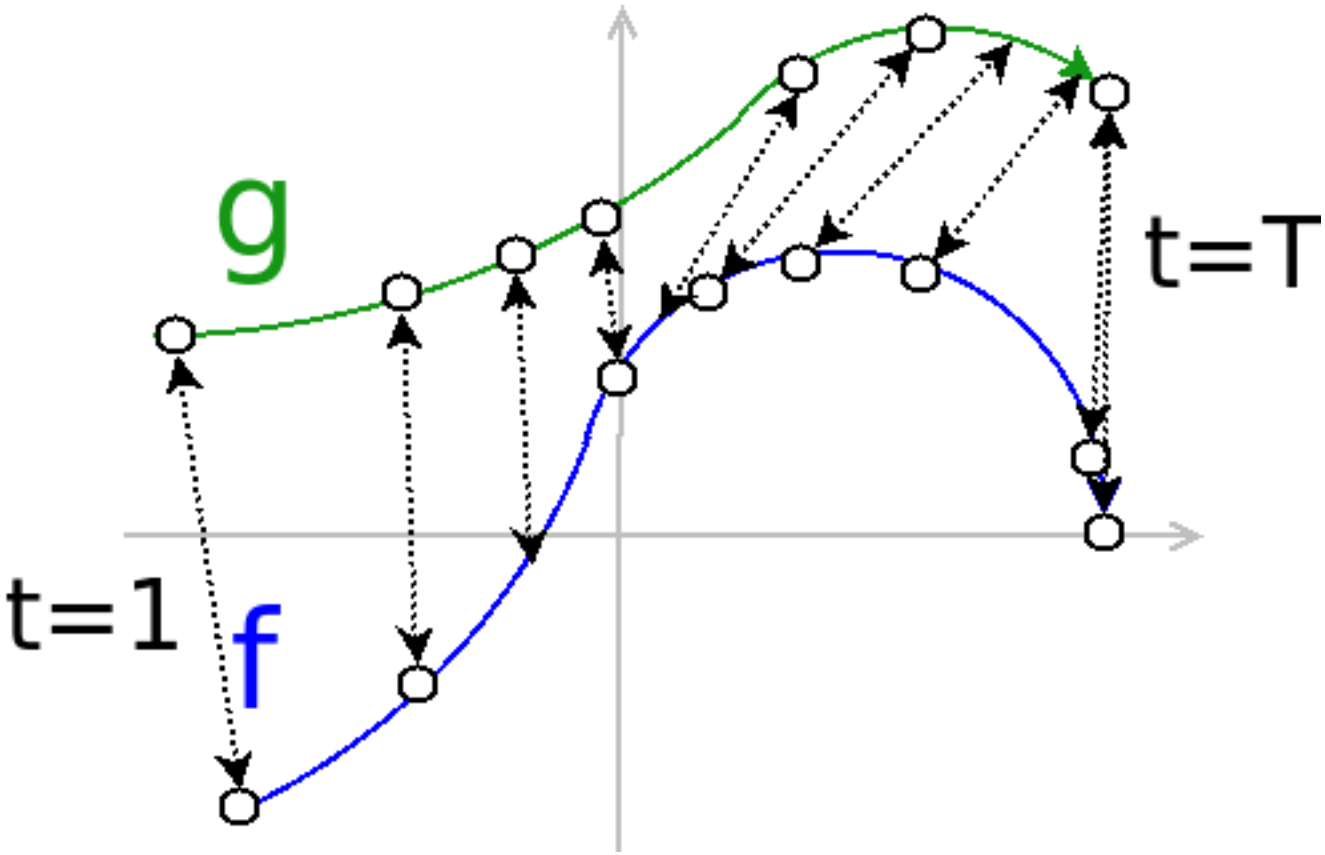
- $\phi_f : [1, T] \rightarrow [1, T_F]$
- $\phi_g : [1, T] \rightarrow [1, T_G]$

such that $f(\phi_f(t)) = f_\phi(t)$ now **corresponds** to $g(\phi_g(t)) = g_\phi(t)$.

The **time warping** $\phi = \langle \phi_f, \phi_g \rangle$ gives the **best matching** for d if

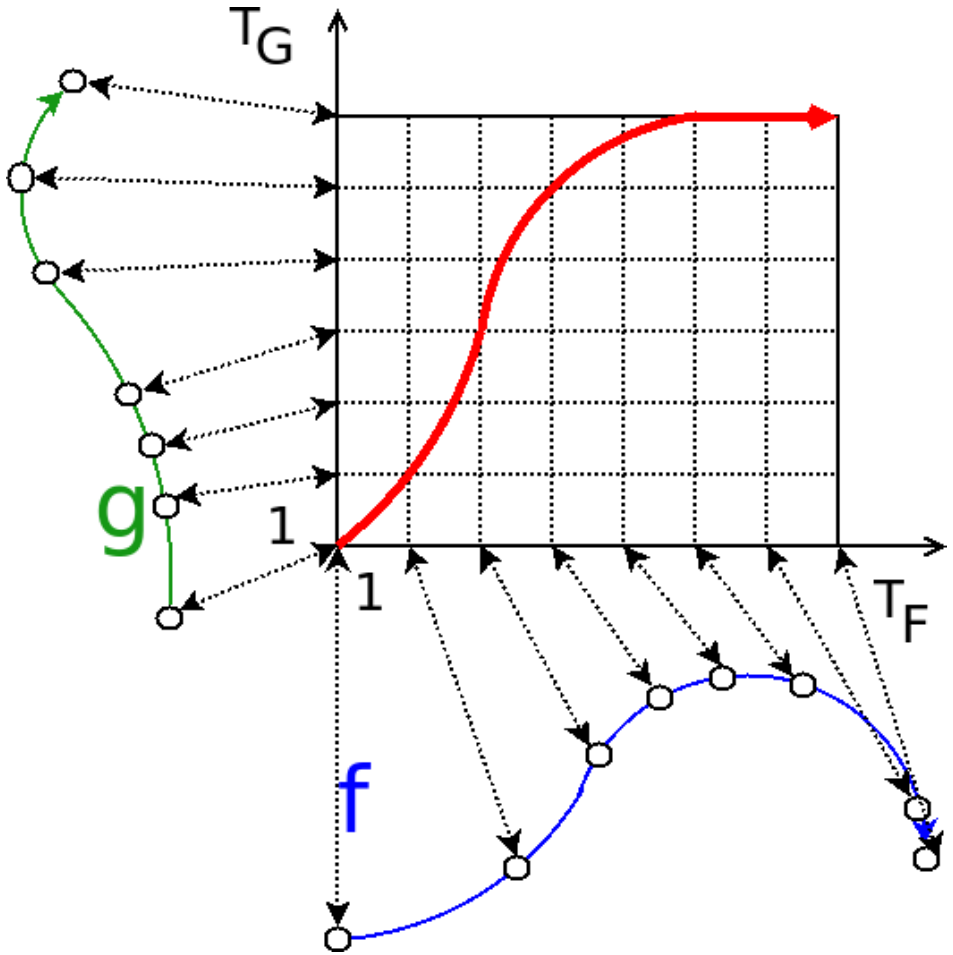
$$\phi = \underset{\phi = \langle \phi_f, \phi_g \rangle}{\operatorname{argmin}} \; d(f \circ \phi_f, g \circ \phi_g)$$

Example of Time Warping



The Warping Plane

The **warping plane** is a **representation** of the warping of f and g:



The **warping path** is the path **drawn** by the **reparametrisation**.

