Workshop on

LIMIT THEOREMS and APPLICATIONS

ABSTRACTS

- **Bernard Bercu** (Université de Bordeaux 1, France)

  Title: Asymptotic results for empirical measures of weighted sums of independent random variables

  Abstract: We investigate the asymptotic behavior of a particular family of weighted sums of independent standardized random variables with uniformly bounded third moments. We prove that the empirical CDF of the resulting partial sums converges almost surely to the normal CDF. It allows us to deduce the almost sure uniform convergence of empirical distribution of the empirical periodogram as well as the almost sure uniform convergence of spectral distribution of symmetric circulant random matrices. In the special case of trigonometric weights, we also establish a central limit theorem and a large deviation principle. It is a joint work with W. Bryc.

- **Hermine Biermé** (Université René Descartes Paris 5, France)

  Title: Self-similar random fields and rescaled random balls models

  Abstract: We study generalized random fields which arise as rescaling limits of spatial configurations of uniformly scattered random balls as the mean radius of the balls tends to 0 or infinity. Assuming that the radius distribution has a power law behavior, we prove that the centered and renormalized random balls field admits a limit with strong spatial dependence. In particular, our approach provides a unified framework to obtain all self-similar, stationary and isotropic Gaussian fields. In addition to investigating stationarity and self-similarity properties, we give L^2-representations of the limiting generalized random fields viewed as continuous random linear functionals. Joint work with A. Estrade (Paris 5) and Ingemar Kaj (Uppsala University)

- **Laure Coutin** (Université René Descartes Paris 5, France)

  Title: Processus multifractaux comme intégrales de Wiener fractionnaires.

  Abstract: Nous définissons une classe de processus multifractaux en intégrant une cascades multiplicative stationnaire contre un mouvement brownien fractionnaire. Les propriétés de scaling sont étudiées ainsi que le formalisme multifractal associé.

  This talk is based on a joint work with P. Abry , P. Chainais et V. Pipiras.

- **Paul Doukhan** (ENSAE, France)

  Title: Weak dependence, Models and Some Applications

  Abstract: The talk will present the basic features of weak dependence defined in Doukhan & Louhichi (1999). A book on the subject coauthored with Dedecker, Lang, León, Louhichi and Prieur appeared as the LNS (Springer) in 2007.

  Our main aim is to propose several new models of time series and random fields for which this theory applies beyond mixing. Nonlinear, nonMarkov stationary models will be proposed.

  We shall derive from a Lindeberg type theorem some applications to resampling and functional estimation.

  References:

  [1] Dedecker, J. and Doukhan, P. (2003) *A new covariance inequality and applications*, Stoch. Proc. Appl. 106-1, 63-80.

  [2] Dedecker, J. and Doukhan, P., Prieur, C., Louhichi, S., Dedecker, J., León, J. R. (2006) *Weak dependence: models, theory and applications* (350 pages) Lecture Notes in Statistics, Springer-Verlag.

  [3] Doukhan, P. (1994) *Mixing: Properties and Examples.* LNS 85. Springer Verlag.

  [4] Doukhan, P., Latour, A.. Oraichi, P. (2006) *Simple integer-valued bilinear time series model.* Advances in Applied Probabilty 38, 559-578.

  [5] Doukhan, P. and Louhichi, S. (1999) *A new weak dependence condition and applications to moment inequalities.* Stoch. Proc. Appl. 84, 313-342.

  [6] Doukhan, P., Teyssière, G. and Winant, P. (2006) *Vector valued ARCH(∞) processes,* in Lecture Note in Statistics 187, Dependence in Probability and statistics (Bertail, P., Doukhan, P. and Soulier, P. editors).

  [7] Doukhan, P., Truquet, L. (2006) *Weakly dependent random fields with infinite memory.*

  [8] Doukhan, P., Wintenberger, O. (2006) *Invariance principle for new weakly dependent stationary models under sharp moment assumptions,* Probab. Math. Statist.

