BATCH LEARNING ALGORITHM OF SOM WITH ATTRACTIVE AND REPULSIVE DATA

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Abstract - *In this paper, we propose a stable batch learning algorithm for SOM doing two learning methods, "attractive learning" and "repulsive learning". The attractive learning and the repulsive learning are caused by successful data and failed data, respectively. A combination of the attractive learning and repulsive learning emulates a trial and error of animals. In order to realize a stable learning, a conversion of the failed data is employed. The converted data are assumed as a successful data which is located at opposite of a reference vector like a mirror image. An effectiveness of the proposed algorithm is verified by some simulations.*

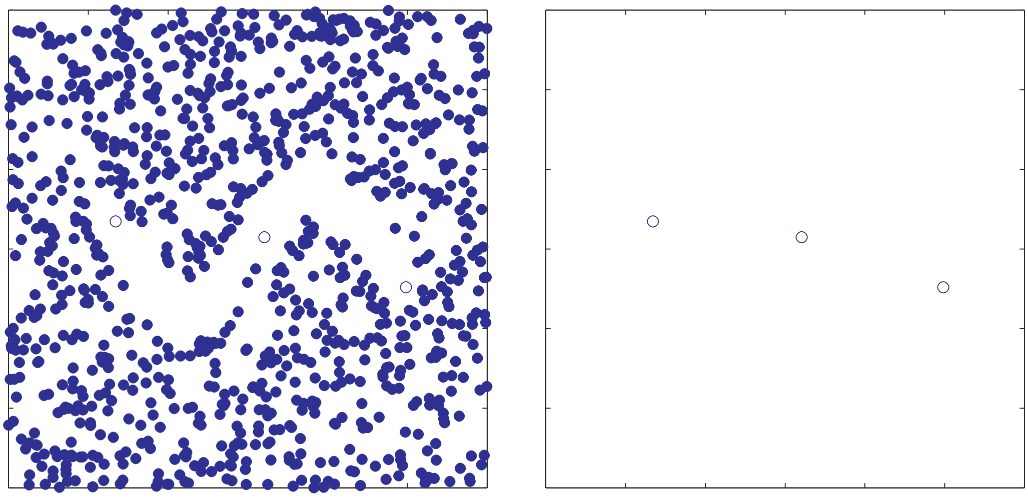
Key words - evaluation, repulsive learning, virtual vector, batch learning

1 Introduction

The aim of our study presented here is to establish an algorithm for generating a self-organizing map, in which the dataset observed from failed trials are respected as well as the one from successful trials. In other words, our purpose is to utilize the failed trials as stepping stones to generate a map of successful data.

To explain our motivation, let us consider the learning process of riding a bicycle. In the early learning stage, we make a lot of mistake; we fall on the ground many times, with making a lot of scratches. However, once getting the knack, suddenly it turns to an easy task. This fact suggests that the failed trials are as important as the successful ones in the learning task. If the failed data, i.e. the experiences of falling down from a bicycle, are not used for the learning, we have to learn from insufficient successful data only. Our goal is to realize such human's learning skill in the artificial controllers by using self-organizing method.

It is important to note that the failed data are usually plentiful while the successful ones are only a few. Therefore, it is important to utilize the failed experiences to grasp the limited chance of successful ones. The situation is abstracted as shown in Figure 1(a). In the figure, there are only three successful data, whereas there are enormous quantity failed data. At a glance, we may find out the place where the map should be drawn. If the failed data



(a) (b)

Figure 1: A abstracted example of failed trials (•) and successful trials (○). (a)A lot of failed trials and a few successful trials. (b)A lot of successful data.

are removed away, it is impossible to estimate the correct way any more (see Figure 1(b)), because the failed data tell the place where the map is prohibited.

Considering above point, we have introduced a repulsive learning in the self-organizing relationship (SOR) network[1]. In the SOR network, each data is evaluated in advance; the successful data are given positive evaluation values, whereas the failed data are corresponded to negative evaluation values. Furthermore, the data with positive evaluations act as the global attractors which draw the best matching unit (BMU) and its neighbors, while the data with negative evaluation act as local repulsers for them. We have applied the SOR network to several control problems such like trailer-truck back-up control[2], and the attempts have succeeded in most cases. However, such repulsive learning sometimes makes the map unstable, especially in the case that failed data is much more than the successful data. In addition, the initial state and the order of presenting data affect the result and stability.