Restrictions on the Warping Functions

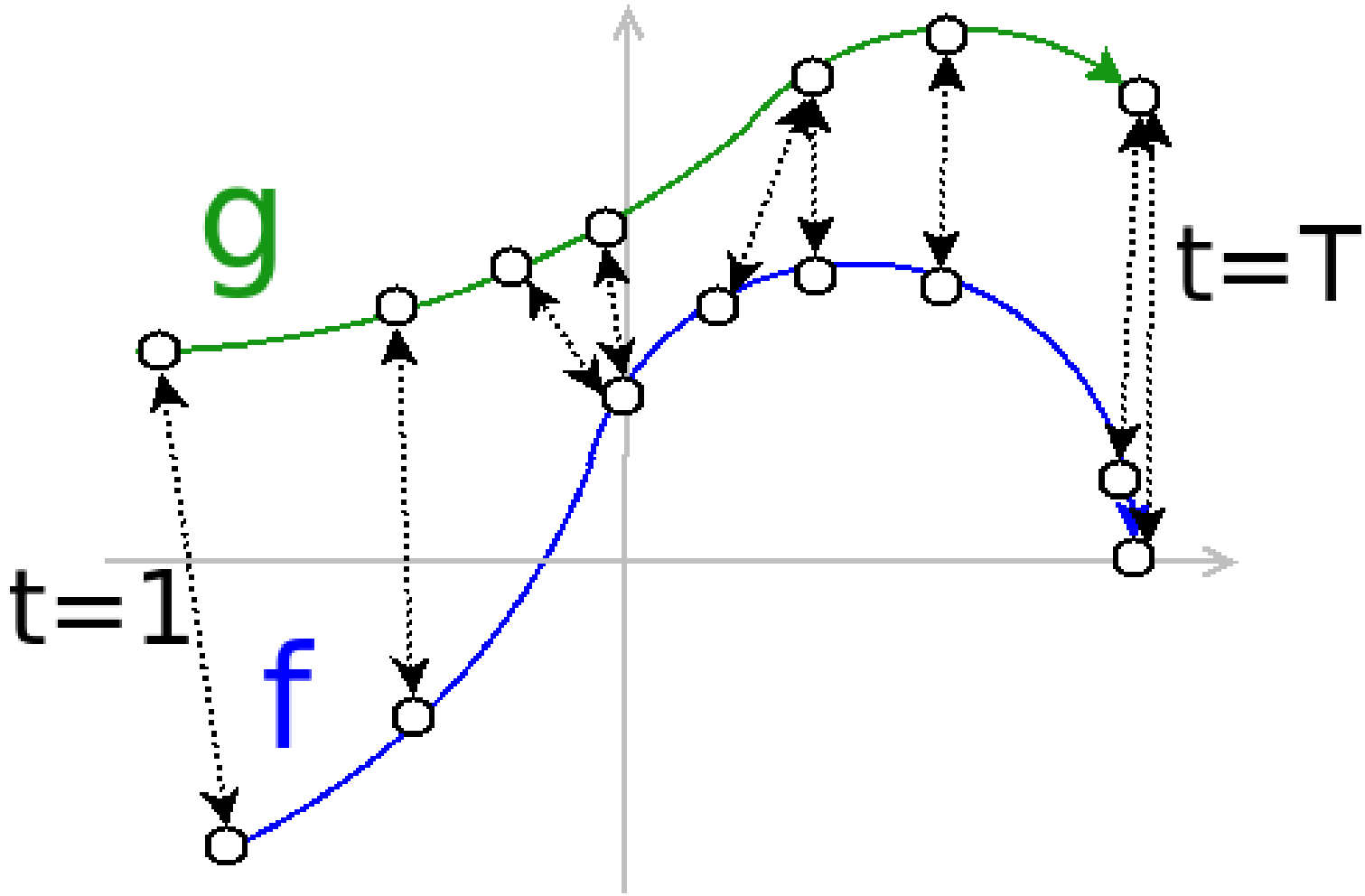
We **restrict** ϕ_f and ϕ_g to be

- **continuous**
- **monotone**

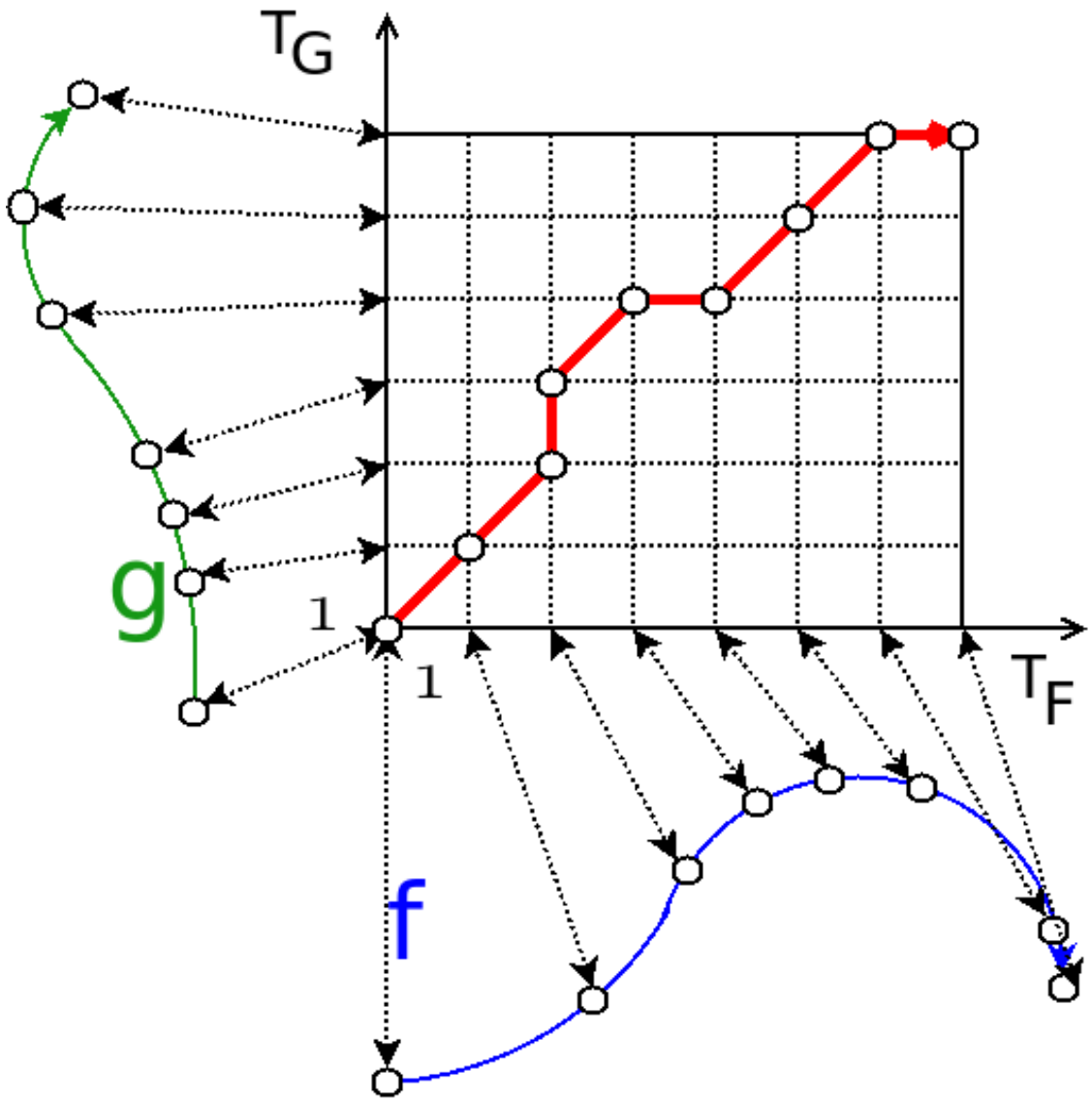
Discrete time warping: a **point** F_i is **always matched** with a **point** G_j.

Dynamic programming can be used to compute the warping!

Example of Discrete Time Warping



Example of Discrete Time Warping - The Warping Space



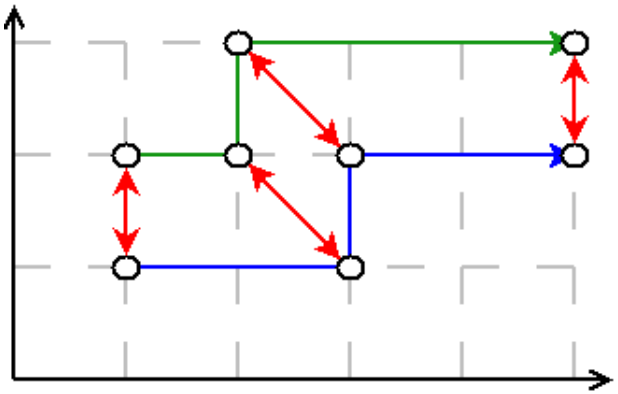
Links Between Warping and Distances

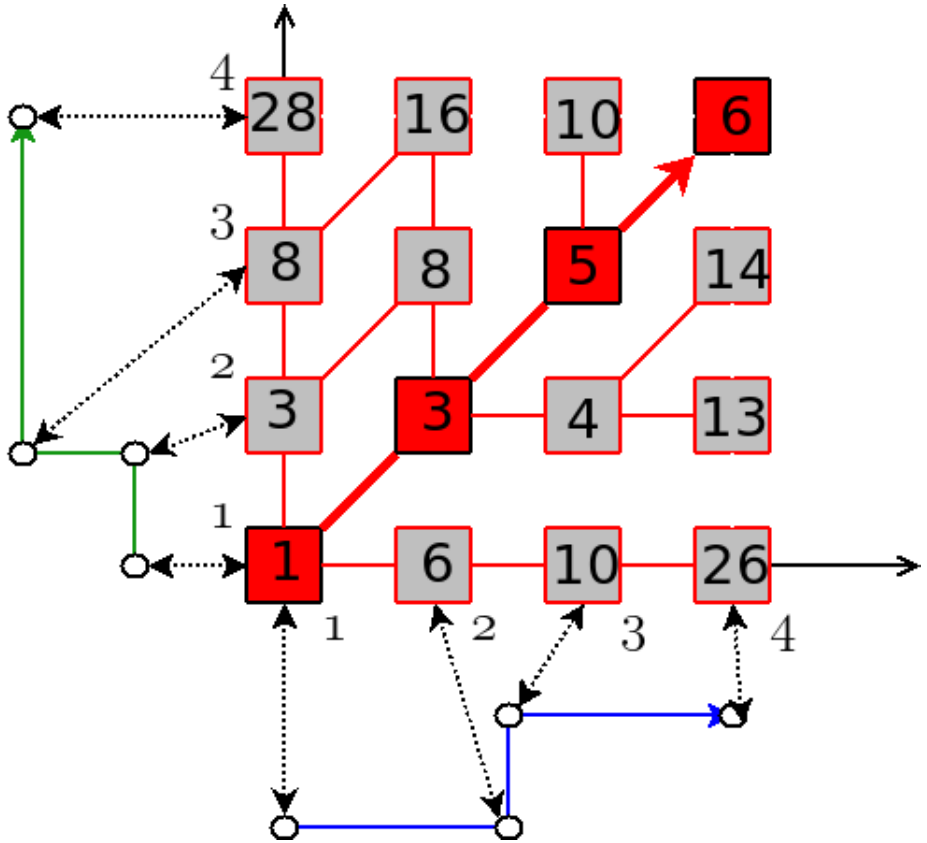
Let us **define** $d(f, g)$ as

$$d(f, g) = \min_{\phi} \sum_{t=2}^{T} c\left(f_\phi(t-1), g_\phi(t-1), f_\phi(t), g_\phi(t)\right)$$

Goal: compute the **distance** $d(f, g)$ and the **corresponding warping**.

