LÉVY PROCESSES AND ITÔ-SKOROHOD INTEGRALS

Khalifa Es-Sebaiy and Ciprian A. Tudor

Abstract. We study Skorohod integral processes on Lévy spaces and we prove an equivalence between this class of processes and the class of Itô-Skorohod process (in the sense of [14]). Using this equivalence we introduce a stochastic analysis of Itô type for anticipating integrals on Lévy spaces.

Introduction

We study in this work anticipating integrals with respect to a Lévy process. The anticipating integral on the Wiener space, known in general as the Skorohod integral (and sometimes as the Hitsuda integral) constitutes an extension of the standard Itô integral to non-adapted integrands. It is nothing else than the classical Itô integral if the integrand is adapted. The Skorohod integral has been extended to the Poisson process and next it has been defined with respect to a normal martingale (see [3]) due to the Fock space structure generates by such processes. Recently, an anticipating calculus of Malliavin-type has been defined on Lévy spaces again by using some multiple stochastic integral with respect to a Lévy process which have been in essence defined in the old paper by K. Itô (see [4]). We refer to [8], [9] or [13] for Malliavin calculus on Lévy spaces and possible applications to mathematical finance.

The purpose of this paper is understand the relation between anticipating Skorohod integral processes and Itô-Skorohod integral process (in the sense of [14] or [11]) in the Lévy case. We recall that the results in [14] and [11] show that on Wiener and Poisson spaces the class of Skorohod integral process with regular integrals coincides with the class of some Itô-Skorohod integrals that have similarities to the classical Itô integrals for semimartingales. The fact that the driven processes have independent increments plays an crucial role. Therefore, it is expected to obtain the same type of results for Lévy processes. We prove here a such equivalent between Skorohod and Itô-Skorohod integrals by using the recent introduced Malliavin calculus for Lévy processes.

Section 2 contains some preliminaries on Lévy processes and Malliavin-Skorohod calculus for them. In Section 3 we prove a generalized Ocone-Clark formula that we will use in Section 4 to prove the correspondence between Skorohod and Itô-Skorohod integrals and to develop an Itô-type calculus for anticipating integrals on Lévy spaces.

Preliminaries

In this section we introduce the basic properties of Malliavin calculus for Lévy processes used in this paper. For more details the reader is referred to [13].

In this paper we deal with a càdlàg Lévy process $X=(X_t)_{0\leq t\leq 1}$ defined on certain complete probability space $\big(\Omega,(F_t^X)_{0\leq t\leq 1},P\big)$, with the time horizon $T=[0,1]$, and equipped with its generating triplet

1991 *Mathematics Subject Classification.* 60H07, 60G20, 60G48, 60J75..

Key words and phrases. Malliavin Calculus, martingale-valued measure, Skorohod integral, Clark-Ocone formula..

(γ, σ^2, ν) where $\gamma \in R$, $\sigma \geq 0$ and $\nu(dz)$ is the Lévy measure on R which, we recall, is such that $\nu(\{0\}) = 0$ and

$$\int_R 1 \wedge x^2 \nu(dx) < \infty$$

Throughout the paper, we suppose that $\int_R x^2\nu(dx) < \infty$, and we use notation and terminologies that is in [1], [13]. By N we will denote the jump measure of X:

$$N(E) = \#\{t : (t, \Delta X_t) \in E\},$$

for $E \in B(T \times R_0)$, where $R_0 = R - \{0\}$, $\Delta X_t = X_t - X_{t^-}$, $\#$ denotes the cardinal.We will note cardinal.We will note $\widetilde{N}$ the compensated jump jump measure:

$$\widetilde{N}(dt, dx) = N(dt, dx) - dt\nu(dx).$$

The process X admits a Lévy-Itô representation

$$X_t = \gamma t + \sigma W_t + \int\int_{(0,t]\times\{|x|>1\}} xN(ds, dx) + \lim_{\varepsilon\downarrow 0} \int\int_{(0,t]\times\{\varepsilon<|x|\leq 1\}} x\widetilde{N}(ds, dx)$$

where W is a standard Brownian motion.