In this paper, we propose a new algorithm of the repulsive learning, which improves the stability drastically, for SOM. The main idea of the proposed algorithm is to introduce the concept of the "*virtual antiparticle*" which is regarded as the virtual mirror image of the data with negative evaluations. More precisely, each data with negative evaluation is converted to the corresponding virtual antiparticle, which acts as a data with positive evaluation located at the symmetrical position across the BMU. In addition, we also employ the batch mode learning[3, 4, 5] to cope with the unstableness caused by dependency of the data order.

The stabilization method by using the virtual antiparticles is introduced first, and then some simulation results that show the effectiveness of the method are reported in the following sections.

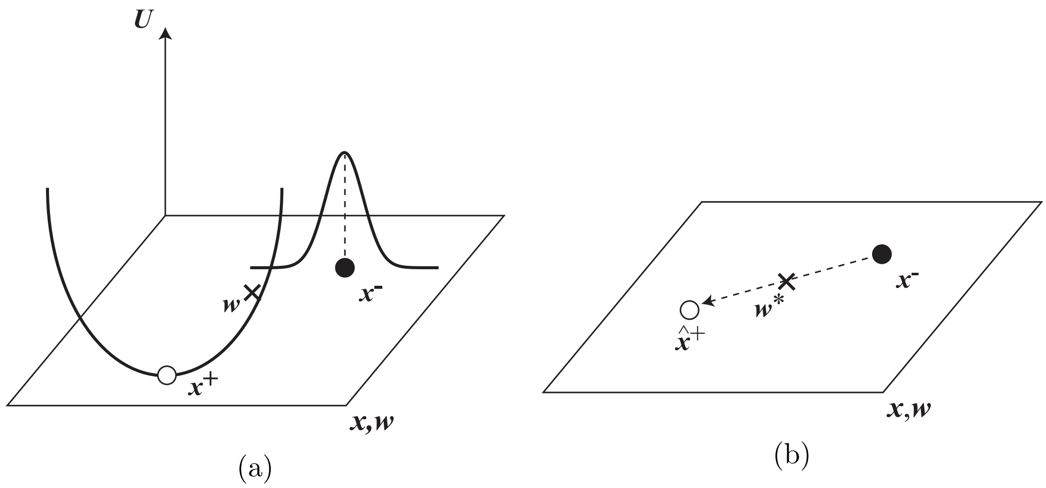


Figure 2: (a)Energy functions of input vectors with evaluations. ∘, • and × show the input vector with positive or negative evaluation and the reference vector, respectively. (b)The conversion of the input vector with the negative evaluation to *a virtual antiparticle*.

2 Stable Learning Using Input Vector with Evaluation

2.1 Evaluation of Input Vectors

In our method, each input vector is supposed to be evaluated in advance depending on the degree of success or failure. Thus, each input vector has its own evaluation value aside from the elements of the vector. The evaluations are subjectively or objectively determined. For example, the evaluations are represented by an impression when some persons see a picture image, by a decrease of control errors in control problems and so on. Here, let $\boldsymbol{x}_l$ be the l-th input vector, E_l be the evaluation value of $\boldsymbol{x}_l$. The evaluation value E_l is assumed to be continuous value between -1 and 1 in this paper. If E_l is positive, i.e. $\boldsymbol{x}_l$ is successful data, $\boldsymbol{x}_l$ attracts the reference vectors, whereas $\boldsymbol{x}_l$ repulses the reference vectors if E_l is negative, i.e. $\boldsymbol{x}_l$ is a failed data. Here, we call these two learning methods as "*attractive learning*" and "*repulsive learning*", respectively.