  [9] Doukhan, P. and Wintenberger, O. (2006) *Weakly dependent chains with infinite memory.*

- **Liudas Giraitis**

  Title: Approximations and limit theory for quadratic forms of linear processes

  Abstract: The paper develops a limit theory for the quadratic form $Q_{n,X}$ in linear random variables $X_1, \ldots, X_n$ which can be used to derive the asymptotic normality of various semi-parametric, kernel, window and other estimators converging at a rate which is not necessarily $n^{1/2}$. The theory covers practically all forms of linear serial dependence including long, short and negative memory, and provides conditions which can be readily verified thus eliminating the need to develop technical arguments for special cases. This is accomplished by establishing a general CLT for $Q_{n,X}$ with normalization $(var[Q_{n,X}])^{1/2}$ assuming only $2+\delta$ finite moments. Previous results for forms in dependent variables allowed only normalization with $n^{1/2}$ and required at least four finite moments. Our technique uses approximations of $Q_{n,X}$ by a form $Q_{n,Z}$ in i.i.d. errors $Z_1, \ldots, Z_n$. We develop sharp bounds for these approximations which in some cases are faster by the factor $n^{1/2}$ compared to the existing results.

- **Jean Jacod** (Université de Paris 6, France)

  Title: Estimating the Degree of Activity of jumps in High Frequency Data

  Abstract: Suppose that a continuous-time process $X = (X_t)_{t\geq 0}$ is observed at finitely many times, regularly spaced, on the fixed time interval $[0, T]$. We suppose that this process is an Itô semimartingale, with a non-vanishing diffusion coefficient, and with jumps. The aim is to estimate the so-called "Blumenthal-Getoor" index of the (partially observed) path on $[0, T]$, which is the (random) infimum of all reals r such that the sum $\sum_{s\leq T} |\Delta X_s|^r t$ is finite (ΔX_s denotes the jump size at time s). When X is a Lévy process, this infimum is non-random, and also independent of T, and has been introduced by Blumenthal and Getoor. Under appropriate assumptions, unfortunately rather restrictive, we provide an estimator, which is consistent when the step size between observations goes to 0, and satisfies in addition a Central Limit Theorem.

  This is a joint paper with Yacine Aït-Sahalia

- **Renaud Marty** (Université de Nancy 1, France)

  Title: Invariance principle, multifractional Gaussian processes and long-range dependence.

  Abstract: We establish an invariance principle where the limit process is a multifractional Gaussian process with a multifractional function which takes its values in (1/2, 1). Some properties, such as regularity and local self-similarity, of this process are studied. Moreover the limit process is compared to the multifractional Brownian motion.

- **Domenico Marinucci** (University of Rome 'Tor Vergata', Italy)

  Title: Group Representations and High-Resolution Central Limit Theorems for Subordinated Spherical Random Fields

  Abstract: We study the weak convergence (in the high-frequency limit) of the frequency components associated with Gaussian-subordinated, spherical and isotropic random fields. In particular, we provide conditions for asymptotic Gaussianity and we establish a new connection with random walks on the the dual of SO(3), which mirrors analogous results previously established for fields defined on Abelian groups. Our work is motivated by applications to cosmological data analysis, and specifically by the probabilistic modelling and the statistical analysis of the Cosmic Microwave Background radiation, which is currently at the frontier of physical research. To obtain our main results, we prove several fine estimates involving convolutions of the so-called Clebsch-Gordan coefficients (which are elements of unitary matrices connecting reducible representations of SO(3)); this allows to intepret most of our asymptotic conditions in terms of coupling of angular momenta in a quantum mechanical system. Part of the proofs are based on recently established criteria for the weak convergence of multiple Wiener-Itô integrals. This is a joint paper by Domenico Marinucci (Rome "Tor Vergata") and Giovanni Peccati (Paris VI).