Itô [4] proved that X can be extended to a martingale-valued measure M of type $(2, \mu)$ on $(T \times R, B(T \times R))$:

For any $E \in B(T \times\ R)$ with $\mu(E) < \infty$

$$M(E) = \sigma \int_{E(0)} dW_s + \lim_{n\to\infty} \int\int_{\{(s,x)\in E:\frac{1}{n}<|x|<n\}} x\widetilde{N}(ds, dx).$$

where $E(0) = \{s \in T : (s, 0) \in E\}$ and

$$\mu(E) = \sigma^2 \int_{E(0)} ds + \int\int_{\{E-E(0)\times\{0\}\}} x^2 ds\nu(dx).$$

Furthermore, M is a centered independent random measure such that

$$E\left(M(E_1)M(E_2)\right) = \mu(E_1 \cap E_2)$$

for any $E_1, E_2 \in B(T \times R)$ with $\mu(E_1) < \infty$ and $\mu(E_2) < \infty$.

Using the random measure M one can construct multiple stochastic driven by a Lévy process as an isometry between $L^2(\Omega)$ and the space $L^2\left((T \times R)^n, B((T \times R)^n), \mu^{\otimes n}\right)$. Indeed, one can use the same steps as on the Wiener space: first, consider a simple function f of the form

$$f = 1_{E_1\times\ldots\times E_n}$$

where $E_1, \ldots, E_n \in B(T \times R)$ are pathwise disjoint and $\mu(E_i) < \infty$ for every i. For a such function define $I_n(f) = M(E_1 \ldots M(E_n))$ and then the operator I_n can be extended by linearity and continuity to an isometry between $L^2(\Omega)$ and the space $L^2\left((T \times R)^n, B((T \times R)^n), \mu^{\otimes n}\right)$.

An interesting fact is that, as in the Brownian and Poissonian cases, M enjoys the chaotic representation property (see [13]), i.e. for every $F \in L^2(\Omega, F^X, P) = L^2(\Omega)$, can be written as an orthogonal sum of multiple stochastic integrals

$$F = E(F) + \sum_{n=1}^{\infty} I_n(f_n)$$

where this converges in $L^2(\Omega)$ and $f_n \in L^2_s((T \times R)^n, B((T \times R)^n), \mu^{\otimes n})$ (the last space is the space of symmetric and square integrable functions on $(T \times R)^n$ with respect to $\mu^{\otimes n}$.)

At this point we can introduce a Malliavin calculus with respect to the Lévy process X by using this Fock space-type structure. If

$$\sum_{n=0}^{\infty} n n! \|f_n\|_n^2 < \infty$$

(here $\|f_n\|_n$ denotes the norm in the space $L^2\left((T \times R)^n, B((T \times R)^n), \mu^{\otimes n}\right)$) then the Malliavin derivative of F is introduced as an annihilation operator (see for example [7])

$$D_z F = \sum_{n=1}^{\infty} n I_{n-1}(f_n(z, .)), \quad z \in T \times R.$$

The domain of derivative operator D is denoted by $D^{1,2}$. It contains the random variable of the above chaotic form such that $\sum_{n=0}^{\infty} n n! \|f_n\|_n^2 < \infty$ holds. We denote by $D^{k,2}$, $k \geq 1$ the domain of the kth iterated derivative $D^{(k)}$, which is a Hilbert space with respect the scalar product

$$\langle F, G \rangle = E(FG) + \sum_{j=1}^{k} E \int_{(T \times R)^j} D_z^{(j)} F D_z^{(j)} G \mu(dz).$$

We introduce now the Skorohod integral with respect to X as a creation operator. Let $u \in H = L^2\left(T \times R \times \Omega, B(T \times R) \otimes F_T^X, \mu \otimes P\right)$, then, for every $z \in T \times R$, $u(z)$ admit the following representation

$$u(z) = \sum_{n=0}^{\infty} I_n(f_n(z, .)).$$

Here we have $f_n \in L^2((T \times R)^{n+1}, \mu^{\otimes^{n+1}})$ and f_n is symmetric in the last n variables. If

$$\sum_{n=0}^{\infty} (n+1)! \|\tilde{f}_n\|_{n+1}^2 < \infty$$

($\tilde{f}_n$ represents the symmetrization of f_n in all its $n+1$ variables) then the Skorohod integral $\delta(u)$ of u with respect to X is introduced by

$$\delta(u) = \sum_{n=0}^{\infty} I_{n+1}(\tilde{f}_n).$$

The domain of δ is the set of processes satisfying $\sum_{n=0}^{\infty} (n+1)! \|\tilde{f}_n\|_{n+1}^2 < \infty$ and we have the duality relationship

$$E(F\delta(u)) = E \int \int_{T \times R} D_z F u(z) \mu(dz), \quad F \in D^{1,2}.$$

