FINANCIAL APPLICATION OF THE SELF-ORGANIZING MAP

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ABSTRACT : We present a financial application of the SOM algorithm. We try to model in a non parametric way the long term evolution of interest rates in order to simulate the distribution of future paths and to choose a risk management policy. Our methodology is based on a double Kohonen classification (for the initial interest rates structure, and for the interest rates shocks, i.e. the deformations of the structures). These classifications are used to approximate the conditional distributions of the shocks given the initial structure. Without assuming any hypotheses on the functional form of the process generating the interest rate structure and its dynamics, we can reproduce long-term evolution compatible with the historical observations.

1. INTRODUCTION

The neural networks in general and the self-organization maps in particular provide a large class of non linear methods and algorithms which are now widely used. They help to solve some difficult problems as pattern recognition, time series prediction, and more generally any statistical problem that involves a large quantity of data. They give a powerful complement to the linear methods when these ones are available and efficient, and allow to deal with complex data in the case the linear methods are not able to model and to analyze the structure of the data.

Financial data, known as very complex, difficult to model, badly fitted by linear models, are clearly candidate to be studied with neural methods as SOM algorithm.

Recently, several new non linear methods have been derived from the original self-organizing algorithm and adapted to data analysis in different fields. They are able to represent multidimensional data in a low dimensional map, to put in evidence the intrinsic relations between two or more than two qualitative variables, to provide a semi-parametric method for forecasting, etc. See for example (Cottrell et al., 1997, Cottrell et Rousset, 1997, Cottrell et al., 1998).

One of the better known characteristics of the SOM algorithm is the topology conservation property : data vectors which are close in the data space are classified in the same class or in neighboring classes. In many applications this property is essential and very useful, as in contextual analysis for example, (Honkela et al., 1997), or biological modeling (Kohonen, 1995). In the applications we are interested in, we do not use the topology conservation property in an explicit way. Contrarily, we would like to emphasize two aspects of the SOM algorithm which are very important for this kind of financial application : first its vector quantization capability, secondly its density approximation property (de Bodt et al., 1997,).

2. VECTOR QUANTIZATION

First of all, the SOM algorithm is a special kind of Vector Quantization algorithm. It can be viewed as an extension of the classical LBG algorithm, k-means algorithm or Simple Competitive Learning (Linde et al, 1980), when the update of code vectors involve also the neighbors of the winning centroid, and not only the winning centroid. If each centroid is linked to ν neighbors, the algorithm is equivalent (Ritter et al. 1992) to the minimization of a generalized distortion function (intra-classes variance extended to neighbor classes) :

$$\boldsymbol{x}_{n,\boldsymbol{n}}(f,\Phi) = \sum_{i=1}^{n} \int_{\underset{k \in V(i)}{\cup} C_k} \left\| x - y_i \right\|^2 f(x)dx$$

where n is the number of units in the Kohonen network, $\boldsymbol{n}$ is the size of the neighborhood in the Kohonen network, f is the density of the data, $\Phi = (y_1, y_2,..., y_n)$ is the vector quantization defined by the code vectors $y_1, y_2, ..., y_n,$ $V(i)$ is the set of indexes in the neighborhood of i, including i, C_k is the k-th. class composed by all the data related with centroid y_k.

We can observe that the generalized distortion is greater than the usual distortion defined by

$$\boldsymbol{x}_n = \boldsymbol{x}_{n,1}(f,\Phi) = \sum_{i=1}^{n} \int_{C_i} \left\| x - y_i \right\|^2 f(x)dx \, ,$$

that the classical methods do minimize. Note that in any case, it is never sure to attain a global minimum.

So the SOM algorithm with $\boldsymbol{n} > 1$ cannot be equivalent to classical Quantization Methods, since it does not minimize the same thing. However, in practical uses, the learning phase of the Kohonen algorithm is concluded with $V(i)=\{i\}$, so that $\boldsymbol{n} =1$. And in fact, the previous learning steps are no more than a good initialization for the classical Quantization algorithm. Although it is not yet rigorously proved, in all the simulations, it appears that in the last phase the algorithm reaches some « better minimum », than the one it reaches when using only $\boldsymbol{n} =1$ from the beginning.

This first property explains why it is valuable to use the SOM algorithm even if the topology preservation property of the map is not useful in the considered application. In fact, it provides a very « good » Vector Quantization of the data, which corresponds to a « good » minimum of the distortion.

3. DENSITY APPROXIMATION

As any Vector Quantization algorithm, the SOM algorithm terminated without neighbor (0-neighbor case) acts as a density approximation algorithm. We know that if $y_1, y_2,...,y_n$ are the centroids after learning, and $C_1, C_2,...,C_n$ the corresponding classes, the following convergence (in law) is guaranteed:

$$\sum_{i=1}^{n} P(C_i)\delta_{y_i} \xrightarrow{law} P$$

when n goes to infinity, and δ_{yi} is a Dirac function on y_i (Pagès, 1993). This is equivalent to say that the empirical measure defined by centroids y_1, y_2,...,y_n, weighted by the frecuencies of the associated classes, converges (in law) to the initial probability P.