- **Florence Merlevède** (Université de Paris 6, France)

  Title: On Moderate Deviations for stationary sequences of bounded random variables

  Abstract: In this talk, I shall present some recent results about the moderate deviations principle (MDP) for stationary sequences of bounded random variables. In a recent work, in collaboration with J. Dedecker, M. Peligrad and S. Utev, by using suitable martingale approximations, we have obtained the MDP under martingale-type conditions. The conditions required allow applications to a large class of examples such as functions of ϕ-mixing sequences, contracting Markov chains, expanding maps of the interval, and symmetric random walks on the circle.
  I shall also present some works in progress with M. Peligrad about the MDP for linear processes with dependent and bounded innovations, and which can exhibited long range dependence.

- **Ivan Nourdin** (Université de Paris 6, France)

  Title: Weighted power variations of fractional and iterated Brownian motions

  Abstract: Let Z be a selfsimilar (with index $0 < H < 1$) process having stationary increments. Assume that $f : R \to R$ is regular enough, and that $\kappa \geq 2$ is an integer. In this talk, we are interested in the so-called *weighted power variations* of Z defined by

  $$V_n^{(\kappa)}(f) = \sum_{k=1}^{2^n} f(Z_{(k-1)2^{-n}})\big(Z_{k2^{-n}} - Z_{(k-1)2^{-n}}\big)^{\kappa}, \quad n \in N.$$

  We will study their asymptotic behaviors in the particular cases where Z is a fractional Brownian motion or an iterated Brownian motion. Moreover, we will relate our results (obtained with Nualart and Tudor on one hand, and Peccati on the other hand) to a recent conjecture by Burdzy and Swanson about the possibility to write an Itô's type formula for quartic Gaussian processes.

- **Giovanni Peccati**(Université de Paris 6, France)

  Title: Stein's method and weak convergence on Wiener space.

  Abstract: We will show that one can combine Malliavin calculus with Stein's method, in order to derive explicit bounds in the Gaussian and Gamma approximations of arbitrary regular functionals of a given Gaussian field (here, the notion of regularity is in the sense of Malliavin derivability). When applied to random variables belonging to a fixed Wiener chaos, our approach generalizes, refines and unifies the central and non-central limit theorems for multiple Wiener-Itô integrals recently proved (in several papers, from 2005 to 2007) by Nourdin, Nualart, Ortiz-Latorre, Peccati and Tudor. We shall discuss some connections with the classic method of moments and cumulants. As an application, we deduce explicit Berry-Esseen bounds in the Breuer-Major Central Limit Theorem for subordinated functionals of a fractional Brownian motion.

  This talk is based on joint works with I. Nourdin (Paris VI).

- **Clémentine Prieur** (Université de Toulouse 1, France)

  Title: Central limit theorem for sampled sums of dependent random variables.

  Abstract: We prove a central limit theorem for linear triangular arrays under weak dependence conditions [1, 3, 4]. Our result is then applied to the study of dependent random variables sampled by a Z-valued transient random walk. This extends the results obtained by Guillotin-Plantard & Schneider [2]. An application to parametric estimation by random sampling is also provided.

  References:

  [1] Dedecker J., Doukhan P., Lang G., Leon J.R., Louhichi S. and Prieur C. (2007). Weak dependence: With Examples and Applications. Lect. notes in Stat. 190. Springer, XIV.

  [2] N. Guillotin-Plantard and D. Schneider (2003). Limit theorems for sampled dynamical systems. *Stochastic and Dynamics* 3, 4, p. 477-497.

  [3] M. Peligrad and S. Utev (1997). Central limit theorem for linear processes. *Ann. Probab.* 25, 1, p. 443-456.

  [4] S. A. Utev (1991). Sums of random variables with φ-mixing. *Siberian Advances in Mathematics* 1, 3, p. 124-155.

