COMPUTING WITH ACTIVITIES, WHU-STRUCTURES AND A QUANTIZATION MODEL OF THE NEURAL NETS

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Abstract – *It is shown that the theories of Computing with Activities and WHU-structures can be combined with a quantum mechanical model of the neural nets. Besides a short introduction in recent experimental results regarding these structures of SOMs the mathematical derivation of the quantum mechanical model of the neural nets is discussed. Furthermore it is shown how a "neural based" quantum mechanic wave function* $\Psi(\vec{x},t)$ *can be declared.*

Key words – Computing with Activities, WHU-structures, Quantum mechanic model of the neural nets

1 Introduction

In 1999 the corresponding author presented a theory about the quantization of neural nets [3], which was based on first ideas of [2], [6] on how to use the mathematical fund of physics to describe the behaviour of SOMs. Even though this theory was widely accepted, two major questions were left unanswered:

- Where can the wave function Ψ be found in a SOM?
- How could "jumps" of the winner neurons be explained by the wave function Ψ?

The first question involves the demand of a kind of time behaviour of the SOM which was not in the focus of most investigations that have been done in the past, the second question is equal to the criticism that the classification results of a SOM, represented by a winner neuron, can jump from one position to another, even if the system to be evaluated is changing only slightly. Concerning these questions we enlarged the theory of the SOMs in two directions: Firstly we introduced the theory of the "Computing with Activities" [4] to solve the problem of discontinuous classification behaviour of the SOMs, secondly we introduced the theory of the WHU-structures [5] to show how time behaviour of SOMs can be defined by a new kind of neighborhood function.

2 Computing with Activities (CWA)

The basic idea of the Computing with Activities is to code complex situation vectors not by the activity of one (winner) neuron but by the activity scheme of a more or less complex set of neurons, resp. the whole SOM. Especially if such a set is placed on a closed SOM (a closed neuron grid), one can expect that all components of a system state vector of higher dimensions (system state vector coding a large number of aspects) can contribute to a scene analysis in an adequate way (resp. having the possibility to create the same or similar neighborhood occurrence), whereby no effects of the margins or no unstable break down effects will occur. This attribute can be seen very clearly if a closed SOM is trained by a simple set of state vectors as shown in Figure 1; as on the closed grid every neuron has the same number of neighbors, the resulting activity pattern of the map shows a highly symmetrical form.

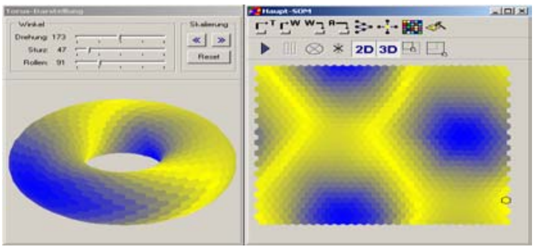

Fig. 1: Classification result of a trained closed SOM (3-Dim. and 2-Dim.)

Certainly this situation will change if complex situations are stored by the activity pattern, as shown in Figure 2, whereby even in that situation no break downs because of margin effects can be observed.

Fig. 2: Activity structure of a closed SOM representing a situation of a robot soccer match

But there is another thing, which can be pointed out from Figure 2: the winner neuron changed, even though the situation changed only slightly. Apart from that the structure of the activity pattern only changed slightly, too, or more globally: If a situation to be classified by a SOM changes slightly, obviously the coordinates of the winner region can: change, not change or jump, whereas the activity pattern will be modified in an adequate way to the change of the system. So we can learn that the winner neuron seems to represent a scene by a "so far unknown" jumping (unstable) classification-scheme but the overall activity pattern represents the scene by itself and its gradient of change accordantly. It sounds hard but the role of the winner neuron will resign for a more global coding if CWA is used while the overall activity pattern of the SOM bears the relevant information. It must be pointed out that also "common" trained SOMs show this behavior, but unfortunately the activity structure is hardly ever analyzed. Otherwise, looking at the behavior of biological neural nets (for example at the so called Place Cells of the Hippocampus) we see that nature chose this special coding long time ago, too, as in all wet nets neuron ensembles act as representatives and not single neurons (grandmother neurons). Also we realize that the power to store information is widely larger by this coding structure as e. g. now by a neuron grid of 30 * 30 neurons about 10^{13} pattern can by stored compared to only 900 of a common SOM.
Slight change in the systems, slight change in the classification behavior, this was one requirement we needed to formulate a wave function Ψ!