We will use the notation

$$\delta(u) = \int_0^1 \int_R u_z \delta M(dz) = \int_0^1 \int_R u_{s,x} \delta M(ds, dx).$$

Remark 1. It has been proved in [13] that if the integrand is predictable then the Skorohod integral coincides with the standard semi-martingale integral introduced in [1].

For $k \geq 1$, we denote by $L^{k,2}$ the set $L^2((T \times R; D^{k,2}), \mu)$. In particular one can prove that $L^{1,2}$ is given by the set of u in the above chaotic form such that

$$\sum_{n=0}^{\infty} (n+1)! \|\tilde{f}_n\|_{n+1}^2 < \infty.$$

We also have $L^{k,2} \subset Dom\delta$ for $k \geq 1$ and and for every $u, v \in L^{1,2}$

$$E(\delta(u)\delta(v)) = E \int \int_{T \times R} u(z)v(z)\mu(dz) + E \int \int_{(T \times R)^2} D_z u(z') D_{z'} v(z) \mu(dz)\mu(dz').$$

In particular

$$E(\delta(u))^2 = E \int \int_{T \times R} u(z)^2 \mu(dz) + E \int \int_{(T \times R)^2} D_z u(z') D_{z'} u(z) \mu(dz)\mu(dz').$$

The commutativity relationship between the derivative operator and Skorohod integral is given by: let $u \in L^{1,2}$ such that $D_z u \in Dom(\delta)$, then $\delta(u) \in D^{1,2}$ and

$$D_z \delta(u) = u(z) + \delta(D_z(u)), \quad z \in T \times R.$$

Generalized Clark-Ocone formula

We start this section by proving some properties of the multiple integrals $I_n(f)$ and how it behaves if it is conditioned by a σ algebra. If $A \in B(T)$ we will denote by F_A^X the σ -algebra generated by the increments of the process X on the set A

$$F_A^X = \sigma(X_t - X_s : s, t \in A).$$

Proposition 1. *The text of the Proposition Let $f_n \in L_s^2((T \times R)^n, \mu^{\otimes^n})$ and $A \in B(T)$. Then*

$$E\left(I_n(f)/F_A^X\right) = I_n(f 1_{(A \times R)}^{\otimes^n}).$$

Proof. By density and linearity argument, it is enough to consider $f = 1_{E_1 \times \ldots \times E_n}$, where $E_1, ..., E_n$ are pairwise disjoint set of $B(T \times R)$ and $\mu(E_i) < \infty$ for every $i = 1, ..., n$. In this case we have

$$E\left(I_n(f)/F_A^X\right) = E\left(M(E_1)....M(E_n)/F_A^X\right)$$

$$E\left(\prod_{i=1}^n \left(M(E_i \cap (A \times R) + M(E_i \cap (A^c \times R))\right)/F_A^X\right)$$

$$= \prod_{i=1}^n M(E_i \cap (A \times R)) = I_n(f 1_{(A \times R)}^{\otimes^n}).$$

And an immediate consequence, we have

Corollary 1. *Suppose that* $F \in D^{1,2}$ *and* $A \in B(T)$. *Then the conditional expectation* $E(F/F_A^X)$ *belongs to* $D^{1,2}$ *and for every* $z \in T \times R$

$$D_z E\left(F/F_A^X\right) = E\left(D_z F/F_A^X\right) 1_{A\times R}(z).$$

Proof. let $F = \sum_{n\geq 0} I_n(f_n)$ with $f_n \in L_s^2\left((T\times R)^n, B((T\times R)^n), \mu^{\otimes n}\right)$. Then by Proposition 1,

$$D_z E\left(F/F_A^X\right) = D_z\left(\sum_{n\geq 0} I_n\left(f_n 1_{A\times R}^{\otimes n}\right)\right)$$

$$= \sum_{n\geq 1} n I_{n-1}\left(f_n(\cdot,z) 1_{A\times R}^{\otimes(n-1)}\right) 1_{A\times R}(z)$$

and it remains to observe that $E\left(D_z F/F_A^X\right) = \sum_{n\geq 1} n I_{n-1}\left(f_n(\cdot,z)1_{A\times R}^{\otimes(n-1)}\right)$.