- **Igor Prünster** (University of Turin, Italy)

  Title: Asymptotics for posterior hazards

  Abstract: A popular Bayesian nonparametric approach to survival analysis consists in modeling hazard rates as kernel mixtures driven by a completely random measure. A comprehensive analysis of the asymptotic behaviour of such models is provided. Consistency of the posterior distribution is investigated and central limit theorems for both linear and quadratic functionals of the posterior hazard rate are derived. The general results are then specialized to various specific kernels and mixing measures, thus yielding consistency under minimal conditions and neat central limit theorems for the distribution of functionals.

  Joint work with P. De Blasi and G. Peccati.

- **Emmanuel Rio** (Université de Versailles, France)

  Title: Rates of convergence for minimal distances in the central limit theorem under projective criteria

  Abstract: In this paper, we give estimates of ideal or minimal distances between the distribution of the normalized partial sum and the limiting Gaussian distribution for stationary martingale differ- ence sequences or stationary sequences satisfying projective criteria. Applications to functions of linear processes and to functions of expanding maps of the interval are given.

  This is a joint paper with J. Dedecker (Paris 6) and F. Merlevède (Paris 6).

- **Philippe Soulier** (Université de Paris 10, France)

  Title: On the existence of some ARCH(∞) processes

  Abstract: A new sufficient condition for the existence of a stationary causal solution of an ARCH(∞) equation is provided. This condition allows to consider coefficients with power-law decay, so that it can be applied to the so-called FIGARCH processes, whose existence is thus proved.

- **Gennady Samorodnitsky** (Cornell University, U.S.A)

  Title: Inverse problems for regular variation, linear filters, functional equations and a cancellation property for σ-finite measures

  Abstract: We study a group of related problems: the extent to which presence of regular variation of the tail of certain σ-finite measures at the output of a linear filter determines the corresponding regular variation of a measure at the input to the filter. This turns out to be related to presence of a particular cancellation property in σ-finite measures, which, in turn, is related to uniqueness of solutions of certain functional equations. The techniques we develop are applied to weighted sums of iid random variables, to products of independent random variables, and to stochastic integrals with respect to Lévy motions. Joint work with Martin Jacobsen, Thomas Mikosch and Jan Rosiński.

- **Tommi Sottinen** (Reykjavik University, Iceland)

  Title: Local continuity

  Abstract: We propose the concept of Local Continuity that is somewhat related to directional continuity.

  DEFINITION: Let X and Y be, say, metric spaces. A function f from X to Y is locally continuous at point x in X if one can find an open set U(x) such that (i) x belongs to the closure of U(x), (ii) if x(n) converges to x in U(x) then f(x(n)) converges to f(x) in Y.

  The set U(x), the local continuity set of f at x, that tells the direction of continuity. If U(x) can be chosen to contain x then f is continuous at x.

  The concept was conceived during our study [Bender, C., Sottinen, T., and Valkeila, E. (2007): Pricing by hedging and no-arbitrage beyond semimartingales (under revision for Finance and Stochastics)] where we considered non-semimartingale pricing models that have non-trivial quadratic variation and a certain "small-ball property". It turned out that in these models one cannot do arbitrage with strategies that are continuous in terms of the spot and some other economic factors such as the running minimum and maximum of the stock. Unfortunately, this result does not extend to even simple strategies, when stopping times are involved. The reason is obvious: Stopping times are typically not continuous. However, local continuity turns out to be just what we need to prove our theorems, and the author is not aware of any reasonable stopping times that are not locally continuous.

  The talk is based on an ongoing joint work with C. Bender (Technical University of Braunschweig), D. Gasbarra (University of Helsinki), and E. Valkeila (Helsinki University of Technology).