3 WHU-structures

Even if the CWA-method demonstrates that a static development of a system can be mirrored in a steady change of an activity pattern, we are still in the need to formulate a model which enables that these changes can be evoked by the SOM itself. It's not only that such models will provide a first temporary based memory structure of a SOM (a short time memory), but also such a model provides the time window we need to describe and interpret the wave function Ψ . Such a model can be done by using two different kinds of 'post-processing strategies'. The first of them defines a bias function, which emphasizes the neurons with the largest activities, the second one a combination of these neurons which are able to hold each other on a higher level of activity over some time regardless of their position on the neural grid. In our experiments we tried both methods and detected that the first one is not an adequate way to deal with 'activities of higher interest' as the postulated bias function has to be case-related or recalculated from classification step to classification step. Indeed the second way to emphasize the activity pattern of interest works more convenient and - as we have to learn from neurology - is closer to the principal of how brain works. For that reason we define a new kind of 'activity-oriented' neighborhood on the SOM called 'Wide Hook Up'-structure (shortly WHU-structure). The resulting net structure with a WHU-structure is more or less a SOM with an arbitrary activity-oriented neighborhood function, which simulates an involved short-time memory structure. Basic assumptions of a WHU-structure are:

- The used neural structure represents a complete graph with an arbitrary neighborhood function, which is defined by the weights of the WHU-structure.
- The modified SOM contains lateral trainable connections, which can change their interneural weights by a special training step.
- During every training or classification step a WHU-structure is build up by those neurons that can maintain or enlarge their activity by the lateral activity transfer. That means that the WHU-structure on the neuron graph forms something like a short cut or 'conversation cycle', which empowers the involved neurons to stabilize and/or intensify his or her own activity during a training or classification step.
- The neurons (and optionally the WHU-structures) abate their activities (weights) in an exponential way over the next calculation steps. In that way a time-dependence of a WHU-structure based short-time memory is defined.

For a better understanding Figure 3 shows the basic principle of a local WHU-structure.

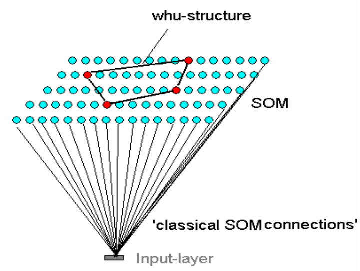

Fig. 3: Schematic SOM with a WHU-Structure

If an activity of a net is calculated under these conditions, structures like those shown in Figure 4 can be expected. Please note that the 'interaction circles' are closed loops as the neuron grid is placed on a closed topological form. So in Figure 4 a triangle can be detected on the left hand side, while a zigzag course can be detected in the middle of the activity pattern. These activities diminish slowly, according to the decay factor that is given. Thus they still exist during the next calculation cycle even if the situation changed.

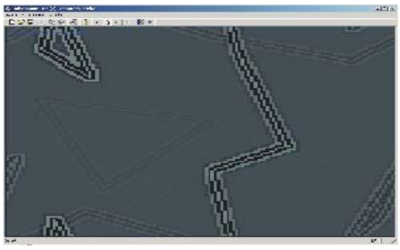

Fig. 4: WHU-structure activity on a closed SOM

CWA by this way, it is logical that activities – caused by complex situations like e.g. the classification of a robot soccer game – influence each other. In addition special situation activity presentations can be intensified or understated by changing the interneural weights of the WHU-structures. Furthermore the underlying activity contributions of the neurons involved in a WHU-structure can boost this structure so far that at the end of a calculation step ‘special activity structures’ will be more dominant than the single activities of the involved neurons. So a special detector of interesting input data constellations can be formed.
CWA and WHU-structures also mean, that these nets show pseudo-oscillations. That means that complex systems will remember the past by the exponential decay of the activity structures over a variable time even if the net is uncoupled from input. To examine this behaviour we designed a hyper-classificator structure of three downstream nets whereby the activity patterns of the lower nets serve as input patterns for the upper nets. If the activity of such a structure is calculated, a WHU-structure is initiated like exemplary shown in Figure 5.