At this point we will state the following version of Ocone-Clark formula on Lévy space which extends a results in [13].

Proposition 2. *[Generalized Clark-Ocone-Haussman formula] Let* F *be a random variable in* $D^{1,2}$. *Then for every* $0 \leq s < t \leq 1$, *we have*

$$F = E\left(F/F_{(s,t]^c}^X\right) + \delta(h_{s,t}(\cdot))$$

where for $(r,x) \in T\times R$ *we denoted by* $h_{s,t}(r,x) = E\left(D_{r,x}F/F_{(r,t]^c}\right) 1_{(s,t]^c}(r)$. *Moreover*

$$F = E\left(F/F_{(s,t]^c}^X\right) + \int\int_{(s,t]\times R} {}^{(p,t)}(D_z F)\ dM_z$$

$$= E\left(F/F_{(s,t]^c}^X\right) + \sigma\int_s^t {}^{(p,t)}(D_{r,0}F)dW_r + \int\int_{(s,t]\times R_0} {}^{(p,t)}(D_{r,x}F)\ \widetilde{N}(dr,dx)$$

where ${}^{(p,t)}(DF)$ *is the predictable projection of* DF *with respect to the filtration* $\left(F_{(r,t]^c}^X\right)_{r\leq t}$.

Proof. Let $F = \sum_{n=0}^{\infty} I_n(f_n)$, where $f_n \in L_s^2\left(([0,1]\times R)^n, \mu^{\otimes^n}\right)$. Firstly, we prove that

$$F = E\left(F/F_{(s,t]^c}^X\right) + \delta\left(h_{s,t}(\cdot)\right).$$

Indeed, for any $s < t \leq 1$ we have

$$E(D_{r,x}F/F_{(r,t]^c}^X)1_{(s,t]\times R}(r,x) = \sum_{n=1}^{\infty} n I_{n-1}\left[f_n((r,x),.)1_{(r,t]^c\times R}^{\otimes^{n-1}}(.)1_{(s,t]\times R}(r,x)\right]$$

Hence, using that $x \in R$ and thus the symmetrization with respect to the variable x has no effect, we obtain

$$\delta(h) = \sum_{n=1}^{\infty} n I_n\left[f_n((t_1,x_1),...,(t_n,x_n))1_{(t_1,t]^c}^{\otimes^{n-1}}(\widetilde{t_2,...,t_n})1_{(s,t]}(t_1)\right]$$

Since

$$1^{\otimes n-1}_{(t_1,t]^c}\widetilde{(t_2,...,t_n)}1_{(s,t]}(t_1)$$

$$= \frac{1}{n!}\sum_{i=1}^{n}\sum_{\sigma(1)=i,\sigma\in S_n}^{n} 1^{\otimes n-1}_{(t_i,t]^c}(t_{\sigma(2)},...,t_{\sigma(n)})1_{(s,t]}(t_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} 1^{\otimes n-1}_{(t_i,t]^c}(t_1,...,\widehat{t_i},...,t_n)1_{(s,t]}(t_i)$$

$$= \frac{1}{n}\left(1-1^{\otimes n}_{(s,t]^c}(t_1,...,t_n)\right).$$

Then

$$\delta\left(h_{s,t}\right) = F - E\left(F/F^X_{(s,t]^c}\right).$$

The second equality in the statement follows from [13] (see also [11]) where the equivalence of the two representations has been proven.

Itô-Skorohod integral calculus

As a consequence of the above results, we will show in this part that every Skorohod integral process of the form

$$Y_t := \delta(u_{.}1_{[0,t]\times R}(\cdot)), \qquad t\in T$$

can be written as an Itô-Skorohod integral in the sense of [14] (this integral has similarities with the standard stochastic integral). We will extend in this way the results of [14] in the Wiener case and of [11] in the Poisson case. The key point of our construction is the fact that the driving process has independent increments.

