From deterministic to stochastic Hodgkin-Huxley models

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The classical Hodgkin-Huxley model, still considered in neuroscience as the best existing model to describe the spiking behaviour of a periodically stimulated neuron, is a deterministic 4-dim dynamical system with a complicated phase diagram. The deterministic model cannot match the fact that real membrane potential data show a quadratic variation. We introduce a stochastic Hodgkin-Huxley model where dendritic input is the only source of noise.

This amounts to consider strongly degenerate stochastic differential equations with smooth coefficients: the diffusion coefficient does not depend on time; the drift depends on both time and spatial position and is periodic in the time argument. We give simple criteria for positive Harris recurrence of such systems. These are formulated in terms of control systems and the support theorem, in terms of one inner point of the state space which is of full weak Hoermander dimension, and in terms of some Lyapunov function.

In the stochastic Hodgkin-Huxley model, positive Harris recurrence allows to describe the spiking behaviour of the neuron in a unified way: there is a unique invariant measure, and we have strong laws of large numbers.

The talk is based on joint work with Eva Löcherbach, Université Paris I, and with Michele Thieullen, Université Paris VI.