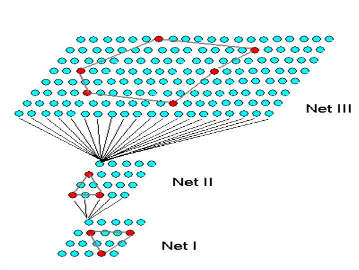

Fig. 5: Exemplary hyper-classificator structure with activated WHU-Structures

An example of the results of our simulator is shown in Figure 6. The dimensions of the networks have been 5*5 neurons of networks number one and two and 25*25 neurons of net number three. Figure 6 shows the initial activity contributions of these three nets. Please note that the upper part of the figure shows the landscape of the all-around activities, while the lower left hand side shows the numerical activity values of the first net. On the lower right hand side the visualized cube shows the activity pattern in that way, that the WHU-structures (with the high activities) are the lines on the top of the cube while the ‘normal’ activities are placed on the bottom of the cube.
As clearly can be pointed out, at the beginning of the experiment all nets show a typical activity structure. At the next step of the experiment the upper net was uncoupled from the lower structures. Now the activity of the larger net structure subsides very rapidly, as can be seen by the little height of the activity structure on the right part of Figure 7. Nevertheless the principal form of the WHU-

structure did not change much which is indicated by the large number of lines still located at the top of the cube shown on the lower part of Figure 7.

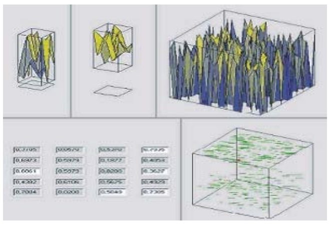

Fig. 6: WHU-structure simulation

Some calculation steps later the activity structure of the hyper-classificator decreased a little bit more, but the principal form of the WHU-structure still 'survived'. It takes some other calculation steps until the WHU-structure totally vanishes.

Fig. 7: WHU-Structure Simulation after the upper net is disconnected

Temporal behavior of the wave function Ψ, this was the second requirement we needed to formulate a quantum mechanical wave function Ψ.

4 Potential oriented description of the neural nets

If we want to use the mathematical fund of quantum mechanics, we first have to define a potential oriented description of the neural net. We do so by defining a so called "classification potential" and "conditioning potential". These potentials are function of the following form

$$E_p = E_p\left(\sum_{n=1}^{N} E_p^n(w_1^1(t),\ldots,w_M^n(t))\right) \qquad U_p = U_p\left(\sum_{n=1}^{N} U_p^n(w_1^1(t),\ldots,w_M^n(t))\right) \tag{1}$$

whereupon n indicates a neuron of the neural net with N neurons and w_m^n indicates the mth degree of freedom of the M degrees of freedom of the neuron n. These degrees of freedom correspond to the inter-neural weights on the SOM (the WHU-structure) and the momentary activity of the neurons evoked by a stimulus given by the input layer at a time t. Now we can say that the net structure is in a steady state for

$$\dot{\vec{w}}(t) = \vec{f}(\dot{\vec{w}}(t)) = \vec{0} \tag{2}$$

and be perturbed under the Lagrange function of the form

$$L = L(\vec{f}(\vec{w}(t))) = E_p(\vec{w}(t)) - U_p(\vec{w}(t)) \neq 0 \tag{3}$$

If and only if the net has adapted in the condition-state asymptotically we call $\vec{w}(t)$ the global representative R_{NET} whereat now L can be identified as a Lyapunov function for which holds

$$\dot{L}(\vec{x}) = \sum_{i=1}^{n} \frac{\partial\ L(\vec{x})}{\partial\ x_i(t)} \frac{\partial\ x_i(t)}{\partial\ t} = \sum_{i=1}^{n} \frac{\partial\ L(\vec{x})}{\partial\ x_i(t)} \dot{x}_i(t) \leq 0 \qquad (4)$$

where the x_i are the transformed coordinates (degrees of freedom) of the vector $\vec{w}(t)$ to an new (local) co-ordination system for which holds

$$\vec{X} = \vec{0} \qquad (5)$$

if the stimulus meets the vector R_{NET}. Depending on these preconditions the conditioning-state of a neural net can now be understood as a non-equilibrium-state of the potential-functions E_p and U_p, which forces a change of the degrees of freedom x_i of the net to adapt the coordinates of the desired net structure $\vec{X} = \vec{0}$. Furthermore a Hamilton-function H of a neural net can be defined, for which holds

$$H = 2E_p(\vec{x}) - L(\vec{x}) = 2E_p(\vec{x}) - (E_p(\vec{x}) - U_p(\vec{x})) = E_p(\vec{x}) + U_p(\vec{x}) \qquad (6)$$

Or briefly for better reading $H=2E_p-L=2E_p-(E_p-U_p)=E_p+U_p$. If we assume that neural nets can be identified as conservative n-particle systems the change of the degrees of freedom is given by

$$\delta\ L = \int_{t_0}^{t_1} (E_p - U_p)dt = 0 \qquad (7)$$

Following the last expression, the actual energy-function H can be separated in two Hamilton-functions

$$H\ = H_K + H_S \qquad (8)$$

where H_S describes the energy, which forces the net to adapt to a new vector R_{NET} by conditioning, and H_K describes the form of the classification potential of the net during a pure classification state.