Proposition 3. *Assume that $u\in L^{k,2}$ with $k\geq 3$, then there exist an unique process $v\in L^{k-2,2}$ such that for every $t\in T$*

$$Y_t := \delta(u_{.}1_{[0,t]\times R}(\cdot)) = \int\int_{(0,t]\times R}{}^{(p,t)}(v_{s,x})\ M(ds,dx).$$

Moreover $v_{s,x}=D_{s,x}Y_s \quad \mu\otimes P$ a.e. on $T\times R\times\Omega$.

Proof. Applying above generalized Clark-Ocone formula we have

$$Y_t = E\left(Y_t/F^X_{t^c}\right) + \int\int_{(0,t]\times R}{}^{(p,t)}(D_zY_t)\ dM_z$$

The process Y satisfies

$$E\left(Y_t - Y_s/F^X_{(s,t]^c}\right) = 0$$

for every $s<t$. Indeed, we take a random variable $F^X_{(s,t]^c}$-measurable F in $D^{1,2}$. According to the duality relationship and Corollary 1, we have

$$E(F(Y_t-Y_s)) = E[\delta(u_{.}1_{(s,t]\times R}(.))] = E\left\langle D_{.}F, u_{.}1_{(s,t]\times R}(.)\right\rangle_{L^2(T\times R,\ \mu)} = 0.$$

Therefore, we obtain

$$E\left(Y_t/F^X_{t^c}\right) = E\left(Y_t - Y_0/F^X_{t^c}\right) = 0,$$

and

$$\delta\left[E^{\ (p,t)}\left(D_{s,x}Y_t\right)1_{[0,t]\times R}(s,x)\right]=\delta\left[E\left(D_{s,x}Y_t/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$=\delta\left[D_{s,x}E\left(Y_t/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$=\delta\left[D_{s,x}E\left(Y_s/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$=\delta\left[E\left(D_{s,x}Y_s/F^X_{(s,t]^c}\right)1_{[0,t]\times R}(s,x)\right]$$

$$=\delta\left[E^{\ (p,t)}\left(D_{s,x}Y_s\right)1_{[0,t]\times R}(s,x)\right]$$

We thus have

$$Y_t=\int\int_{(0,t]\times R}{}^{(p,t)}(D_{s,x}Y_s)\ M(ds,dx).$$

Set $v_{s,x}=D_{s,x}X_s$. To obtain the desired Itô-Skorohod representation it is sufficient to prove that $v\in L^{k-2,2}$. By using the property of commutativity between D and δ and the inequalities for the norms of anticipating integrals we have

$$\|v\|^2_{1,2}\le\|u\|^2_{1,2}+\|\delta(D_{s,x}u_{.}1_{[0,s]\times R}(.))\|^2_{1,2}$$

$$\le\|u\|^2_{1,2}+E\int\int_{T\times R}\left(\delta(D_{s,x}u_{.}1_{[0,s]\times R}(.)\right)^2\mu(ds,dx)$$

$$+E\int\int_{(T\times R)^2}\left(D_{r,y}\delta(D_{s,x}u_{.}1_{[0,s]\times R}(.))\right)^2\mu(ds,dx)\mu(dr,dy)$$

$$\le\|u\|^2_{1,2}+3E\int_{T\times R}\int_{T\times R}\left(D_{z_2}u_{z_1}\right)^2\mu(z_1)\mu(z_2)$$

$$+3E\int_{T\times R}\int_{T\times R}\int_{T\times R}\left(D_{z_3}D_{z_2}u_{z_1}\right)^2\mu(z_1)\mu(z_2)\mu(z_3)$$

$$+2E\int_{T\times R}\int_{T\times R}\int_{T\times R}\int_{T\times R}\left(D_{z_4}D_{z_3}D_{z_2}u_{z_1}\right)^2\mu(z_1)\mu(z_2)\mu(z_3)\mu(z_4)\le 4\|u\|^2_{3,2}.$$