Provided that centroids are adequately weighted, this results shows that it is possible to reconstruct the initial law, and the result is exact when the number of centroids goes to infinity. In (Pagès, 1993), the author also showed that the speed of convergence is better than with data obtained by independent random drawings. So it is very efficient to use the SOM algorithm in order to compute a discretization of the initial density of the data.

THE FINANCIAL DATA

The data come from the US bonds market. They are daily interest rates structures for maturity from 1 to 15 years. The interest rate for each maturity has been calculated by JP Morgan from the prices of US T-Bills and T-Bonds. The sample covers the period from 1/5/1987 to 5/10/1995, altogether 2088 entries. From these data, we compute the deformations (or shocks) between the observed term structure at time t (that is a 15-dimensional vector) and the previous one at time (t-10), (over working days). See (Cottrell et al., 1996) for more detailed presentation and (De Bodt et al., 1997) for the general method used for fitting and forecasting.

The method can be summarized as below
1) We classify the interest rates structures using a one-dimensional SOM network, with 9 units. The mean profiles are presented in figure 1. Note the conservation of the neighborhood between the different shapes from one unit to the next one. In fact we do not use this property, but it provides a nice representation. The Fisher and Wilks statistics are all significant and the resulting classification is very satisfactory.
2) We classify the interest rates shocks using a 30-units one dimensional SOM network
3) Considering that the two classifications provide two discretizations into respectively 9 and 30 values, we estimate the nine frequency distributions of the interest rate shocks classes conditionally to the class of the initial interest rate structure. We check that the 9 distributions are all different using χ^2 tests. So the existence of a relation between shocks and initial interest rate is empirically confirmed.
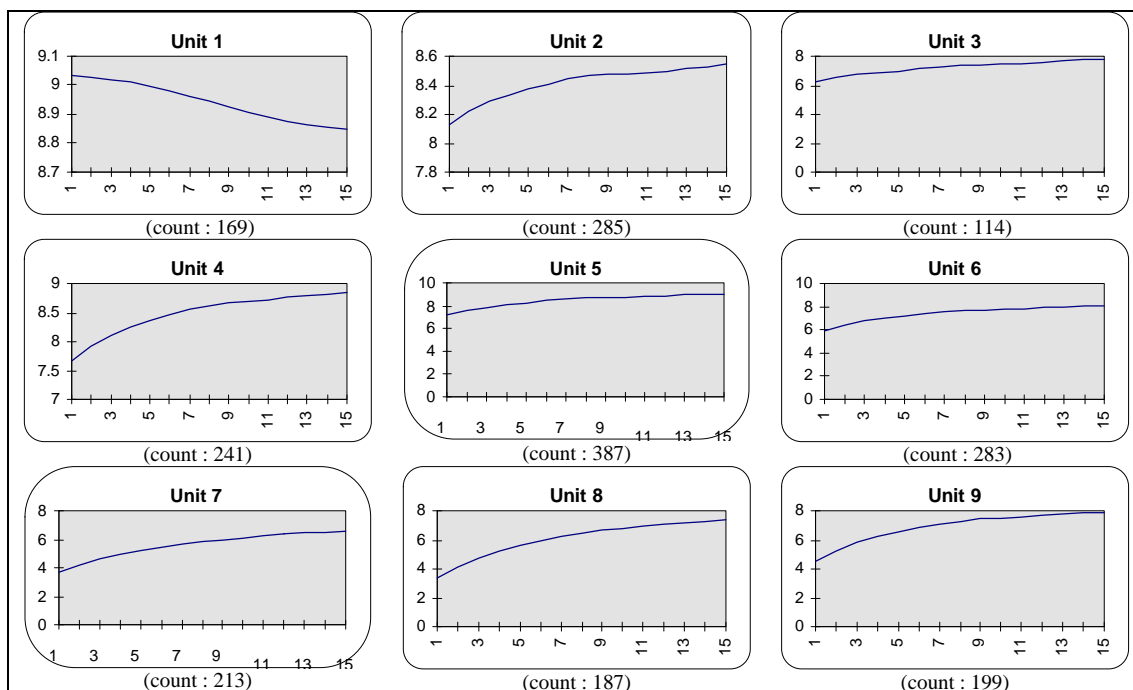

Fig. 1 : mean profiles of the clustered interest rate structures, using daily data coming from the US market.