If a Laurin serial develops H_S, the adaptation to the new vector R_{NET} follows the formulas

$$\dot{\vec{x}} = -(E_p - U_p)\vec{x}_0 \equiv \dot{\vec{w}} = -(E_p - U_p)\vec{w}_0 \qquad (9)$$

so the Lagrange function L is the generator of the net modifying operator

$$\Gamma(\vec{x},t) = \Gamma(\vec{w},t) = e^{-Lt} \qquad (10)$$

So we see, that in both coordinate systems, defining the w_i and x_i, a change will take place accordantly.

Now we define (a momentary, only at the time t existing) activity function $\psi(\vec{x},t)$ that will act as follows: if the classification concept (represented by the potential E_p) is met by a stimulus for the winner and the other neurons hold

$$\left|\psi_{n=Winner}(\vec{x},t)\right|^2 \approx 1 \quad \left|\psi_{n \neq Winner}(\vec{x},t)\right|^2 \leq 1 \qquad (11)$$

Surely a possible form for $\psi(\vec{x},t)$ will be given by

$$\psi(\vec{x},t) = e^{-\sqrt{\left|\vec{E}_p(\vec{x})\right|}\vec{x} \; -(E_p - U_p)t} \tag{12}$$

whereby the expression $e^{-\sqrt{\left|\vec{E}_p(\vec{x})\right|}\vec{x}}$ describes the time-independently classification behaviour of the net (represented by the configuration-vector R_{NET}) and $e^{-(E_p - U_p)t}$ the disturbance of the net activity $\psi(\vec{x},t)$ under conditioning. Furthermore $e^{-\sqrt{\left|\vec{E}_p(\vec{x})\right|}}$ equals the so called "impulse vector" of a neuron/neural net thus describes the change of a stimulus by the configuration of the weights. These impulses will never be equal to zero! Otherwise for a met classification holds $\vec{X} = \vec{0}$ and for that reason holds $\psi(\vec{x},t) = 1$ claimed by formula 11. If $\Delta\vec{x}(t)$ denotes the flow of the vector $\vec{x}(t)$ and if $F_i = \dot{x}_i$ stands for the conditioning-impulse of the degree of freedom x_i the conditioning of a neural net can be described as the shift of the potential of the R_{NET} forced by the Lyapunov-Lagrange function $L = E_p - U_p \neq 0$ as shown in Figure 8.

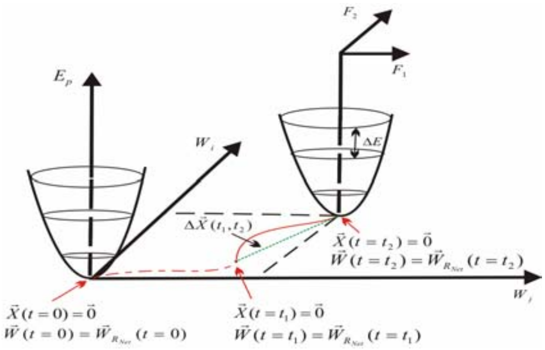

Fig. 8: Translation of the potential E_p forced by $L=E_p-U_p \neq 0$

5 Quantum mechanical description of the neural nets

Formula 10 describes the behaviour of a neural net in its classical image only, so $\Psi(\vec{q},t)$ equals a probability function of the net activity at a given point t in time.
Next we remember that the Schrödinger equation

$$-\frac{\partial^2 \Psi(\vec{q},t)}{\partial \vec{q}^2} + \frac{2m}{\hbar^2} V_{pot}(\vec{q})\Psi(\vec{q},t) = \frac{i2m}{\hbar}\frac{\partial \Psi(\vec{q},t)}{\partial t} \tag{13}$$

is the quantum mechanical image of the Hamilton-Jacobi equation

$$H\left(\frac{\partial S}{\partial q}, q\right) = E \tag{14}$$

which describes the net behaviour in its classical image. Here for S holds

$$S = \int L(q,\dot{q},t)dt \tag{15}$$

whereby the new coordinate q corresponds to x.

