URBAN-SEGREGATION MODELS (WITH A STATISTICAL-PHYSICS FLAVOUR)

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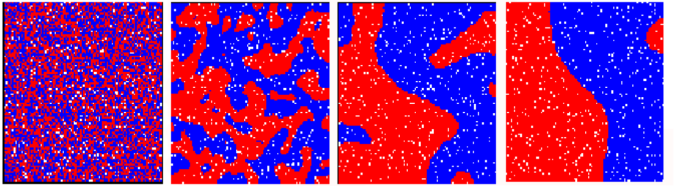
Monday, June 4th

Segregation

Can we "understand" racial and/or social segregation patterns in cities?



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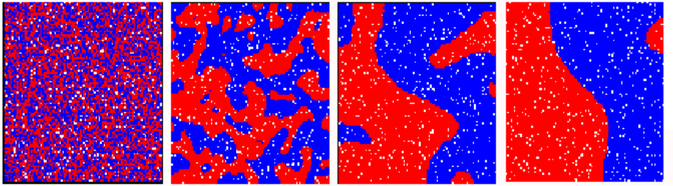

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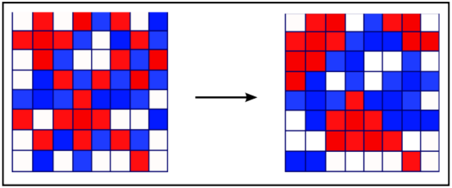
A Complex Phenomenon

- Key factors: (in)tolerance, prices ?
- Micro- or macroscopic drive to segregation ?
- Schelling's multi-agent model(s)

Ref: Schelling, *J. Math. Sociol.*, 1, 143 (1971)

Schelling's Model



- Initial configuration on a checkerboard of size $L \times L$, incl. density of vacant sites ρ.
- Satisfaction of agents given by:

$$N_d < T(N_d + N_s),$$

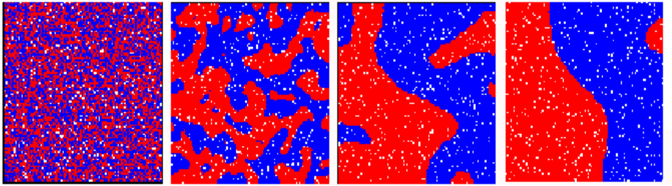
where T is the "tolerance".

- Dynamics incl. choice of moving agent and rules for moving.
- Definition and measure of segregation:

$$\mathcal{S} = \frac{2}{(L^2(1-\rho))^2} \sum_{\{C\}} n_C^2,$$

where n_C is the size of cluster C.

Segregation



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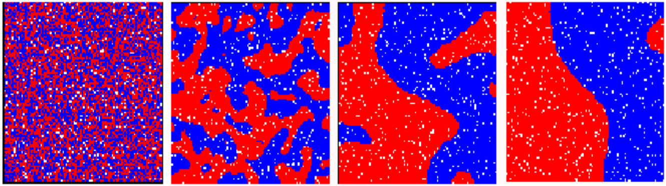

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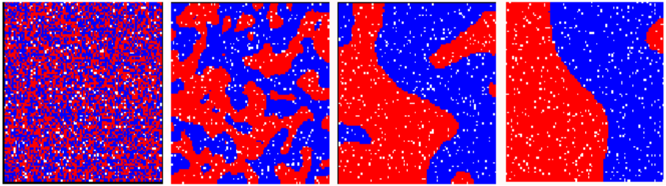

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A Statistical-Physics Problem?