D-DTW: Illustration





Practical Considerations and Distances Measures

Examples of Distances

The **result depend on the choice** of d, e.g. the sum of

- the **distances** between matched points, i.e.

$$d(f,g) = \min_{\phi} \sum_{t=2}^{T} \left\| \overrightarrow{f_\phi(t) g_\phi(t)} \right\|^2$$

- the **variations of the distance** between matched points, i.e.

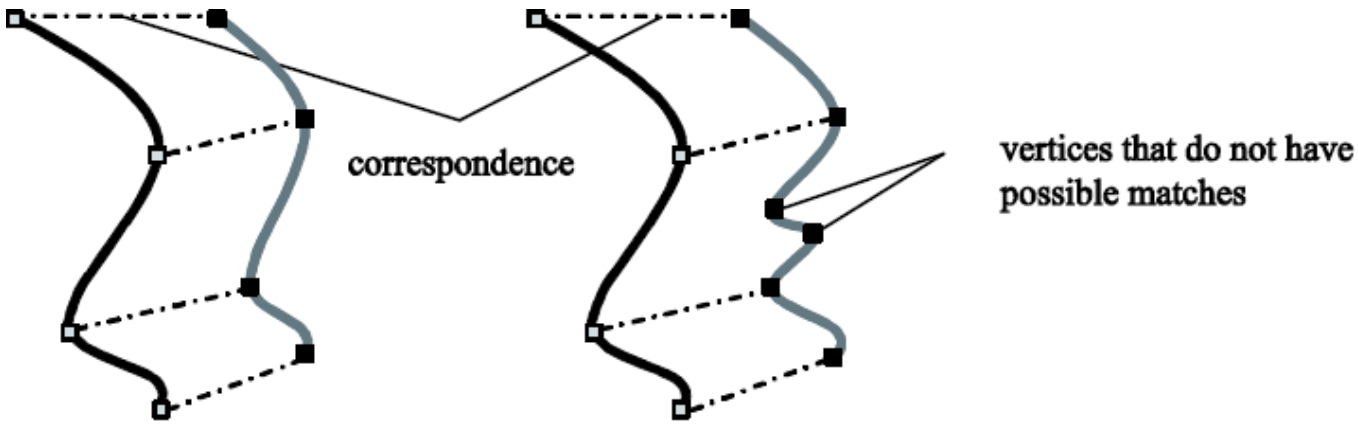
$$d(f,g) = \min_{\phi} \sum_{t=2}^{T} \left\| \overrightarrow{f_\phi(t) g_\phi(t)} - \overrightarrow{f_\phi(t-1) g_\phi(t-1)} \right\|^2$$

- ...

Skip Unmatchable Points

No match on the other curve ?

- just skip the points and impose them a penalty !

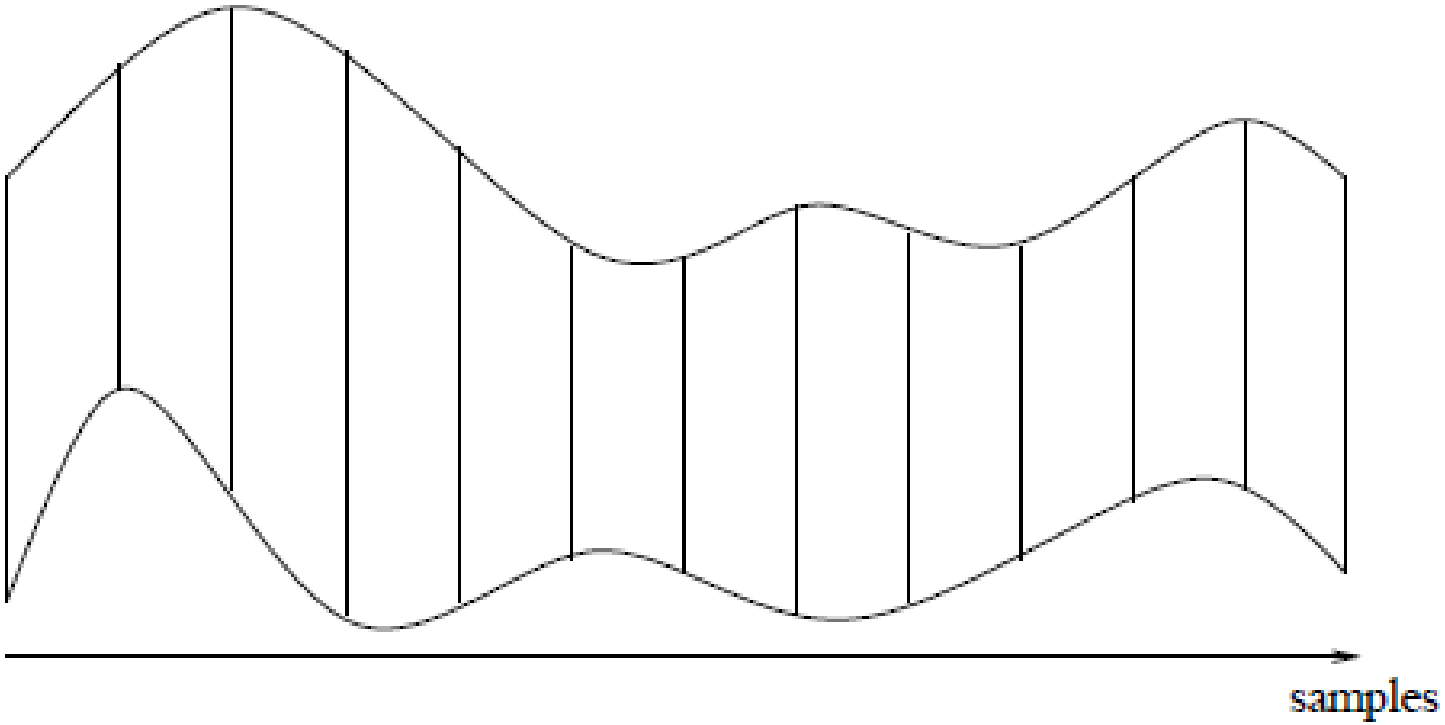


Source: *Relative Curve Orientation in the Alignment of Inconsistent Linear Datasets*

Examples of Curve Alignment

Examples of Curve Alignment (1)