For transforming this equation into its quantum mechanical image, the net parameters have to be transformed into operators as follows

$$q \to \hat{q}, \quad p \to \hat{p} = \frac{\partial}{\partial q}, \; E \to \frac{i}{\hbar}\frac{\partial}{\partial t}, \; \hat{H} = -\frac{\partial^2}{\partial \vec{q}^2} + \frac{2m}{\hbar^2} U_p \tag{16}$$

As $H_G = H_K + H_S$ we get for the Hamilton-operator $\hat{H}_K$

$$\hat{H}_K = -\frac{\partial^2}{\partial \vec{q}^2} + \frac{2m}{\hbar^2} U_p(\vec{q}) \tag{17}$$

and for $\hat{H}_S$

$$\hat{H}_S = -\frac{\partial^2}{\partial \vec{q}^2} - \frac{2m}{\hbar^2} U_p(\vec{q}) \tag{18}$$

which leads to the following form of the Schrödinger equation

$$\left(-\frac{\partial^2 \Psi(\vec{q},t)}{\partial \vec{q}^2} + \frac{2m}{\hbar^2} U_p(\vec{q}) \Psi(\vec{q},t) \right) + \left(-\frac{\partial^2 \Psi(\vec{q},t)}{\partial \vec{q}^2} - \frac{2m}{\hbar^2} U_p(\vec{q}) \Psi(\vec{q},t) \right) = \frac{i2m}{\hbar} \frac{\partial \Psi(\vec{q},t)}{\partial t} \tag{19}$$

By separating the activity-function $\Psi(\vec{q},t)$ into a classification-related activity function $\Psi_K(\vec{q},t)$ and a conditioning-related function $\Psi_S(\vec{q},t)$

$$\Psi(\vec{q},t) = \Psi_K(\vec{q},t) \Psi_S(\vec{q},t) e^{i\hbar^{-1}\left(\sqrt{|\vec{E}_q|}\vec{q}\right)} \tag{20}$$

two Schrödinger equations will result

$$\left(-\frac{\partial^2 \Psi_K(\vec{q},t)}{\partial \vec{q}^2} + \frac{1}{\hbar^2} U_p(\vec{q}) \Psi_K(\vec{q},t) \right) = \frac{i2m}{\hbar} \frac{\partial \Psi_K(\vec{q},t)}{\partial t} = \frac{1}{\hbar^2} E_K \Psi_K(\vec{q},t) \tag{21}$$

$$\left(-\frac{\partial^2 \Psi_S(\vec{q},t)}{\partial \vec{q}^2} - \frac{1}{\hbar^2} U_p(\vec{q}) \Psi_S(\vec{q},t) \right) = \frac{i2m}{\hbar} \frac{\partial \Psi_S(\vec{q},t)}{\partial t} = \frac{1}{\hbar^2} E_S \Psi_S(\vec{q},t) \tag{22}$$

With (10) we get for $\Psi_K(\vec{q},t)$ and $\Psi_S(\vec{q},t)$

$$\Psi_K(\vec{q},t) = e^{i\hbar^{-1}\left(-\sqrt{|\vec{E}_q|}\vec{q} - (E_p + U_p)t/2m\right)}, \quad \Psi_S(\vec{q},t) = e^{i\hbar^{-1}\left(-\sqrt{|\vec{E}_q|}\vec{q} - (E_p - U_p)t/2m\right)} \tag{23}$$

whereby for the mass of the neurons holds 2m=1.

As the expression

$$\Psi'(\vec{q},t) = e^{i\hbar^{-1}\left(-\sqrt{|\vec{E}_q|}\vec{q}\right)} \tag{24}$$

is equal for both activity functions, we will get for the generalised activity function $\Psi(\vec{q},t)$

$$\Psi(\vec{q},t) = e^{i\hbar^{-1}\left(-\sqrt{|\vec{E}_q|}\vec{q}\ -(E_p+U_p)t-(E_p-U_p)\cdot t\right)} \cdot e^{-dk} \tag{25}$$

where e^{-dk} denotes the decay of the activity over a calculation step in the time interval *k*. As for the time-independently Schrödinger equation holds

$$\left(-\frac{\partial^2 \Psi_K(\vec{q})}{\partial \vec{q}^2} + \frac{1}{\hbar^2} U_p(\vec{q})\Psi_K(\vec{q})\right) = \frac{1}{\hbar^2} E_K \Psi_K(\vec{q}, t) \tag{26}$$

in the equilibrium state E_p-U_p=0 the Schrödinger equation will be solved by the function

$$\Psi_S(\vec{q},t) = e^{i\hbar^{-1}\left(-\sqrt{|\vec{E}_q|}\vec{q}\right)} \tag{27}$$