2.2 Attractive and Repulsive Learning Algorithm

First, let us consider the simplest case in which there are only one reference vector $\boldsymbol{w}$ and one input vector with positive evaluation $\boldsymbol{x}^+$ (see Figure 2(a)). In this case, the input vector $\boldsymbol{x}^+$ attracts $\boldsymbol{w}$ wherever it is located. In other words, $\boldsymbol{x}^+$ is regarded as a global attractor, and the energy function U^+ of which is represent by:

$$U^+ = \frac{E^+}{2} \parallel \boldsymbol{x}^+ - \boldsymbol{w} \parallel^2 . \tag{1}$$

Here, E^+ denotes the positive evaluation value of input vector $\boldsymbol{x}^+$. Note that the energy function is same as the ordinary SOM[6, 7, 8] excluding the evaluation. In this case, $\boldsymbol{w}$ converges to $\boldsymbol{x}^+$.

In the case that the input vector has a negative evaluation, the input vector informs that the probability of success around the failed data is low. Consequently, $\boldsymbol{w}$ should be repulsed from the input vector locally. Thus, the energy function U^- of the input vector with the negative evaluation $\boldsymbol{x}^-$ is assumed to be described by:

$$U^- = \alpha \mid E^- \mid \exp(-\frac{\parallel \boldsymbol{x}^- - \boldsymbol{w} \parallel^2}{2\sigma_r^2}). \tag{2}$$

Here, E^- denotes the negative evaluation value of the input vector $\boldsymbol{x}^-$, α and σ_r are the parameters that determine the strength and the region of the repulsion, respectively.
Now let us suppose that there are L input vectors $\mathcal{X} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_L\}$. $\mathcal{X}^+ = \{\boldsymbol{x}_1^+, \cdots, \boldsymbol{x}_{L^+}^+\}$ is dataset with positive evaluation and $\mathcal{X}^- = \{\boldsymbol{x}_1^-, \cdots, \boldsymbol{x}_{L^-}^-\}$ is the dataset with negative evaluation. Here, $L^+ + L^-$ is L. Furthermore, suppose that there are N reference vectors $\mathcal{W} = \{\boldsymbol{w}_1, \cdots, \boldsymbol{w}_N\}$. In this case, the energy function of $\boldsymbol{w}_i$ is represented as follows:

$$U(\boldsymbol{w}_i) = \sum_{j=1}^{L^+} \frac{m_j^+}{2} \parallel \boldsymbol{x}_j^+ - \boldsymbol{w}_i \parallel^2 + \sum_{k=1}^{L^-} \alpha \mid m_k^- \mid \exp(-\frac{\parallel \boldsymbol{x}_k^- - \boldsymbol{w}_i \parallel^2}{2\sigma_r^2}). \tag{3}$$

Here, m_j^+, m_k^- denote the neighboring weights which are represented by product of the evaluation values of $\boldsymbol{x}_j^+$ or $\boldsymbol{x}_k^-$ and the neighborhood function of the BMU, respectively. Taking the derivative of equation (3) with respect to the $\boldsymbol{w}_i$, and putting it equal to zero. A convergent point for the reference vector $\boldsymbol{w}_i$ is obtained as follows:

$$\boldsymbol{w}_i = \frac{\sum_{j=1}^{L^+} m_j^+ \boldsymbol{x}_j^+ - \sum_{k=1}^{L^-} \mid m_k^- \mid f_k(\boldsymbol{w}_i) \boldsymbol{x}_k^-}{\sum_{j=1}^{L^+} m_j^+ - \sum_{k=1}^{L^-} \mid m_k^- \mid f_k(\boldsymbol{w}_i)}, \tag{4}$$

$$f_k(\boldsymbol{w}_i) = \frac{\alpha}{\sigma_r^2} \exp(-\frac{\parallel \boldsymbol{w}_i - \boldsymbol{x}_k^- \parallel^2}{2\sigma_r^2}). \tag{5}$$

In the case that the energy of the repulsive learning is larger than that of the attractive learning, the reference vector can not be converged to the optimum solution, because the denominator of equation (4) becomes negative and a center of gravity of the input vectors is not obtained. Thus, the learning including the repulsive learning becomes unstable. Concretely, the reference vectors should be moved out from the input space.
To solve this problem, let us introduce the idea of “a virtual antiparticle”. By employing this idea, all input vectors with negative evaluations are converted to input vectors with positive evaluations, which are located at the symmetrical position across the BMU for the data.