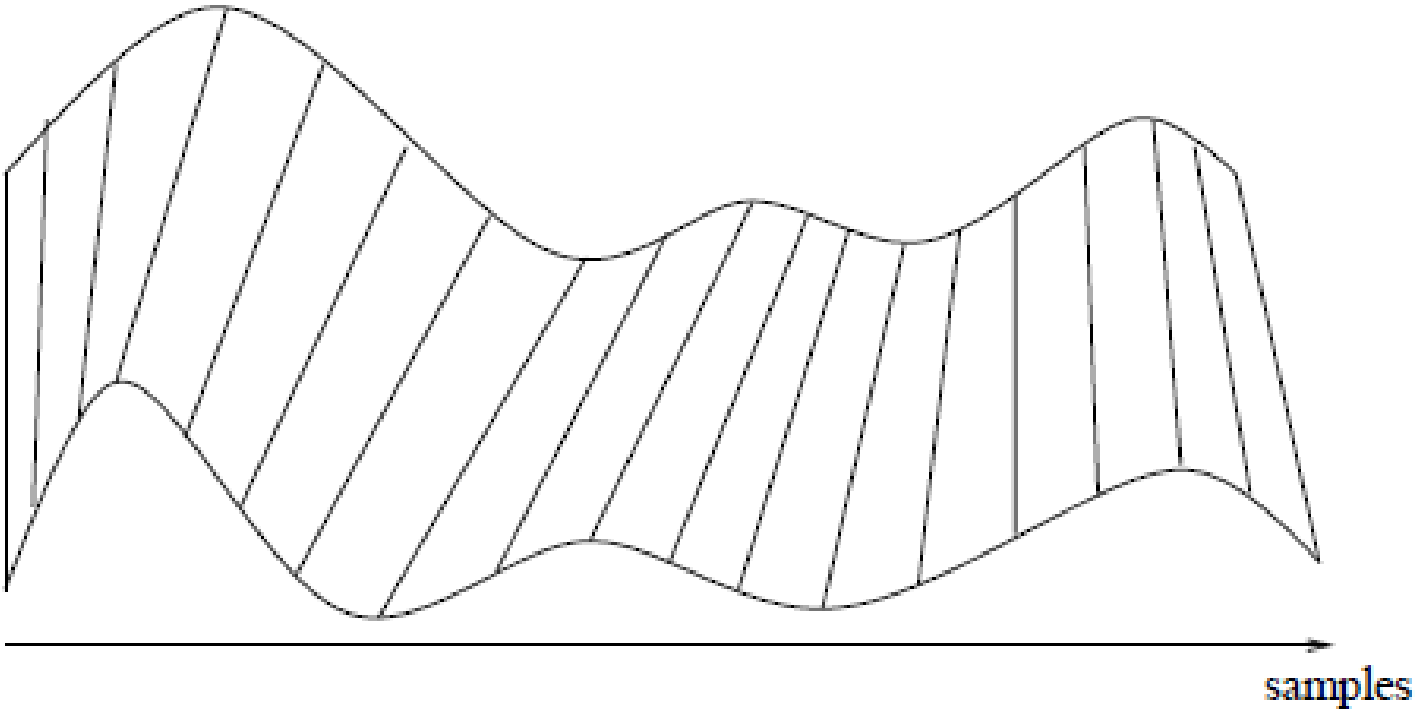
Curve alignment by dynamic time warping.



Source: *Word Image Matching Using Dynamic Time Warping*

Examples of Curve Alignment (1)

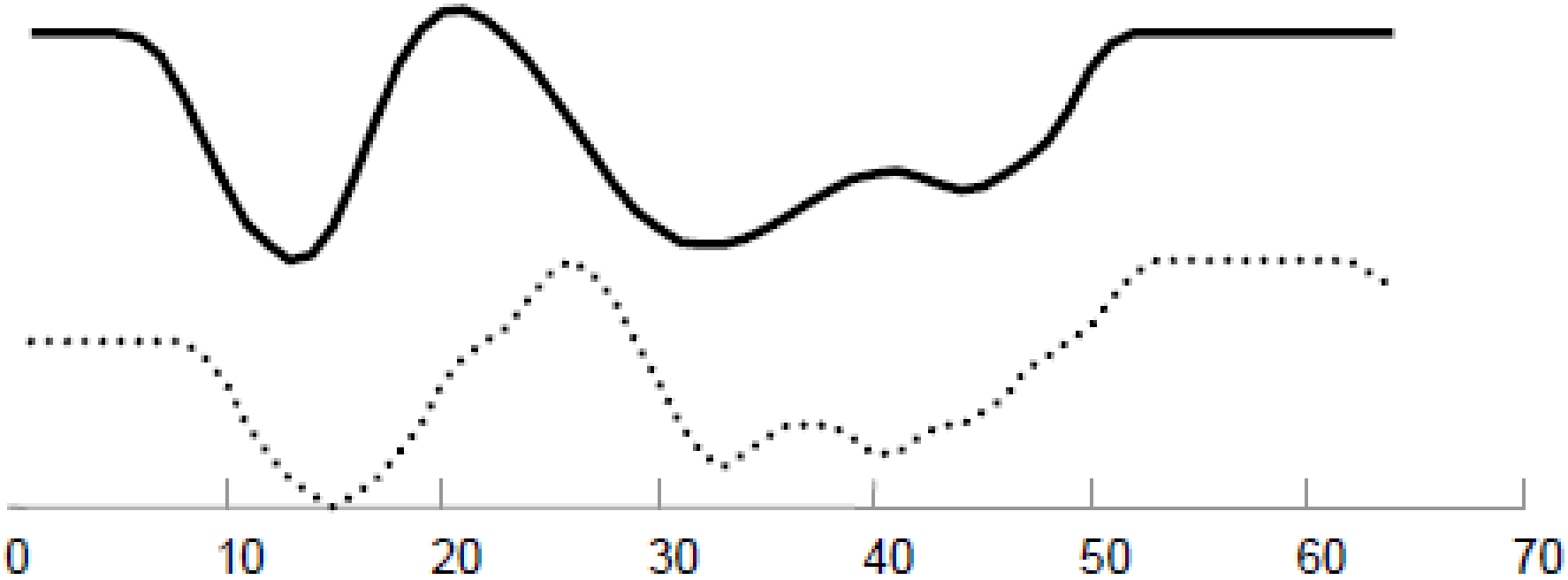
Curve alignment by dynamic time warping.



Source: *Word Image Matching Using Dynamic Time Warping*

Examples of Curve Alignment (2)

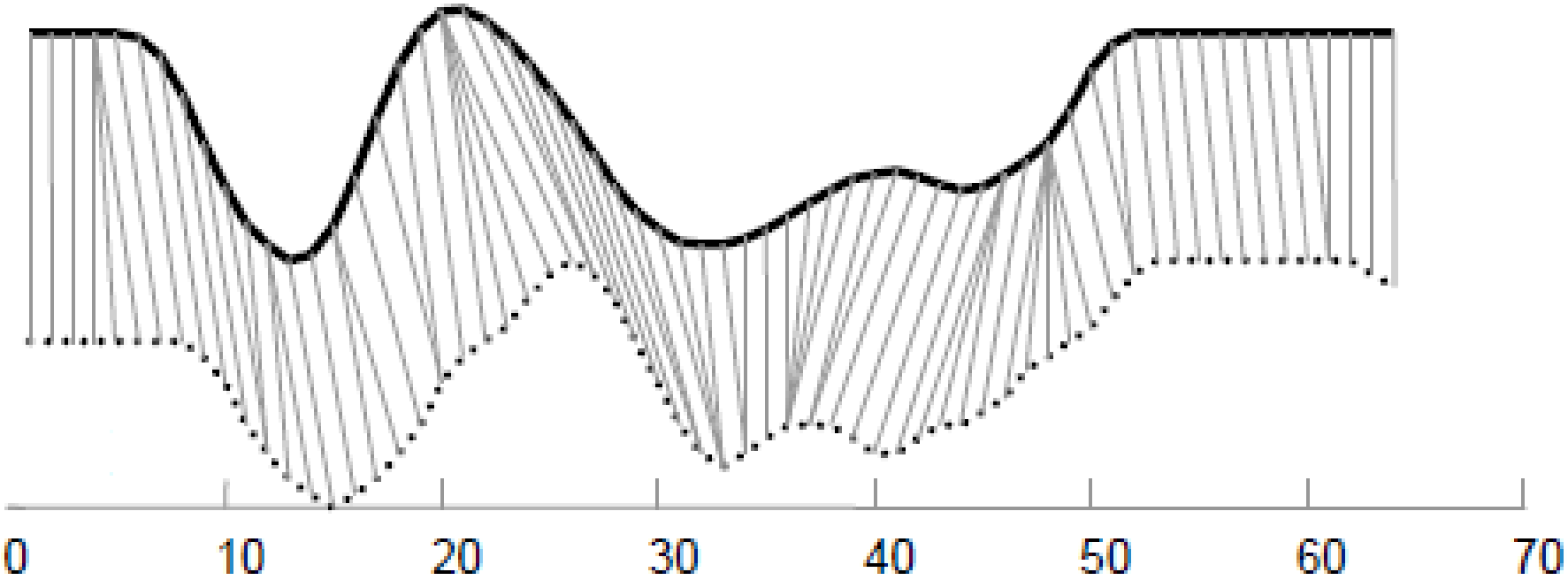
Curve alignment by derivative dynamic time warping.



