Some known facts about financial data

Eric de Bodt[1], Joseph Rynkiewicz[2], Marie Cottrell[2]

[1] Université Catholique de Louvain, IAG-FIN, 1 pl. des Doyens,
B-1348 Louvain-la-Neuve, Belgium
and

Université Lille 2, ESA, Place Deliot, BP 381,
F-59020 Lille, France

[2] Université Paris I, SAMOS-MATISSE, UMR CNRS 8595
90 rue de Tolbiac,
F-75634 Paris Cedex 13, France

Abstract :
Many researchers are interesting in applying the neural networks methods to financial data. In fact these data are very complex, and classical methods do not always give satisfactory results. They need strong hypotheses which can be false, they have a strongly non-linear structures, and so on. But neural models must also be cautiously used. The black box aspect can be very dangerous. In this very simple paper, we try to indicate some specificity of financial data, to prevent some bad use of neural models.

1. Introduction

The aim of this communication is to synthesize the main known facts on the analyze and the modeling of financial data. The econometric literature has developed an important stream of research on this topics and it seems to us absolutely necessary to be aware of the results accumulated since the last 30 years in this field in order to identify the potential applications of neural networks to financial data. The textbooks of Campbell, Lo and MacKinlay [1997] and Greene [2000] are certainly useful starting points[1]. Let us emphasized that we will certainly not pretend to be exhaustive in this survey. We only present a subjective selection of important points that should, at least we hope so, convinced to reader to go ahead in this literature before trying his *magic box* on financial data.

Potential application fields of neural networks are in fact numerous. Let us just quote Lo & MacKinlay [1997 – p. 467] on this subject: "Many aspects of economic behavior may not be linear. Experimental evidence and casual introspection suggest that investors' attitudes towards risk and expected return are nonlinear. The terms of many financial contracts such as options and other derivative securities are non
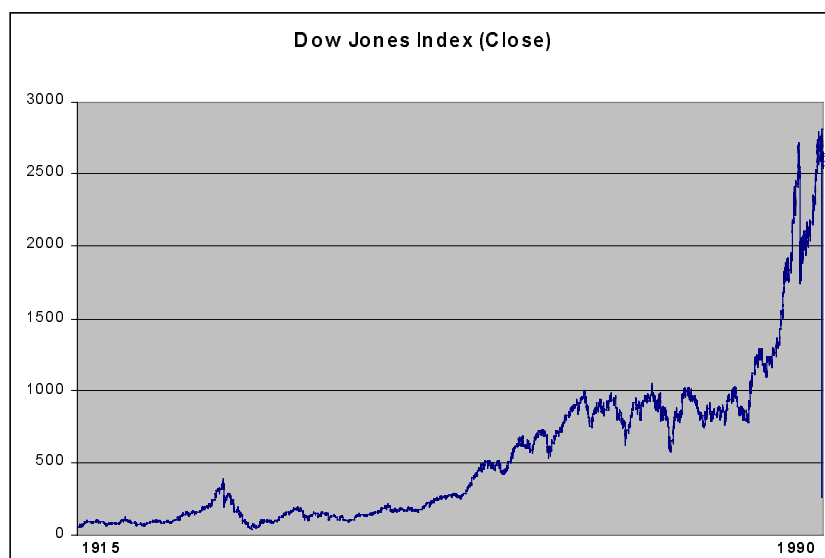
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[1] Lots of developments presented here are inspired from them.

linear.". A brief review of the books of Deboeck and Kohonen [1998], Azoff [1994] and Refenes [1995] shows a large panel of them : non-linear arbitrage pricing theory, non-linear tests of the efficient market hypothesis, tracking of stock indexes, forecasting of prices, bond rating, bankruptcy prediction, prediction of corporate mergers, risk management, diversification, trading systems, interest rates simulations with application to Value-at-Risk, stock picking, … In 1994, Hutchinson, Lo and Poggio present an application of neural networks (both MLP and RBF networks) to the problem of pricing and hedging of derivatives securities in the Journal of Finance. Since then (and even before but maybe not in the highest scientific journals in finance), numerous applications of neural networks have been proposed. All this shows clearly that the potential fields of application of neural networks in finance are the ones that financial research has always been interested in. Two remarks are yet worth to be done:

- the non linear nature of financial data remains, in the absence of theoretical models, mainly an hypothesis that have to be tested. Several propositions have been done in literature. The neural networks research community is used to use the notion of correlation dimension introduced by Grassberger and Procaccia [1993] or the BDS statistics (Brock, Dechert and Scheinkman [1987]). This is not by far the only proposition and we find, for example, in Greene [2000], several more classical approaches, based on polynomial regression analysis.