$$\hat{\boldsymbol{x}}_k^+ = \boldsymbol{w}_k^* + (\boldsymbol{w}_k^* - \boldsymbol{x}_k^-) = 2\boldsymbol{w}_k^* - \boldsymbol{x}_k^-, \tag{6}$$

$$\hat{m}_k^+ = \mid m_k^- \mid f_k(\boldsymbol{w}_k^*). \tag{7}$$

Here, $\hat{\boldsymbol{x}}_k^+$, $\hat{m}_k^+$ are the vector and the neighboring weight of the virtual antiparticle of $\boldsymbol{x}_k^-$, and $\boldsymbol{w}_k^*$ is the reference vector of the BMU of $\boldsymbol{x}_k^-$. Thus, the convergent point represented in equation (4) is modified to:

$$\boldsymbol{w}_i = \frac{\sum_{j=1}^{L^+} m_j^+ \boldsymbol{x}_j^+ + \sum_{k=1}^{L^-} \hat{m}_k^+ \hat{\boldsymbol{x}}_k^+}{\sum_{j=1}^{L^+} m_j^+ + \sum_{k=1}^{L^-} \hat{m}_k^+}$$
$$= \frac{\sum_{j=1}^{L^+} m_j^+ \boldsymbol{x}_j^+ + \sum_{k=1}^{L^-} m_k^- f_k(\boldsymbol{w}_k^*)\boldsymbol{x}_k^- + 2\sum_{k=1}^{L^-} \mid m_k^- \mid f_k(\boldsymbol{w}_k^*)\boldsymbol{w}_k^*}{\sum_{j=1}^{L^+} m_j^+ + \sum_{k=1}^{L^-} \mid m_k^- \mid f_k(\boldsymbol{w}_k^*)}. \quad (8)$$

In the equation (8), the first, second and third term in the numerator denote a components for attraction, repulsion and compensation. The unstable point which arises from negative denominator can be avoided by the conversion of the input vectors with negative evaluations, thus the center of gravity of the input vectors can be obtained. Therefore, a stable learning including the attraction and the repulsion is realized.

A simple simulation of the proposed learning algorithm is achieved to confirm the convergence. One input vector with positive evaluation and ten input vectors with negative evaluations shown in Figure 3(a) are prepared. The initial value of a reference vector $\boldsymbol{w}$ is 0.5. The total energy function is shown in Figure 3(b), where, parameters of the repulsive learning are $\alpha = 1$ and $\sigma_r = 0.2$. The changes of the energy during 50 learning steps in which the reference vector is updated by equation (4) and (8) are shown in Figure 3(c). In the case that the update of the reference vector is based on equation(4) the learning is very unstable. On the other hand, in the case of learning with equation (8), the stable learning is achieved.

2.3 Batch Learning Algorithm

The proposed stable batch learning algorithm is summarized as follows.

Step 0 All reference vectors $\boldsymbol{w}_i$ $(i = 1, \cdots, N)$ are initialized by random numbers.

Step 1 A winner unit l^* for the input vector $\boldsymbol{x}_l$ is selected by the smallest *Euclidean Distance* by:

$$l^* = \arg\min_i \parallel \boldsymbol{x}_l - \boldsymbol{w}_i \parallel . \quad (9)$$

Step 2 A coefficient of neighboring effect $\phi_{i,l}$ and a coefficient of the repulsive learning ψ_i are calculated as follows:

$$\phi_{i,l} = \begin{cases} E_l h(i, l^*) & \text{if } E_l \geq 0 \\ E_l \frac{\alpha}{\sigma_r^2} \exp(-\frac{\parallel \boldsymbol{w}_{l^*} - \boldsymbol{x}_l \parallel^2}{2\sigma_r^2}) h(i, l^*) & \text{if } E_l < 0 \end{cases}, \quad (10)$$