Source: *Derivative Dynamic Time Warping*

Examples of Curve Alignment (2)

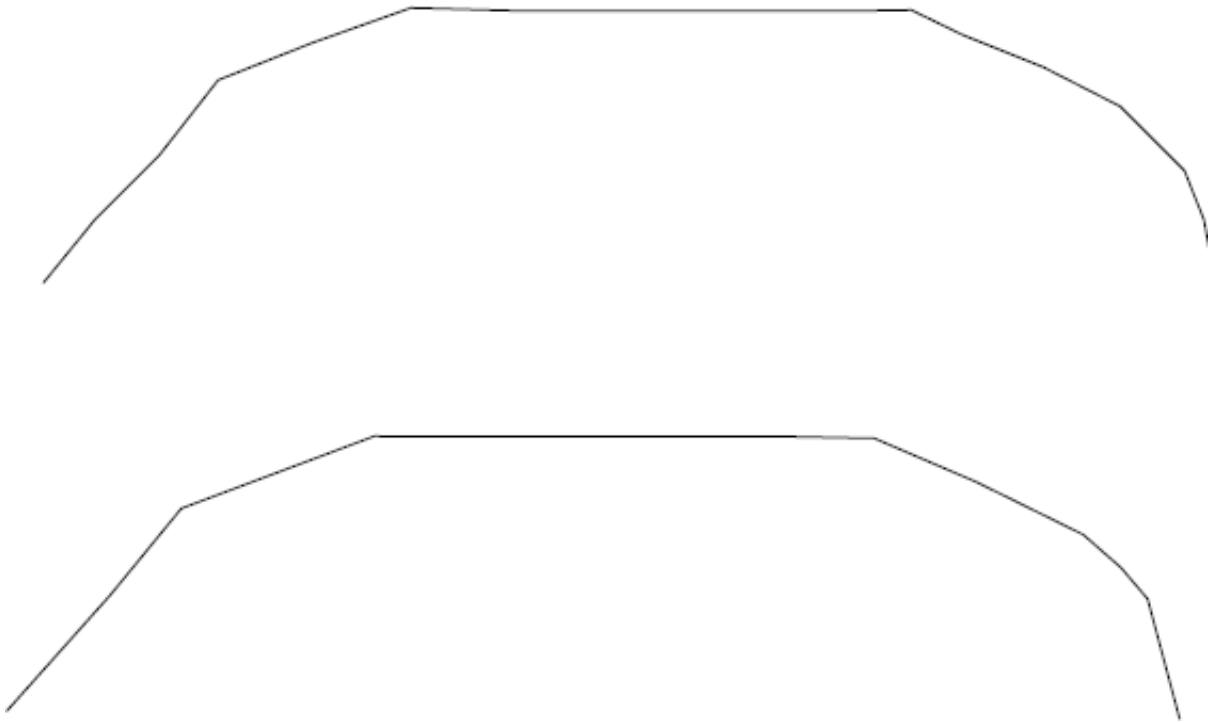
Curve alignment by derivative dynamic time warping.



Source: *Derivative Dynamic Time Warping*

Examples of Curve Alignment (3)

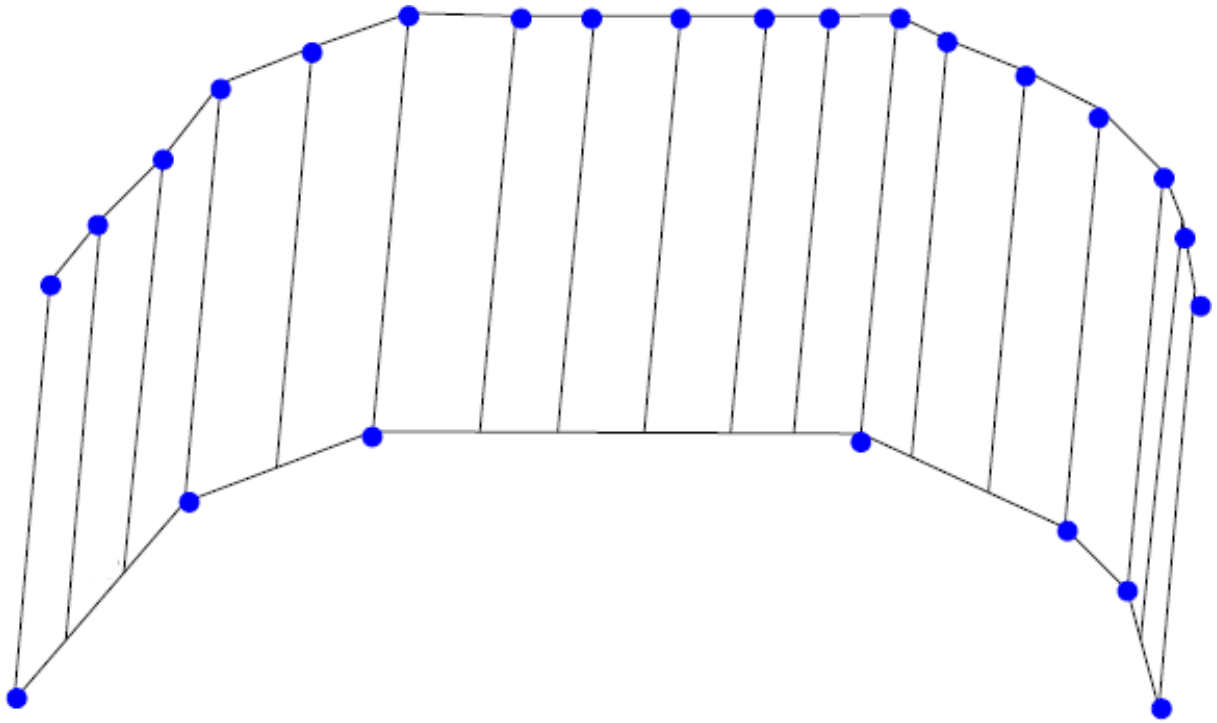
Curve alignment by continuous dynamic time warping.



Source: *Curve Matching, Time Warping, and Light Fields: New Algorithms for Computing Similarity between Curves*

Examples of Curve Alignment (3)

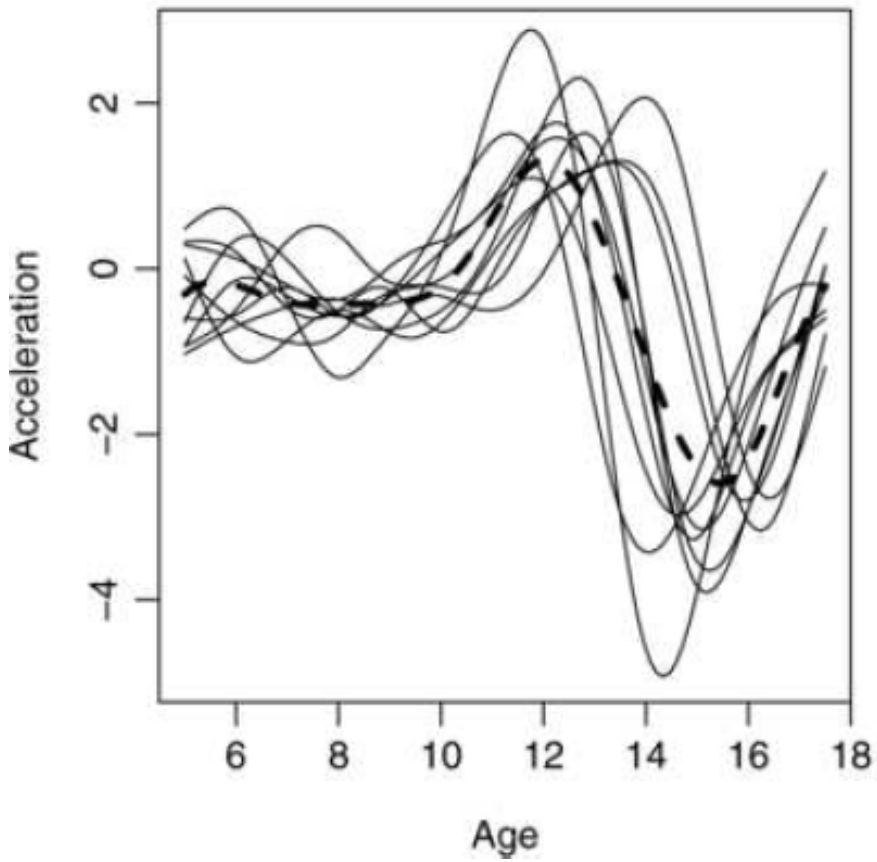
Curve alignment by continuous dynamic time warping.



Source: *Curve Matching, Time Warping, and Light Fields: New Algorithms for Computing Similarity between Curves*

Examples of Curve Alignment (4)

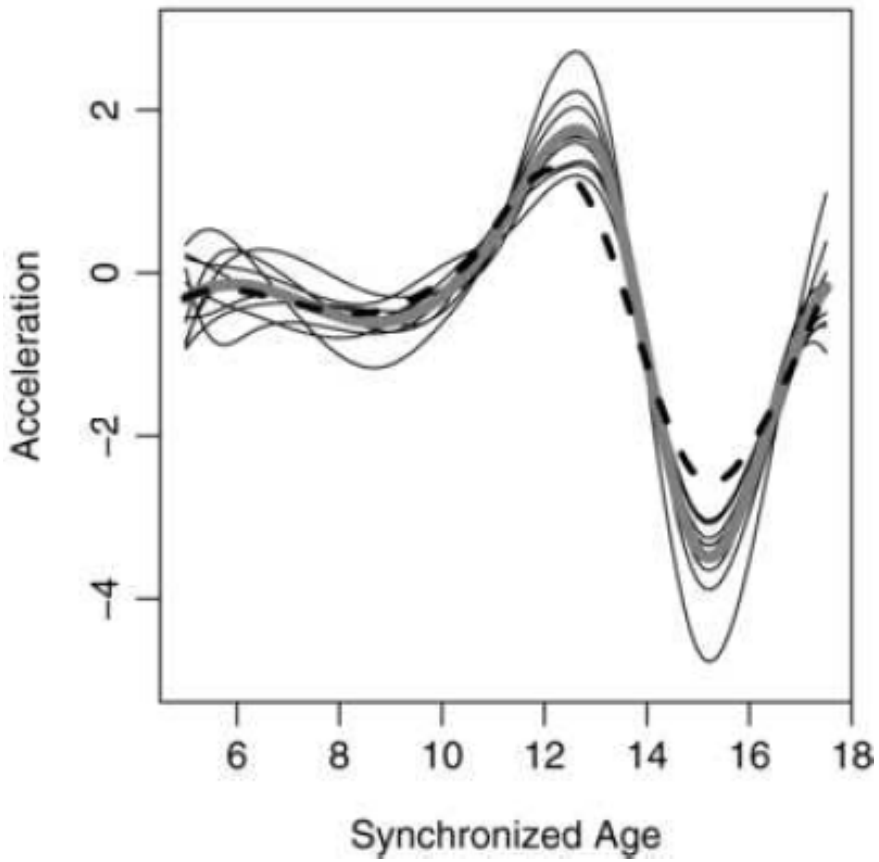
Curve alignment by equating the moments of a given set of curves.



