An estimator for the quadratic variation of mixed Brownian Fractional Brownian motion

Esko Valkeila

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Joint work with Ehsan Azmoodeh (TKK).

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References

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Azmoodeh, E., and Valkeila, E. (2008).
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Hedging robustness of Black & Scholes model

Basic observation

Assume that X is a continuous process defined on some probability space $(\Omega, \mathcal{F}, \mathrm{P})$ with finite quadratic variation $[X, X]$:

$$[X, X]_t = \lim_{|\pi| \to 0} \sum_{t_k \in \pi} \left(X_{t_k} - X_{t_{k-1}} \right)^2 ,$$

where $\pi = \{t_k : 0 = t_0 < t_1 < \cdots < t_n = t\}$ is a sequence of partitions of the interval $[0, t]$, $|\pi| = \max\{t_k - t_{k-1} : t_k \in \pi\}$, and the limit is either in probability or P – almost surely. The process $[X, X]$ is continuous and increasing.

The following famous observation belongs to Hans Föllmer:

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Hedging robustness of Black & Scholes model
Basic observation, II

If $F \in C_{1,2}(\mathbb{R}_+, \mathbb{R})$, the we have the Itô formula

$$F(t, X_t) = F(0, X_0) + \int_0^t F_t(s, X_s)ds + \int_0^t F_x(s, X_s)dX_s + \frac{1}{2}\int_0^t F_{xx}(s, X_s)d[X, X]_s. \quad (1)$$

If the quadratic variation exists as almost sure limit, then the stochastic integral $\int_0^t F_x(s, X_s)dX_s$ in (1) can be interpreted as pathwise Riemann-Stieltjes integral.
Shoenmakers and Kloeden used this observation to show that hedging of European type of options is the same in all pricing models, where the quadratic variation of the stock price has the same form as a functional of the stock price as in the classical Black & Scholes model.

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In [BSV] we consider classes of pricing models, where the continuous stock price S has the following quadratic variation:

$$d[S,S]_t = \sigma^2(S_t)dt, \qquad (2)$$

where $\sigma : \mathbb{R} \to \mathbb{R}$ is a continuously differentiable function of linear growth.

Typical examples of this kind of stock price models are the classical Black & Scholes model, where the stock price $\tilde{S}$ is given by

$$\tilde{S}_t = s_0 e^{\sigma W_t - \frac{1}{2}\sigma^2 t},$$

and so-called mixed Brownian - fractional Brownian motion pricing models, where the stock price S is given by

$$S_t = s_0 e^{X_t - \frac{1}{2}\sigma^2 t},$$

here $X_t = \sigma W_t + \eta B_t^H$, B^H is a fractional Brownian motion with Hurst index $H > \frac{1}{2}$, independent of W, and η is a constant.

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It is known that we have

$$d[\tilde{S},\tilde{S}]_t = \sigma^2 \tilde{S}_t^2 dt \text{ and } d[S,S]_t = \sigma^2 S_t^2 dt,$$

and for both price processes the bracket has the functional form of (2), and in the terminology of [BSV] both price processes belong to the same *model class*.
We have shown that within a fixed model class the hedging has the same functional form for a big class of options, which includes European options, and path dependent options like lookback options and Asian options.
We want to give an estimator for the quadratic variation of the process X, which can be a semimartingale or a non-semimartingale, depending weather $H > \frac{3}{4}$ or $H \in (\frac{1}{2}, \frac{3}{4}]$.

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Spectral characterization of the bracket

The process X is a semimartingale

In [DS] Dzhaparidze and Spreij gave another estimator for the bracket $[X, X]$. Let $\mathbb{F}^X$ be a filtration of X.

Let τ be a stopping time and $\lambda \in R$. Define the *periodogram* $I_\tau(X; \lambda)$ of X by

$$I_\tau(X; \lambda) := |\int_0^\tau e^{i\lambda s} dX_s|^2;$$

here $i = \sqrt{-1}$.

Let $L > 0$ and let ξ be a symmetric random variable with a density g_ξ and real characteristic function φ_ξ, and $\xi \perp\!\!\!\perp \mathbb{F}^X$. Define the randomized periodogram by

$$\mathsf{E}_\xi\, I_\tau\,(X; L\xi) = \int_{\mathbb{R}} I_\tau\,(X; Ly)\, g_\xi(y) dy. \qquad (3)$$

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If the stopping time τ is finite and the characteristic function φ_ξ has bounded variation , then Dzhaparidze and Spreij have shown that we have the following characterization of the bracket as $L \to \infty$

$$\mathsf{E}_\xi \, I_\tau (X; L\xi) \xrightarrow{\mathsf{P}} [X, X]_\tau . \qquad (4)$$

If the process X is a continuous Gaussian martingale and the stopping time τ is a constant T, $\tau = T$, then Dzhaparidze and Spreij can drop the assumption that the characteristic function φ_ξ has a bounded variation.