- Large interacting-particle systems
- Micro-dynamics with macroscopic effects
- Pattern formation and criticality

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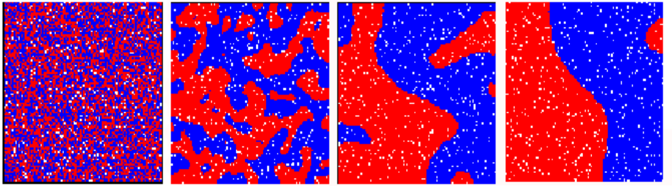

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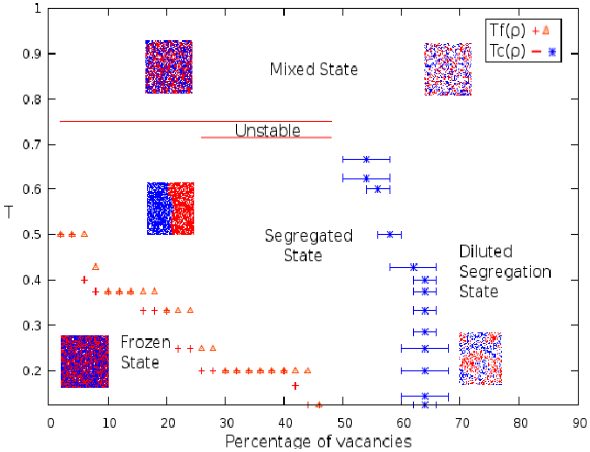
Can one establish fruitful analogies?

Phase Diagram of the Gauvin-Vannimenus-Nadal Model



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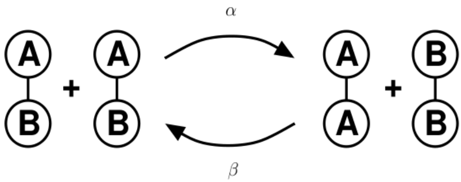
A Theoretical Physicist's Approach: Start With the Bare Bones



- Network "abstracted" away: simply $\frac{N}{2}$ pairs
- Satisfaction of agent at site i is u if her unique neighbour is of the same type, v if she is different ($u > v$)
- Transfer matrix: $T_{ij}(\sigma) = \frac{1}{N^2}(1 - s_i)s_j^{(i,j)}$

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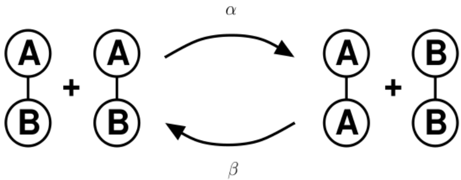

- Time evolution of the interface density (fraction of inhomogeneous pairs), in the large-N limit:

$$\frac{dx}{dt} = \beta(1 - x(t))^2 - \alpha x(t)^2, \quad x(0) = \frac{1}{2},$$

with $\beta = 2(1 - u)v$, $\alpha = 2(1 - v)u$

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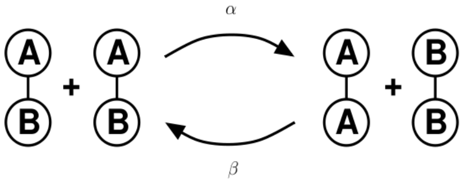

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- Interface density at time t:

$$x(t) = \frac{\sqrt{\alpha\beta} + \alpha \tanh(t\sqrt{\alpha\beta})}{2\sqrt{\alpha\beta} + (\alpha + \beta)\tanh(t\sqrt{\alpha\beta})}$$

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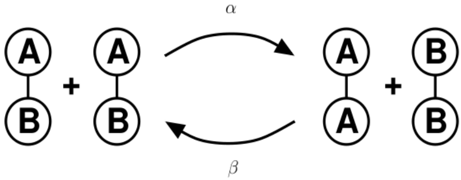

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Emergence of Segregation: the Rogers-MacKane Model

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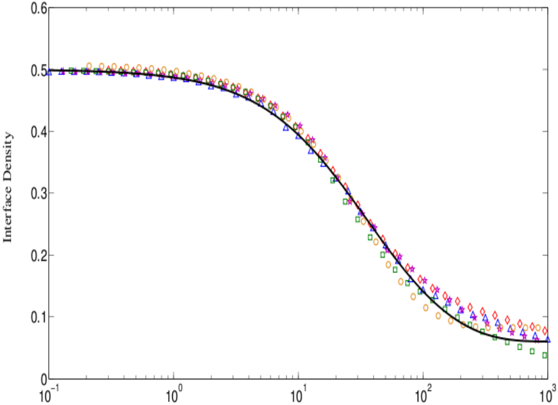