In the same manner, we found that

$$\|v\|^2_{k-2,2}\le C(k)\|u\|^2_{k,2},$$

where $C(k)$ is a positive constant depending of k. To conclude our proof we have to show the uniqueness of these processes. We assume that there exist v and v' in $L^{k-2,2}$ such that

$$Y_t=\int\int_{(0,t]\times R}{}^{(p,t)}(v_{s,x})\ M(ds,dx)=\int\int_{(0,t]\times R}{}^{(p,t)}(v'_{s,x})\ M(ds,dx).$$

Using again the property of commutativity, we have

$$E\left(w_{s,x}/F^X_{[s,t]^c}\right)1_{[0,t]\times R}(s,x)+\int\int_{(s,t]\times R}E\left(D_{s,x}w_{r,x}/F^X_{[r,t]^c}\right)\;M(dr,dx)=0,$$

where $w_{s,x}=v_{s,x}-v'_{s,x}$. Conditioning by $F^X_{[s,t]^c}$, we obtain

$$E\left(w_{s,x}/F^X_{[s,t]^c}\right)1_{[0,t]\times R}(s,x)=0,\quad s\leq t, x\in R.$$

By letting t goes to s we get that $w_{s,x}=0$ in $L^2(\Omega)$ for every $(s,x)\in\ T\times\ R$. We can thus conclude that $v=v'$ in $L^{k-2,2}$.

Using the correspondence between Skorohod and Itô-Skorohod integrals proved above, we can derive an Itô formula for anticipating integrals on Lévy space. As far as we know, this is the only Itô formula proved for these class of processes.

Proposition 4. *[Itô's formula] Let v be a process belong to $L^2(T\times R\times\Omega,\mu\otimes P)$. Let us consider the following stochastic process*

$$Y_t=\int\int_{(0,t]\times R}E\left(v_{s,x}/F^X_{[s,t]^c}\right)M(ds,dx)$$

and let f be a C^2 real function. Then

$$f(Y_t)=f(0)+\int\int_{(0,t]\times R}f'(Y_t^{s^-})^{(p,t)}(D_{s,x}Y_s)\;M(ds,dx)$$

$$+\frac{1}{2}\int\int_{(0,t]\times R}f''(Y_t^{s^-})(^{(p,t)}(D_{s,0}Y_s))^2\;ds$$

$$+\sum_{0<s\leq t}\left(f(Y_t^s)-f(Y_t^{s^-})-f'(Y_t^{s^-})(Y_t^s-Y_t^{s^-})\right)$$

where $Y_t^s:=\int\int_{(0,s]\times R}E\left(v_{s,x}/F^X_{[s,t]^c}\right)M(ds,dx)$ and $Y_t^{s^-}=\lim_{r\to s^-}Y_t^r$ for all $0<s\leq t$.

Proof. Fix $t\in(0,T]$. We define $Z_s=Y_t^s$ if $s\leq t$ and $Z_s=Y_t$ if $s>t$. Also let $(G_s)_{s\geq0}$ be a filtration given as follows $G_s=F^X_{[s,t]^c}$ if $s\leq t$ and $G_s=F^X_1$, if $s>t$.

It is easy to see that $(Z_s)_{s\geq0}$ is a square integrable cadlag martingale with respect to $(G_s)_{s\geq0}$ and therefore the standard stochastic calculus for jump processes can be applied to it.

Applying Itô's formula (see [12], Theorem 32, p. 71) we obtain for every $s>0$

$$f(Z_s)=f(0)+\int_{(0,t]}f'(Z_{s^-})dZ_s+\frac{1}{2}\int_{(0,t]}f''(Z_{s^-})d[Z,Z]^c_s$$

$$+\sum_{0<s\leq t}\left(f(Z_s)-f(Z_{s^-})-f'(Z_{s^-})(Z_s-Z_{s^-})\right)$$

where $[Z,Z]^c$ is the continuous part of quadratic variation process $[Z,Z]$ of Z. It is well known that $[N,N]^c_s = 0$. From Proposition 4 of [13], we see that for every $s \leq t$

$$[Z,Z]^c_s = [Y,Y]^c_s = \sigma^2 \int_{(0,s]} E\left(v_{r,0}/F^X_{[r,t]^c}\right)^2 dr.$$

