Relaxation Methods for Mathematical Programs with Complementarity Constraints

Haddou, Mounir
IRMAR-INSA Rennes, mounir.haddou@insa-rennes.fr

Abstract

We consider the Mathematical Program with Complementarity Constraints

$$\begin{array}{ll} \min\limits_{x \in \mathbb{R}^n} & f(x) \\ \text{s.t.} & g(x) \leq 0, h(x) = 0, \\ & 0 \leq G(x) \perp H(x) \geq 0, \end{array} \tag{1}$$

with $f : \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}^m$, $g : \mathbb{R}^n \to \mathbb{R}^q$ and $G, H : \mathbb{R}^n \to \mathbb{R}^c$ that are assumed continuously differentiable. The notation $0 \leq u \perp v \geq 0$ for two vectors u and v in $\mathbb{R}^c$ is a shortcut for $u_i \geq 0,\ v_i \geq 0$ and $u_i v_i = 0$ for all $i \in \{1, \ldots, c\}$.

We propose a new family of relaxation schemes for mathematical programs with complementarity constraints that extends the relaxations converging to an M-stationary point [1, 2, 3]. We discuss the properties of the sequence of relaxed non-linear programs as well as stationarity properties of limiting points. We prove under a new and weak constraint qualification, that our relaxation schemes have the desired property of converging to an M-stationary point.

Unfortunately, in practice, relaxed problems are only solved up to approximate stationary points and the guarantee of convergence to an M-stationary point is lost (c.f [4]).

We define a new strong approximate stationarity condition and prove that we can maintain our guarantee of convergence and attain the desired goal of computing an M-stationary point.

A comprehensive numerical comparison between existing relaxations methods is performed and shows promising results for our new methods.

We also propose different extensions to tackle MPVC (vanishing constraints) and MOCC (cardinality constraints) problems.

References

[1] Flegel, Michael L and Kanzow, Christian, *Abadie-type constraint qualification for mathematical programs with equilibrium constraints* , Journal of Optimization Theory and Applications, 2005, V.124 (3), pp 595–614 .

[2] Kadrani, Abdeslam and Dussault, Jean-Pierre and Benchakroun, Abdelhamid, *A new regularization scheme for mathematical programs with complementarity constraints*, SIAM Journal on Optimization, 2009, V. 20 (1), pp 78–103.

[3] Schwartz, Alexandra, *Mathematical programs with complementarity constraints: Theory, methods, and applications*, Ph. D. dissertation, Institute of Applied Mathematics and Statistics, University of Wrzburg, 2011.

[4] Kanzow, Christian and Schwartz, Alexandra, *The Price of Inexactness: Convergence Properties of Relaxation Methods for Mathematical Programs with Complementarity Constraints Revisited* , Mathematics of Operations Research, 2015, V.40(2), pp 253–275.