- **Donatas Surgailis** (Vilnius Institute of Mathematics and Informatics, Lithuania)

  Title: Lévy-stable behavior of squares

  Abstract: We introduce a new modification of Sentana's (1995) Quadratic ARCH (QARCH), the Linear ARCH (LARCH) (Giraitis et al., 2000, 2004) and the bilinear models (Giraitis and Surgailis, 2002), which can combine the following properties:

  (a.1) conditional heteroskedasticity

  (a.2) long memory

  (a.3) the leverage effect

  (a.4) strict positivity of volatility

  (a.5) Lévy-stable limit behavior of partial sums of squares

  Sentana's QARCH model is known for properties (a.1), (a.3), (a.4), and the LARCH model for (a.1), (a.2), (a.3). Property (a.5) is new.

  References:

  [1] Giraitis, L., Robinson, P.M., Surgailis, D. (2000) A model for long memory conditional heteroscedasticity, *Ann. Appl. Probab.* **10**, 1002–1024.

  [2] Giraitis, L., Surgailis, D. (2002) ARCH-type bilinear models with double long memory, *Stoch. Process. Appl.* **100**, 275–300.

  [3] Giraitis, L., Leipus, R., Robinson, P.M., Surgailis, D. (2004) LARCH, leverage and long memory, *J. Financial Econometrics* **2**, 177–210.

  [4] Sentana, E. (1995) Quadratic ARCH models, *Rev. Econ. Stud.* **3**, 77–102.

- **Murad Taqqu** (Boston University, U.S.A)

  Title: Central limit theorem for functions of dependent stable variables

  Abstract: The talk is divided in three parts:

  1. Bounds for the covariance of a large class of functions of infinite variance stable random variables, including unbounded functions such as the power function and the logarithm. These bounds involve measures of dependence between the stable variables, some of which are new.

  2. Use the bounds to establish a Central Limit Theorem for unbounded functions of stable moving average time series.

  3. Use the Central Limit Theorem to establish the asymptotic normality of wavelet-based estimators of the self-similarity parameter in fractional stable motions. This is joint work with Vladas Pipiras and Patrice Abry.

- **Soledad Torres** (Universidad de Valparaiso, Chile)

  Title: Numerical scheme for RBSDE.

  Abstract: We propose a method for numerical approximation of Reflected Backward Stochastic Differential Equations. Is based in the approximation for the Brownian motion by a simple random walk. We prove a weak convergence.

  The talk is based on joint work with Miguel Martinez and Jaime San Martin

- **Esko Valkeila** (Helsinky University of Technology, Finland)

  Title: An estimator for the quadratic variation of mixed Brownian fractional Brownian motion

  Abstract: In the work [1] we show that it is possible to extend the classical Black & Scholes hedging for a class of models, where the quadratic variation is identical to the Black & Scholes model. Dzhaparidze and Spreij show in [2], that the periodogram constructed from the process estimates the quadratic variation in the semimartingale setting. We show that the periodogram estimates the quadratic variation for the mixed Brownian fractional Brownian motion, too.

  The talk is based on joint work with Ehsan Azmoodeh.

  References:

  [1] C. Bender, T. Sottinen, and E. Valkeila (2006), No-arbitrage pricing beyond semimartingales. WIAS Preprint 1110.

  [2] K. Dzhaparidze, and P. Spreij (1994), Spectral characterization of the optional quadratic variation process. Stochastic Process. Appl., 54, 165-174.

- **Harry Van Zanten** (Vrije Universiteit, Amsterdam, The Netherlands)

  Title: Contraction rates of posteriors based on Gaussian process priors

  Abstract: Contraction rates of posterior distributions on nonparametric models are derived for Gaussian process priors. We show that the convergence rate depends on the small ball probabilities of the Gaussian process and on the position of the true parameter relative to the reproducing kernel Hilbert space of the Gaussian process. Explicit examples are given for various statistical settings, including density estimation, nonparametric regression, and classification. We also discuss how rescaling of the prior process affects the contraction rates and how random rescaling can yield rate-adaptive procedures. This is based on joint work with Aad van der Vaart.