- some scanning of the econometric literature quickly shows also that neural networks are not the only candidate to tackle the non linear nature of financial data. For time series data, polynomial models, proposed by Volterra [1959] (more than 40 years ago), piecewise-linear models, Markov-switching models, … are potential competitors.

The main "producers" of financial data are the financial markets. They generate, at each moment millions and millions of prices of financial securities. Those are of different kinds. The most popular ones are securities that represent a piece of ownership of a firm (the classical share) and are exchanged on stock markets. But bonds, currencies, futures, options, warrants, commodities, … are all traded today on specific financial markets, with specific rules governing their quotation. All these data are mainly times series but the specificities of the security that they represent and of the environment in which they are formed are to be taken into account if we want to seriously analyze it. Moreover, not all financial data are prices (e.g., volumes, dividends, ratios, …) and not all financial data are coming from financial markets (e.g., data coming from financial statements, forecasts of financial analysts, …). It is clearly out of the scope of this contribution to cover all those kinds of financial data and we will concentrate our attention on stock market prices. This is a choice, partly justified by the very specific and historic interest that these markets have raised in the financial literature.

To put into light the main features of stock market data, we will try to respond to five questions: Must we work on prices or on returns? Are they Gaussian? Are they stationary? Is the variance constant through time? Is it possible to forecast them ? In order to avoid a too classical review of literature, we will base our development on the analysis of the Dow Jones index[2] (there after, DJI) series on the period 1915 - 1990[3]. The figure 1 presents it. This represents 18840 days of quotation. The index starts at 55.4 on the 4/01/1915 to end at 2596.86 on the 20/2/1990. The inspection of figure 1, at least for eyes used to examine time series, reveals lots of things[4]. Let us discover them together.

Figure 1

2. Must we work on prices or returns ?

At first sight, it could be tempting to work directly on prices (on the case of the DJI, to work on the index level). Let us try, for example, to estimate a first order auto-regressive model on the DJI :

$$DJI_t = \alpha_0 + \alpha_1 DJI_{t-1} + \varepsilon_t,$$

where ε_t is a centered noise. An extract of the results[5] is presented at table 1. The model seems very significant, with an impressive R-Squared[6]. A closer look to it will allow us to show that it is of no interest. Based on the estimated parameters, we have computed the number of times the model allows us to correctly estimated the sign of the variation of the index. The result is 9524 out of 18839 observations (the first one is lost to compute the initial variation), that is to say 50.5% of the time. Not really better than the result obtained by tossing a coin !

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Std. error of regression = 26.1347
R-squared = .997617

          Estimated      Standard
Variable  Coefficient    Error           t-statistic
Alpha_0   .581189        .271015         2.14449      **
Alpha_1   .999177        .355801E-03     2808.25      ***
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** is for significant at 5%, *** is for significant at 1%;
Table 1

The theoretical reasons of this result will be exposed later but, at this stage, this result is sufficient to put into light that any attempt to model or forecast stock market must be based on successive variations of price and not on the prices themselves. The classical measure of successive variations is the return, either calculated in discrete time as $r_t = \dfrac{p_{t+1} - p_t}{p_t}$ or as $r_t = \ln(\dfrac{p_{t+1}}{p_t})$. Figure 2 presents the returns of DJI. The very noisy nature of the data comes immediately to light.

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[5] All estimations presented in this paper have been realized with TSP 4.5.
[6] Readers used to time series analysis will immediately suspect that the process is not stationary but we will defer this point up to section 4.
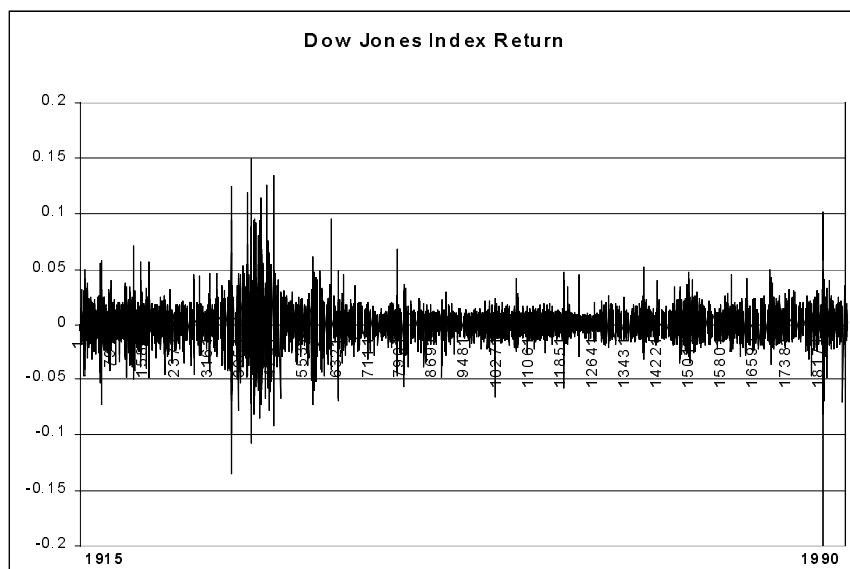
Figure 2