So that fort he classification act holds

$$\Psi(\vec{q},t) = \Psi_K(\vec{q},t) \cdot e^{i\hbar^{-1}\left(-\sqrt{|\vec{E}_q|}\vec{q}\right)} \cdot e^{i\hbar^{-1}\left(\sqrt{|\vec{E}_q|}\vec{q}\right)} = \Psi_K(\vec{q},t) \tag{28}$$

To proceed in our analysis of $\Psi_K(\vec{q},t)$, we remember that the following equation holds

$$\Psi_K(\vec{q},t) = e^{-\frac{i}{\hbar}(\sqrt{|\vec{E}_p|}\vec{p}+(E_p+U_p)t)} = \frac{1}{\hbar}\cos(\sqrt{|\vec{E}_p|}\vec{p} + (E_p + U_p)t) - \frac{i}{\hbar}\sin(\sqrt{|\vec{E}_p|}\vec{p} + (E_p + U_p)t) \tag{29}$$

which can be regarded as a complex potential of the form

$$\Omega(\Psi_K(\vec{q},t)) = \ln(\Psi_K(\vec{q},t)) \tag{30}$$

where the imaginary expression $-\frac{i}{\hbar}\sqrt{|\vec{E}_q|}\vec{q}$ describes the (semidefinit) streamlines of the complex potential and the expression (E_p+U_p)t describes the quantized equi-potential-lines of the complex potential $\Omega(\Psi_K(\vec{q},t))$ [3]. The expression

$$\hbar^{-1}(E_p + U_p)t = const. \tag{31}$$

describes an information particle wave-front for the eigenvalue E_p + U_p (with the energy E_p + U_p), which emanates out of the source 'neuron' into the neighborhood of the neuron and ‘induces’ the classification potential E_p of R_{NET}.

Coming back to CWA and WHU-structures we now understand, that we can use the quantum mechanical principles to describe both: The overall activity pattern described by CWA can be understood as superposition of the wave functions $\Psi_n(\vec{q},t=t_0)$ at a given time t_0. Therefore it’s the solution of the time-independent Schrödinger equation, whereas the WHU-structures describe the dynamical behaviour of $\Psi_{K,n}(\vec{q},t=t_0)$, resp. $\Psi_{K+S,n}(\vec{q},t)$ the solution of the time-dependent Schrödinger equation. It is important to mention that the $\Psi_n(\vec{q},t)$ are guiding fields and therefore are not observable. Only its square is observable as it corresponds to the probability of a local value of the activity. Also it must be pointed out, that we are dealing with a quantum mechanical model on a grid, what gives advices to the final form of $\Psi_n(\vec{q},t)$, E_p and U_p.

Final question is: What are the benefits of such a theory?

Final answer is: it's a bridge between the theory of the simulated neural nets to neurology and cybernetics, as in the context for the description of the biological neural nets of Helmuth Benesch [1] carrier-activity pattern-signification (which means that the momentary material state of the brain chooses an dedicated activity pattern which represents an dedicated (self aware) signification) it describes the activity pattern level. I am sure if we will understand this level in the same way as we understood the carrier level, defined by the architecture of our simulated SOMs, we can get the chance to understand (and therefore will be able to simulate) the signification level.

5 Conclusions

In the context carrier-activity pattern-meaning a quantization model of neural nets can be derived that describes a neural classifier as an n-particle-system with a well-defined potential behaviour and quantized energy rates. The classification-act as well as the change of the net under conditioning can be interpreted by the model as the disturbance of the equilibrium state between a classification potential E_p and a conditioning potential U_p. So every neural net can be regarded as a dynamical system, where the quantization parameter $\hbar$ rules the classification behaviour of the net. Similar to the theory of brain activation states the storing and the evaluation of information by dry-neural nets can now be understood as two kinds of one activation principle, which is ruled by the equilibrium state of the potentials of the degrees of freedoms of the net. The change of these system-variables can be described in the classical image by the Hamilton-Jacobi equations. On the other hand these changes can be described in the quantum mechanical image of neural nets by the Schrödinger equation too, where the same potential structure as in the classical phase-space-image can be observed. According to this theory we can understand a classifying neuron as a source emitting classification waves in the context of a new model of quantum neurodynamics.

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