Source: *Curve alignment by moments*

Examples of Curve Alignment (4)

Curve alignment by equating the moments of a given set of curves.

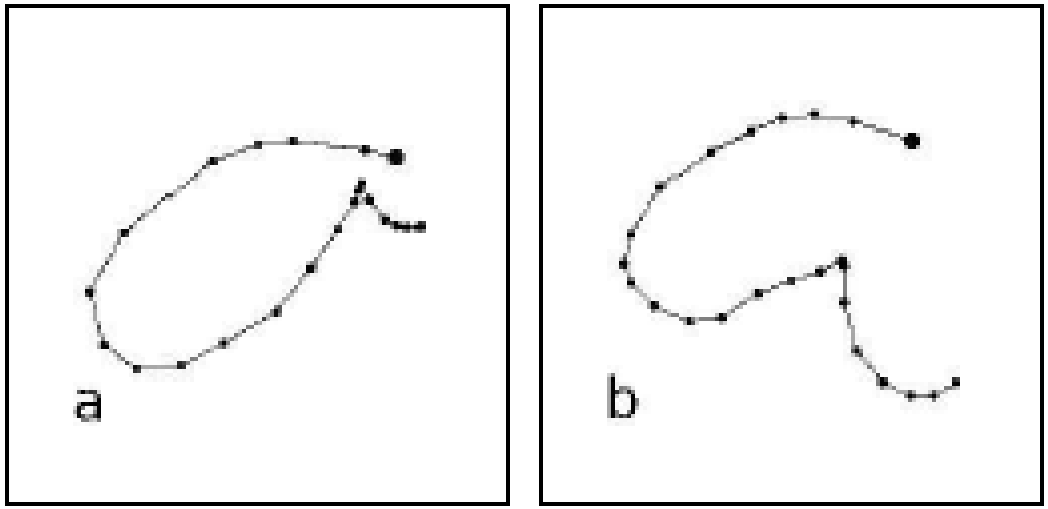


Source: *Curve alignment by moments*

Applications for Curve Alignment

Application: Handwriting Recognition

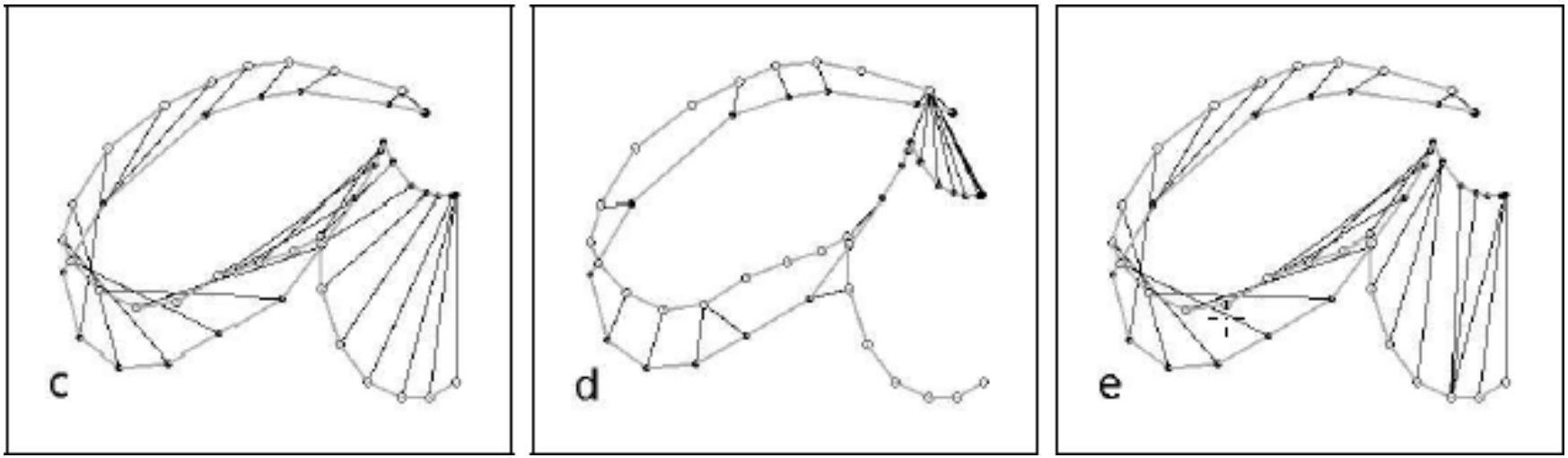
Handwriting can be represented using 2D parametric curves.



Source: *Using Dynamic Time Warping for Intuitive Handwriting Recognition*

Application: Handwriting Recognition

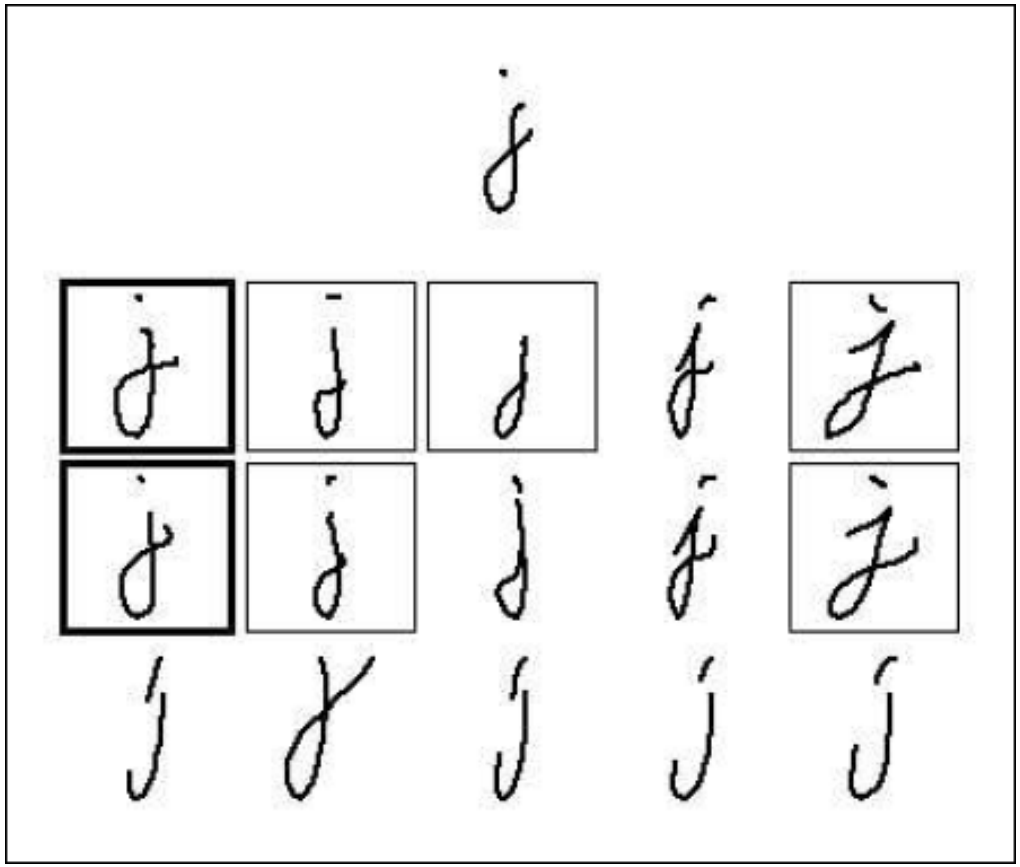
Curve alignment techniques can be used to match e.g. letters.