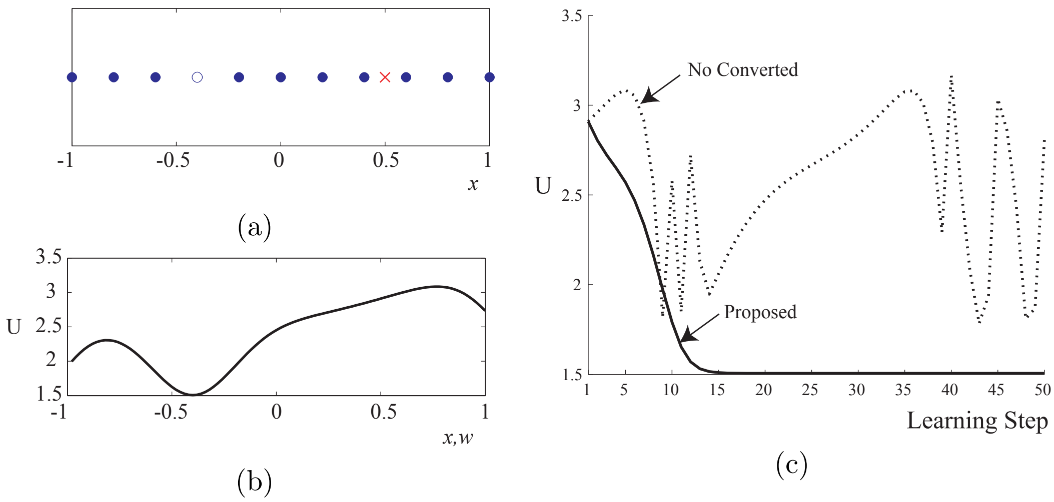


Figure 3: (a)One input vector with positive evaluation (∘) and ten input vectors with negative evaluations (•). (b)A energy function, where, parameters of the repulsive learning are $\alpha = 1$ and $\sigma_r = 0.2$. (c)A decrease of the energy function during 50 learning steps. The line and broken line show a case when the input data with negative evaluations are converted and a case when the input data are not converted, respectively.

$$\psi_i = 2 \sum_{l \in \mathcal{X}_i^-} \mid \phi_{i,l} \mid, \tag{11}$$

$$h(i, l^*) = \exp(-\frac{\| r_i - r_{l^*} \|^2}{2\sigma(t)^2}) \tag{12}$$

Here, r_i, r_{l^*} are coordinates of the i-th and the winner unit on the competitive layer, respectively. $\sigma(t)$ is a width of neighboring function at learning step t. $\mathcal{X}_i^-$ is defined as a set of the input vectors with negative evaluation in the voronoi region of the reference vector $\boldsymbol{w}_i$.

Step 3 After all input vectors are applied, the reference vectors are updated as follows:

$$\boldsymbol{w}_i(t+1) = (1-\varepsilon)\boldsymbol{w}_i(t) + \varepsilon \frac{\sum_{l=1}^{L} \phi_{i,l}\boldsymbol{x}_l + \sum_{i=1}^{N} \psi_i \boldsymbol{w}_i(t)}{\sum_{l=1}^{L} \mid \phi_{i,l} \mid}. \tag{13}$$

Here, ε is a learning rate.

Step 4 Steps 1 to 3 are repeated decreasing the width of neighboring function $\sigma(t)$.

3 Simulation Results

Figure 4(a) shows 1000 input vectors that include only two input vectors with positive evaluation. The input space is filled by a lot of input vectors with negative evaluations. The