Thus, and in particular for $s = t$(in the sense of the limit almost sure or in L^2))

$$f(Z_t) = f(Y_t) = f(0) + \int\int_{(0,t]\times R} f'(Y_t^{s^-}) E\left(v_{s,x}/F^X_{[s,t]^c}\right) \; M(ds,dx)$$

$$+\frac{1}{2}\int\int_{(0,t]\times R} f''(Y_t^{s^-}) \left[E\left(v_{s,0}/F^X_{[s,t]^c}\right)\right]^2 \; ds$$

$$+ \sum_{0<s\leq t} (f(Y_t^s) - f(Y_t^{s^-}) - f'(Y_t^{s^-})(Y_t^s - Y_t^{s^-})).$$

So that the result holds.

Another consequence of Proposition 3 is the following Burkholder inequality which gives a bound for the L^p norm of the anticipating integral.

Proposition 5. *[Burkholder's Inequality] Let Y be a process of Itô-Skorohod form as above and $2 \leq p < \infty$. Then there exist a universal constant $C(p)$ such that*

$$E|Y_t|^p \leq C(p) E\left(\sigma^2 \int_{(0,t]} E\left(v_{r,0}/F^X_{[r,t]^c}\right)^2 dr + \int\int_{(0,t]\times R_0} x^2 E\left(v_{r,x}/F^X_{[r,t]^c}\right)^2 \; N(dr,dx)\right)^{p/2}$$

Proof. The proof of this proposition is straightforward from Theorem 54, P. 174 in [12] and the approximation procedure used along the paper.

References

1. D. Applebaum, *Lévy processes and stochastic and stochastic calculs*, Cambridge University Press, Cambridge, 2004.
2. F. E. Benth, G. Di Nunno, A. Lokka, B. Oksendal, F. Proske, *Explicit representations of the minimal variance portfolio in markets driven by Lvy processes*, Mathematical Finance **13** (2003), 55-72.
3. C. Dellacherie, B. Maissoneuve, P.A. Meyer, *Probabilités et Potentiel, Chapitres XVII à XXIV.*, Hermann, 1992.
4. K. Itô, *Spectral type of the shift transformation of differential processes with stationary increments*, Trans. Am. Math. Soc. **81** (1956), 252-263.
5. A. Lokka, *Martingale representations and functionals of Lvy processes*, Preprint series in Pure Mathematics, University of Oslo 21 (2001).
6. D. Nualart, *Malliavin calculs and related topics*, Springer, 1995.
7. D. Nualart, J. Vives, *Anticipative calculus for the Poisson space based on the Fock space*, Séminaire de Probabilités XXIV, LNM 1426, edited by Springer, 1990, pp. 154-165.
8. G. Di Nunno, B. Oksendal, F. Proske, *White Noise Analysis for Lévy processes*, Journal of Functional Analysis **206** (2004), 109-148.
9. G. Di Nunno, Th. Meyer-Brandis, B. Oksendal, F. Proske, *Malliavin calculus and anticipative Itô formulae for Lévy processes*, Preprint series in Pure Mathematics, University of Oslo (2004).
10. B. Oksendal, F. Proske, *White Noise of Poisson random measure*, Potential Analysis **21** (2004), 375-403.

11. G. Peccati, C.A. Tudor, *Anticipating integrals and martingales on the Poisson space*, Random Operators and Stochastic Equations, to appear (2007).
12. Ph. Protter, *Stochastic integration and differential equations: a new approach*, Springer, 1992.
13. J. L. Solé, F. Utzet, J. Vives, *Canonical Lévy process and Malliavin calculus*, Stochastic Processes and their Applications **117** (2007), 165-187.
14. C.A. Tudor, *Martingale type stochastic calculus for anticipating integral processes*, Bernouilli **10** (2004), 313-325.

Department of Mathematics, Faculty of Sciences Semlalia, Cadi Ayyad University 2390 Marrakesh, Morocco
E-mail address: k.essebaiy@ucam.ac.ma

SAMOS/MATISSE, Centre d'Economie de La Sorbonne, Université de Panthéon-Sorbonne Paris 1, 90, rue de Tolbiac, 75634 Paris Cedex 13, France.
E-mail address: tudor@univ-paris1.fr