Source: *Using Dynamic Time Warping for Intuitive Handwriting Recognition*

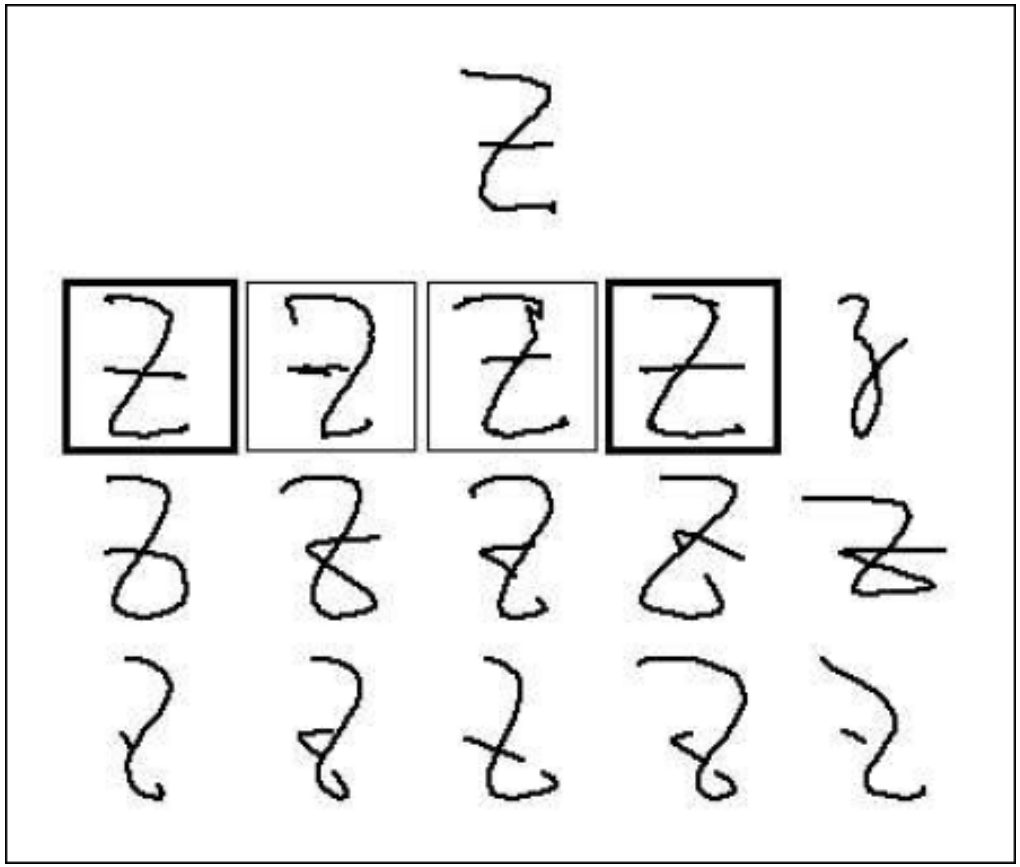
Warning: distance measure matters !

Application: Handwriting Recognition



Source: *Using Dynamic Time Warping for Intuitive Handwriting Recognition*

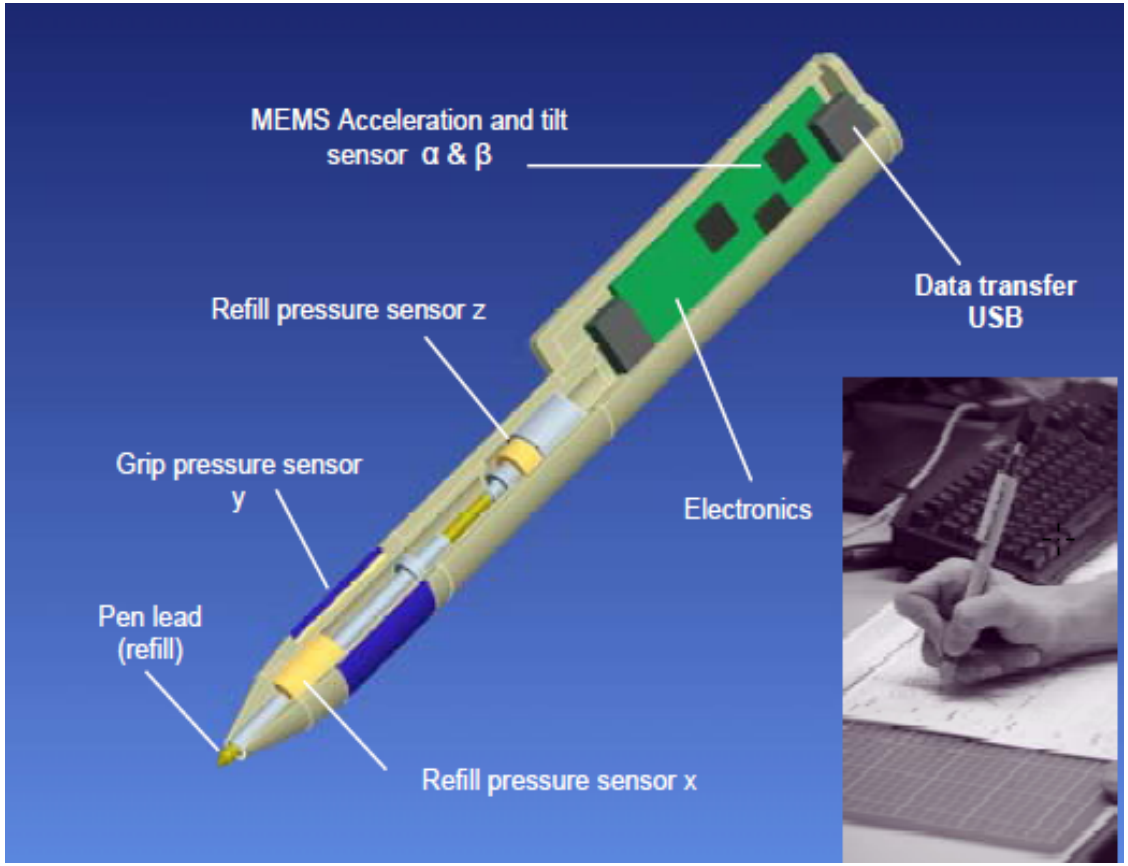
Application: Handwriting Recognition



Source: *Using Dynamic Time Warping for Intuitive Handwriting Recognition*

Application: Handwriting Recognition

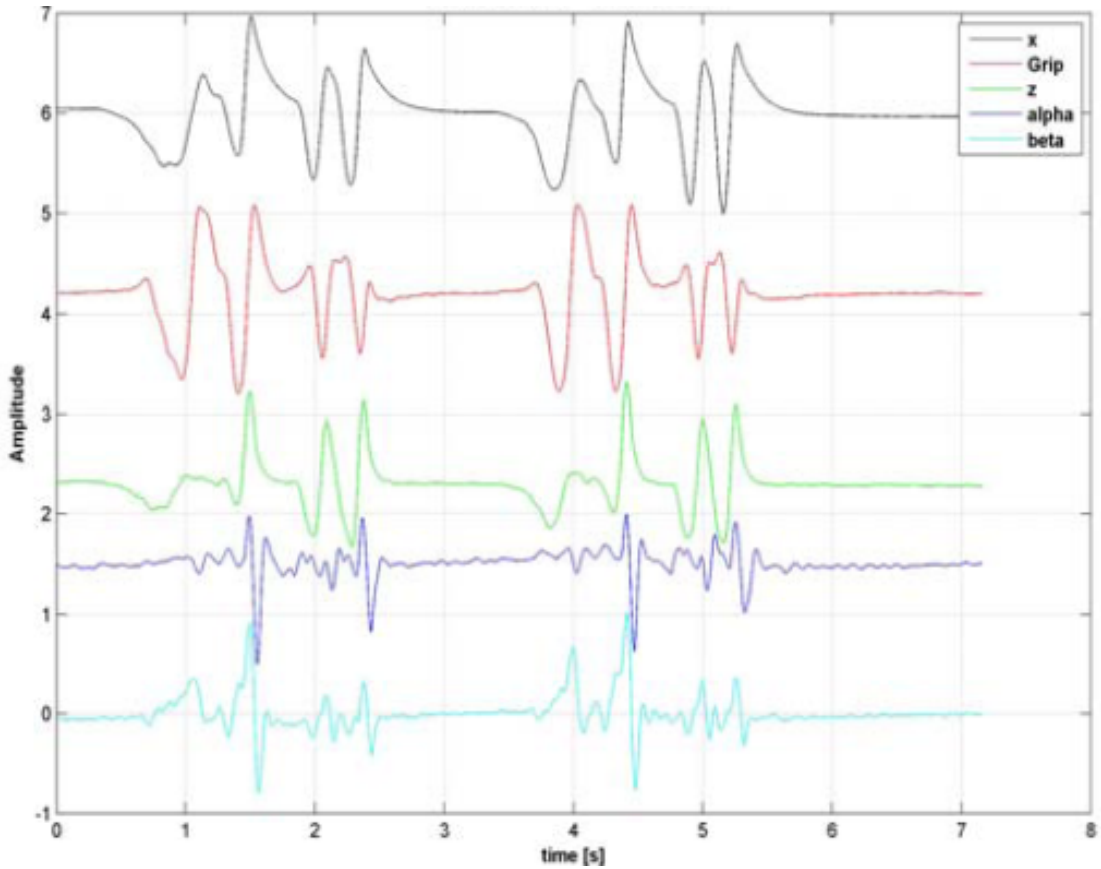
Different measures/curves can be matched simultaneously.