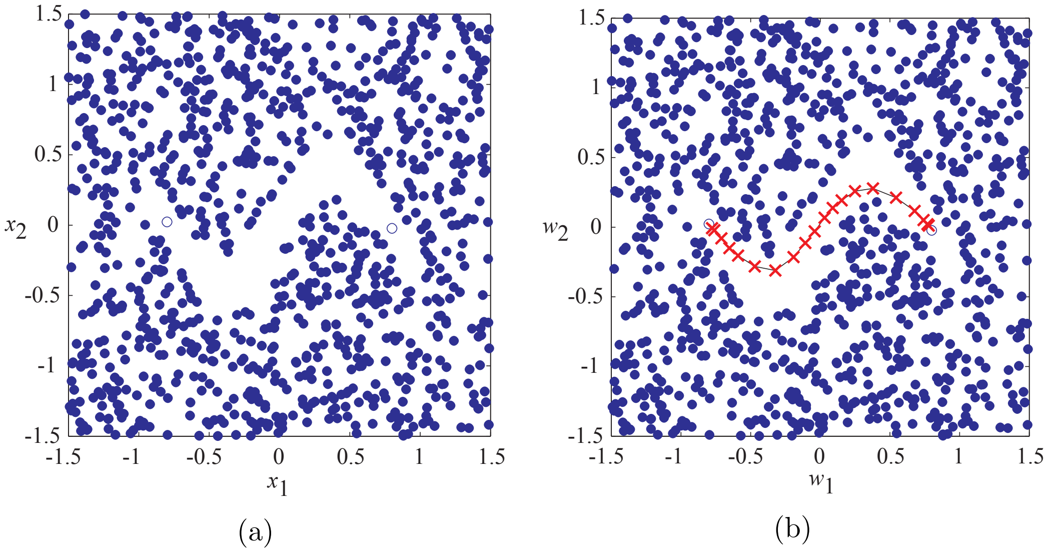


Figure 4: (a)Two input vectors with positive evaluations (∘) and many input vectors with negative evaluations (•). (b)A distribution of the reference vectors after a batch learning of 200 steps.

learning with the proposed algorithm is achieved using these input data for getting a topographic map avoiding the region of the negative evaluations. Figure 4(b) shows the topological map is obtained by 20 units (20×1). In the simulation, we take $\alpha = 0.01$, $\sigma_r = 0.1$, $\varepsilon = 0.1$, and decrease σ from 10 to 1 exponentially. The topological map is generated to avoid the input vector with negative evaluation and to preserve the topology of the input vector with positive evaluation. Although a ratio of the input vector with negative evaluation over the whole input vector is very high, the stable learning is achieved by the proposed batch learning algorithm.

4 Conclusions

In this paper, we proposed the batch learning algorithm for SOM with attractive and reclusive learning. In the proposed algorithm, each input vector is supposed to be evaluated in advance depending on the degree of success or failure. "*Attractive learning*" which attracts the reference vectors to input vectors with positive evaluations and "*repulsive learning*" which repulses the reference vector from input vectors with negative evaluations are introduced by emulating a trial and error of animals. A problem of stability of learning caused by repulsive learning is solved by introduction of the virtual antiparticle vector which is located at the symmetrical position. The proposed algorithm can generate the topological map avoiding the input vector with negative evaluation. An effectiveness of the proposed algorithm was verified by simulations. Furthermore, the improvement of the design of controllers can be expected by applying the proposed algorithm to the SOR network.

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References

[1] T. Yamakawa and K. Horio (1999), "Self-Organizing Relationship (SOR) Network", *IEICE Transaction on Fundamentals*, **vol. E82-A, no. 8** pp. 1674-1677.

[2] T. Koga, K. Horio and T. Yamakawa (2004), "Self-organizing Relationship (SOR) Network with Fuzzy Inference Based Evaluation and Its Application to Trailer-Truck Back-Up Control", *Proc. of International Conference on Neural Information Processing (ICONIP'04)*, pp. 368-374.

[3] T. Kohonen (1993), "Things you haven't heard about the self-organizing map", *Proc. of the International Conference on Neural Networks (ICNN'93)*, pp. 1147-1156.

[4] T. Kohonen (1995), *Self-Organizing Maps*, Berlin, Springer-Verlag.

[5] T. Kohonen (1998), "The self-organizing map", *Neurocomputing*, **vol. 21** pp. 1-6.

[6] E. Erwinm, K. Obermayer and K. Schulten (1992), "Self-organizing maps: Ordering, convergence properties and energy functions", *Biological Cybernetics*, **vol.67**, pp. 47-55.

[7] T. M. Heskes, and B. Kappen (1993), "Error potentials for self-organization", *Proc. of the International Conference on Neural Networks (ICNN'93)*, pp.1219-1233.

[8] Y. Cheng (1997), "Convergence and Ordering of Kohonen's Batch Map", *Neural Computation*, **vol.9** pp.1667-1676.