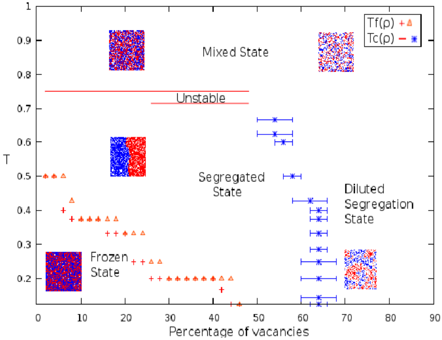

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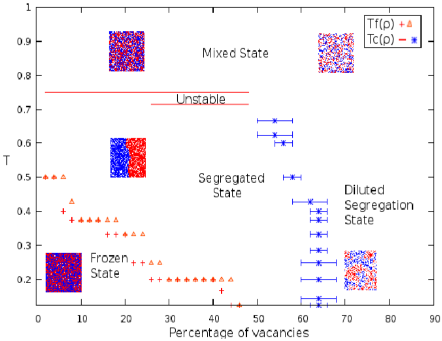

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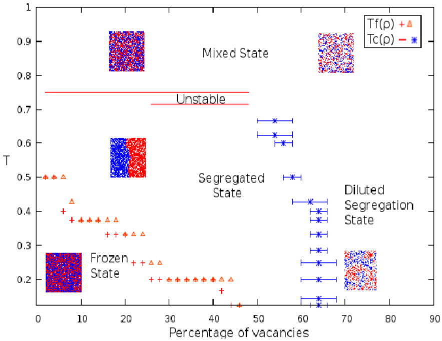
Phase-space dynamics

- What are the relevant parameters to tune?
- What parameters can actually be tuned?
- ⇝ urban renovation policies? (sustainibility of enforced mixity)



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Perturbating the social grid

- Introduce *C*-type agents which can flip from type *A* to type *B*
- Faster (de)segregation? Phase diagram?
- ⇝ rôle of social mobility? (mixing dynamics & time scales)

THANK YOU FOR YOUR ATTENTION!

Analogy with Spin-Models

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$$E = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j - K(T) \sum_{\langle i,j \rangle} \sigma_i^2 \sigma_j^2$$

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- E interpreted as the sum over all sites of $N_d - T(N_d + N_s)$, provided that substituting

$$T = \text{ temperature} \leftrightarrow T = \text{ tolerance}$$

makes sense...

A General Framework: Schelling-Class Models

A common mathematical description for all variants of Schelling's model?

- agents of two types, A and B;
- N sites joined by *some* edges;
- $\sigma_i = 1$ (resp. -1) if site i occupied by an A (resp. B)-type agent, 0 if site i is vacant (NB the convenience of the product $\sigma_i \sigma_j$);
- fraction of vacant sites: $\rho = \frac{1}{N} \sum_i (1 - |\sigma_i|)$
- satisfaction s_i of agent at site i depends only on:

$$x_i = \frac{\sum_{j \in \partial i} (|\sigma_i \sigma_j| - \sigma_i \sigma_j)}{2 \sum_{j \in \partial i} |\sigma_i \sigma_j|}$$

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- write $\sigma = (\sigma_1, \ldots, \sigma_N)$ for the state vector, and $\mathbf{s} = (s_1, \ldots, s_N)$ for the satisfaction vector ; and write $\sigma^{(i,j)}$, $\mathbf{s}^{(i,j)}$ for the new vectors resulting from interchanging $i \leftrightarrow j$.

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- measure segregation via:

$$x = \frac{\text{number of edges between agents of opposite types}}{\text{number of edges between agents of any type}}.$$