We prove that if the process X is a mixed Brownian - fractional Brownian motion, $T > 0$ is a fixed time and the symmetric random variable ξ has *finite second moment*, then the randomized periodoram of X also satisfies (4).

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Estimator for the bracket

Preparations

Assume that X is a mixed Brownian - fractional Brownian motion and $\lambda \in \mathbb{R}$. Define the process Y by

$$Y_t = \int_0^t e^{i\lambda s} dX_s \qquad \text{for } t \leq T.$$

Then

- $[Y, \overline{Y}]_t = t$, where $\overline{Y}_t$ is the complex conjugate of Y_t.
- Itô formula (1) gives

$$I_T(X; \lambda) = |Y_T|^2 = T + 2\text{Re} \int_0^T \int_0^t e^{i\lambda(t-s)} dX_s dX_t.$$

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Estimator for the bracket

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Define the randomized periodogram by

$$\mathrm{E}_\xi \, I_T(X; L\xi) = T + 2\mathrm{Re} \int_{\mathbb{R}} \left[\int_0^T \int_0^t e^{iLy(t-s)} dX_s dX_t \right] g_\xi(y) dy,$$

where ξ is a symmetric random variable independent of W, B^H. As a first step of the proof we must prove a Fubini theorem which allows us to compute

$$\int_{\mathbb{R}} \left[\int_0^T \int_0^t e^{iLy(t-s)} dX_s dX_t \right] g_\xi(y) dy = \int_0^T \int_0^t \varphi_\xi(L(t-s)) dX_s dX_t,$$

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Preparations, III

Note that we have the following limit

$$\lim_{L\to\infty} \varphi_\xi(L(t-s) = \delta_{\{0\}}(t-s).$$

After we have the Fubini theorem we use this limit to prove that

$$\mathrm{P} - \lim_{L\to\infty} \int_0^T \int_0^t \varphi_\xi(L(t-s))dX_s dX_t = 0. \tag{5}$$

To prove the Fubini theorem and also the limit (5) we decompose X as $X = W + B^H$.

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Fubini theorem

Recall first the p- variation of X: the process X has finite p variation, $p > 1$ if

$$\sup_{t_k \in \pi} \sum_{t_k} |X_{t_k} - X_{t_{k-1}}|^p < \infty \quad \mathrm{P} \text{ -a.s.}$$

Fubini Theorem Assume that Y^y is a parametric family of processes having finite p- variation, dominated independently of the parameter y, and X is a processes with finite q variation with $\frac{1}{p} + \frac{1}{q} > 1$,

$$\int_{\mathbb{R}} \mathrm{E}\,|Y_t^y| g(y) dy < \infty.$$

Then

$$\int_{\mathbb{R}} \int_0^T Y_s^y dX_s g(y) dy = \int_0^T \int_{\mathbb{R}} Y_s^y g(y) dy dX_s.$$

Some comments on the proof of the Fubini theorem.

- The above theorem is a small improvement of corresponding theorem by Krvavych and Mishura (2001), and proved similarly.
- The proof is based on Young inequality, which allows to discretize both sides of the above inequality. For the finite discrete time sums the equality is true, and then one can pass to the limit.

When we integrate the iterated integrals with respect to the parameter y, we have four integrals: In the case of $dW_s dW_t$ we use standard stochastic Fubini theorem, and interchange the order of the first integral and the integral over R; in the other three cases we can use the above theorem to justify the changing of the order of the integrals.

Fubini theorem
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Fubini theorem
On the proof, II

To finish the proof of the identity

$$\int_{\mathbb{R}} \left[\int_0^T \int_0^t e^{iLy(t-s)} dX_s dX_t \right] g_\xi(y) dy = \int_0^T \int_0^t \varphi_\xi(L(t-s)) dX_s dX_t,$$

we use standard Fubini theorem for the four Wiener integrals, and combine the terms.
The last step is to show that

$$\int_0^T \int_0^t \varphi_\xi(L(t-s)) dX_s dX_t \to 0$$

as $L \to \infty$. We consider again four different integrals.

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The periodogram limit theorem
Last part of the proof

- To justify the limit for the integral $\int_0^T \int_0^t \varphi_\xi(L(t-s))dW_s dW_t$ one can use Itô isometry.
- To show that the integral $\int_0^T \int_0^t \varphi_\xi(L(t-s))dB_s^H dW_t \to 0$ one can use the independence of B^H and W. The same reasoning applies to the integral,, where we change the order of B^H and W.
- Finally, to show that the integral $\int_0^T \int_0^t \varphi_\xi(L(t-s))dB_s^H dB_t^H \to 0$, we proceed indirectly. We use the the connection of Riemann-Stieltjes integrals to divergence integrals, estimate the moments of divergence integrals as well as the correction term to verify this last limit.

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Discussion

- The talk is based on manuscript [AV], which means that we are still working with several details.
- Can one allow more general martingales M instead of W? Apparently, the problem will be in the verification of the Fubini theorem.
- Can one drop the independence of W and B^H? Probably yes, but the last part of the proof will be more analytical.

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