4. SIMULATION OF LONG TERM EVOLUTION

Using these empirical conditional distributions of frequencies, we use a Monte-Carlo procedure to simulate the interest rates structure evolution. The procedure is :

1. draw randomly an initial interest rate structure;
2. determine the number of the Kohonen class of this interest rate structure;
3. draw randomly a shock according to the conditional distribution of frequencies of the interest rate shocks;
4. apply the shock to the interest rate structure;
5. repeat the procedure 125 times to construct an interest rate structure evolution on a 5 years horizon (125 times the 10 days covered by the interest rate shock);
6. for each simulation, repeat the procedure 1000 times to build the distribution of probability of interest rate structures, starting from the same initial interest structure.

Figure 2a and 2b respectively show the distribution of the short-rate and the long-rate for three simulations. The first two have been realized using the same interest rate initial shape (for which unit 6 is the winning one). The third one has been done using an initial interest rate structure attached to unit 1 (the only inverted interest rates structure mean profile). Based on these figures, we see that the procedure is stable and that, on a five years basis, the initial interest rates structure mainly influences the short rate level. We also see that, for all simulations, the level of the short-rate and the long-rate are compatible with the historical one and that the curves are well shaped. Fig 3 presents two interest rate structures obtained by the simulation. This property has been verified in all the results. We should also mention that in all simulations and at all steps, all forward interest rates are positive.
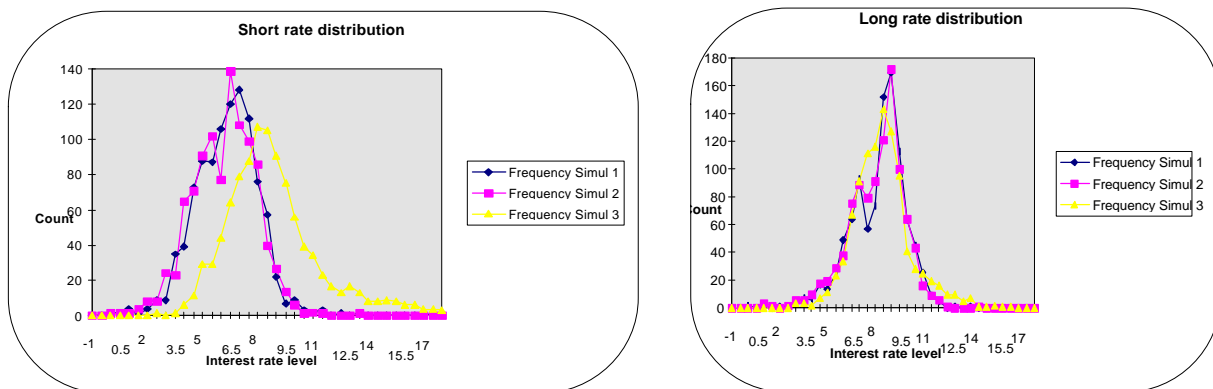

Fig. 2a **:** The short rate distributions produced by simulation 1 and 2 (starting from the same initial interest rate structure) highlight the stability of simulation procedure. Fig 2b **:** The long-rate distributions produced by simulation 1 and 2 (starting from the same initial interest rate structure) highlight the stability of simulation procedure.
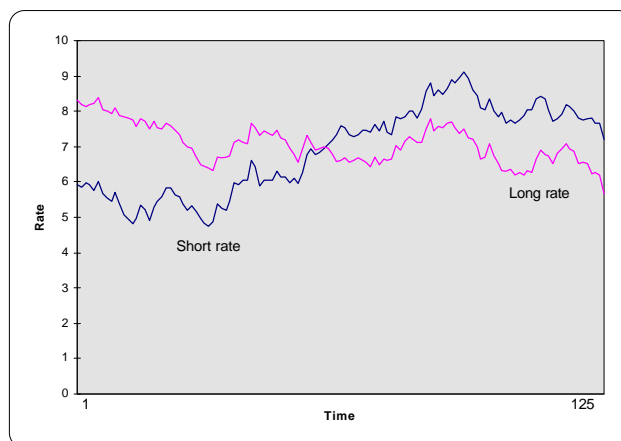

Fig. 3 : One trajectory of the short and long rate over 5 years, chosen among the 1000 produced by simulation 1.

5. CONCLUSION

Among the many open questions that remains about the approach that we propose here, one of the most important is the notion of *compatibility* of the simulated paths with the historical data set used. By *compatibility* we mean that the simulated paths will, on average, exhibit the same statistical properties than the process underlying the historical data set. We have tested this property and we have shown that the procedure does not generate explosive path, even on a long term horizon. To test the accuracy of the procedure, we have generated a set of vector of interest rates by a theoretical model. The data were generated by the well known Cox Ingersoll Ross interest rate model (Cox, Ingersol & Ross, 1985) and we have used the General Method of Moment (Hansen, 1982) to verify if the simulated paths have the same properties than the theoretical path. The results are encouraging and confirm that our procedure respects the nature of generating process of the interest rate structure.

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