Source: *Reduced Dynamic Time Warping for Handwriting Recognition Based on Multidimensional Time Series of a Novel Pen Device*

Application: Handwriting Recognition

Different measures/curves can be matched simultaneously.



Source: *Reduced Dynamic Time Warping for Handwriting Recognition Based on Multidimensional Time Series of a Novel Pen Device*

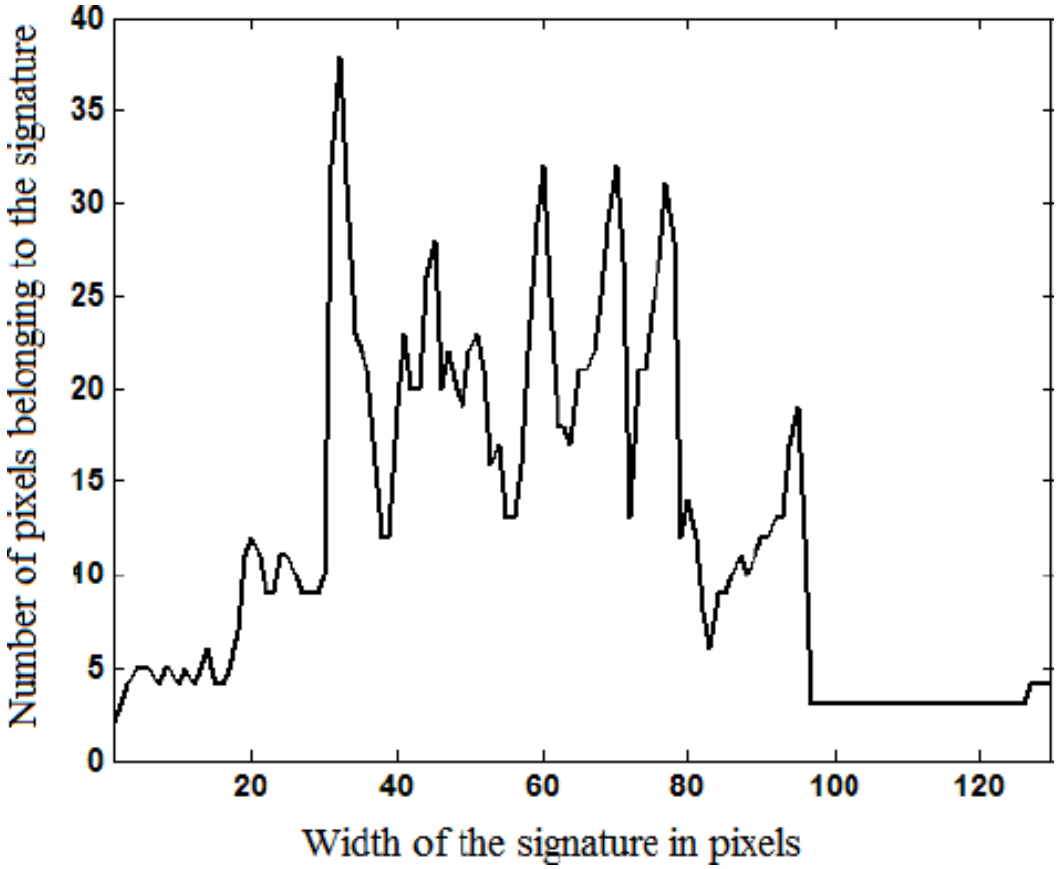
Application: Signature Verification

Raw images can also be compared if a proper transformation is used.

Source: *Dynamic Time Warping Based Static Hand Printed Signature Verification*

Application: Signature Verification

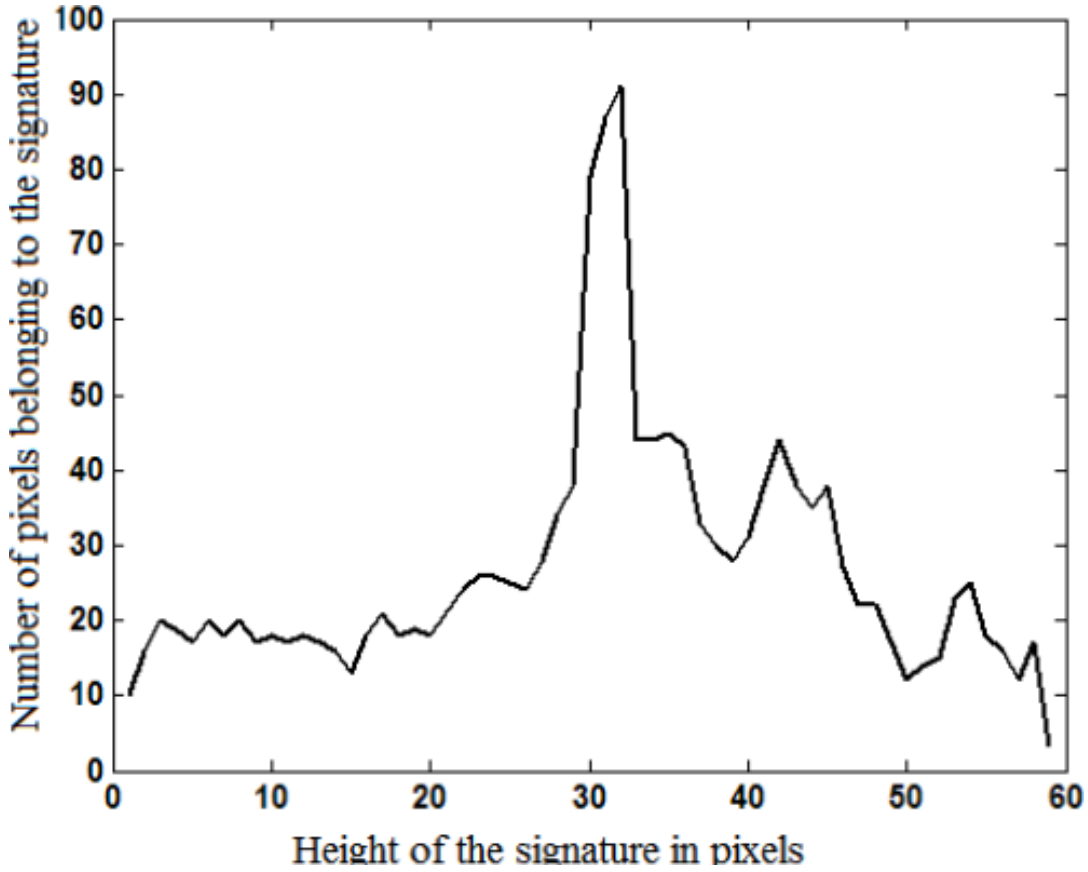
Example: projection of the image on horizontal/vertical axis.



Source: *Dynamic Time Warping Based Static Hand Printed Signature Verification*

Application: Signature Verification

Example: projection of the image on horizontal/vertical axis.



Source: *Dynamic Time Warping Based Static Hand Printed Signature Verification*

Dynamic Programming is not Limited to Curves

Dynamic time warping belongs to the dynamic programming methods.

Dynamic programming can also be used to align sequences of symbols.

T – – T C A T A
T G C T C G T A

Source: *What is Dynamic Programming ?*

Dynamic programming can use the costs for edit/match/delete operations.

Conclusion

Curve Alignment: a Powerful Tool

Curve alignment methods allow comparing curves

- with different number of points
- with different variation rates
- with slight dynamic differences

The key for success is to define a meaningful distance measure !

If necessary, extract features from the curve (e.g. derivative).

Different techniques exist, but dynamic time warping is a good solution.

Thank you for
your Attention !

References I


M Bashir and J Kempf.
Reduced dynamic time warping for handwriting recognition based on multi-dimensional time series of a novel pen device.
International Journal of Electrical and Computer Engineering, 3(8), 2008.


Sean R Eddy.
What is dynamic programming ?
Nat Biotech, 22, 2004.


Alon Efrat, Quanfu Fan, and Suresh Venkatasubramanian.
Curve matching, time warping, and light fields: New algorithms for computing similarity between curves.
Journal of Mathematical Imaging and Vision, 27(3):203–216, 2007.

References II


Gareth M James.
Curve alignment by moments.
Annals of Applied Statistics, 1(2):480–501, 2007.


Eamonn J. Keogh and Michael J. Pazzani.
Derivative dynamic time warping.
In *First SIAM International Conference on Data Mining (SDM'2001*, 2001.


Ralph Niels and Louis Vuurpijl.
Using dynamic time warping for intuitive handwriting recognition.
In *Proc. IGS2005, 2005. In*, pages 217–221, 2005.


Jayadevan R, Satish R Kolhe, and Pradeep M Patil.
Dynamic time warping based static hand printed signature verification.
Journal of Pattern Recognition Research, 4(1), 2009.

References III


Toni M. Rath and R. Manmatha.
Word image matching using dynamic time warping.
In *CVPR (2)'03*, pages 521–527, 2003.


David N. Siriba, Daniel Eggert, and Monika Sester.
Relative curve orientation in the alignment of inconsistent linear datasets.
In *Proceddings of the 14th AGILE International Conference on Geographic Information Science*, 2011.