3. Are they Gaussian?

The Gaussian hypothesis on the distribution of returns is at the foundation of numerous theoretical developments in finance, such as the efficient frontier of Markoviz [1959] or the Capital Asset Pricing Model of Sharpe [1964] and Lintner [1965b], to quote only two of the most famous. It is needless to say that the Gaussian hypothesis makes considerably easer lots of mathematical et statistical developments. But is it a reasonable hypothesis? From a theoretical point of view, it is possible to build on the Central Limit Theorem to justify it, as Fama [1965] suggests it. Returns are indeed the result of the sum of the variations of prices from transaction to transaction on the market. If variations of prices from transaction to transaction can be considered as random outcome of probability distribution with finite mean and variance, then, returns should be Gaussian[7]. Which is the empirical evidence ? We present, for the DJI returns series, at figures 3a, 3b and 3c, the distribution of returns calculated on a daily basis, weekly basis and monthly basis.

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[7] It should be noted that this assertion remains valid in the context of unequal variance between the distributions from which the variations of prices are coming.

Figure 3a	*Figure 3b*	*Figure 3c*
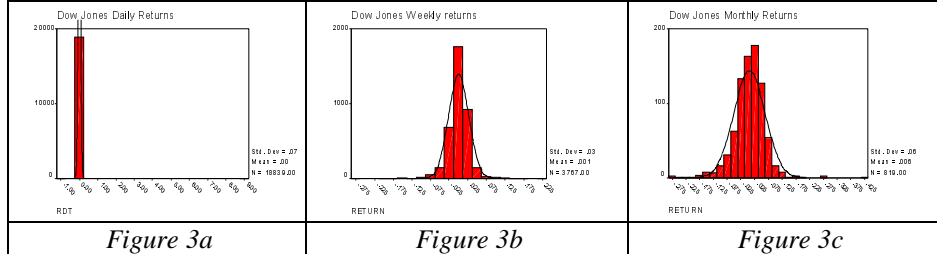
As awaited from the theoretical point of view (from the Central Limit Theorem), the more you aggregate returns, the more the distribution seems to be Gaussian. The same phenomenon can be observed concerning the aggregation of returns to form portfolios (the larger the number of securities in the portfolio, the closer are the returns to a Gaussian distribution). Empirical tests show that returns of individual securities are a lot less Gaussian than returns of portfolios of them.

4. Are they stationary ?

A time process y_t is said weakly stationary (or covariance stationary) if $E[y_t]$ and $Var[y_t]$ are finite constants independent of time, and $Cov[y_t, y_s]$ is a finite function of the absolute value $|t\text{-}s|$. Does the returns of the DJI returns series respond to these conditions ? The development of econometric test of stationarity has been a very active field and is closely related to the concept of random walk[8]. It is out of the scope of this paper to go into the details of this problematic. We will limit ourselves here to a presentation of the concept of random walk, to show that a random walk process is not stationary and, last but not least, to introduce the Dickey-Fuller test of the presence of a random walk. A random walk process is defined as

$$y_t = \mu + y_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is an i.i.d. sequence of random variables, with $E[\varepsilon_t] = 0$ and $Var[\varepsilon_t] = $ Cst.

Suppose that this process can be stationary, then we can reformulate the stationary solution of (1) as

$$y_t = \sum_{i=0}^{\infty} (\mu + \varepsilon_{t-i}) .$$

But this solution cannot be a weakly stationary process since
- If μ is not null, y_t has an infinite expectation, (if μ is null, $E(y_t)=0$)
- Even if μ is null, the variance of y_t is infinite as an infinite summation of variance terms $Var(\varepsilon_t)$.

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[8] In Econometry, this means that the process has an unit root.

The usual test of the presence of a random walk is the Dickey-Fuller test. The test is constructed on the regression equation

$$y_t - y_{t-1} := \Delta\, y_t = \gamma\, y_{t-1} + \varepsilon_t \qquad (2)$$

The presence of a random walk is revealed if γ is equal to 0. The classical test in this case is the Dickey-Fuller test and we have to use Dickey-Fuller critical values[9].

We consider again the DJI returns series. As a previous study, we plot the auto-correlation function up to lag 30 (fig. 4). The auto-correlation function clearly indicates that it seems to have no significant auto-correlation in the series of returns of DJI, except for the first lag, which most probably is due to asynchronous trading, overall in the first part of the century[10]. This observation seems to indicate that the returns are not correlated and advocates for the presence of random walk (equation (2), with $\gamma = 0$).
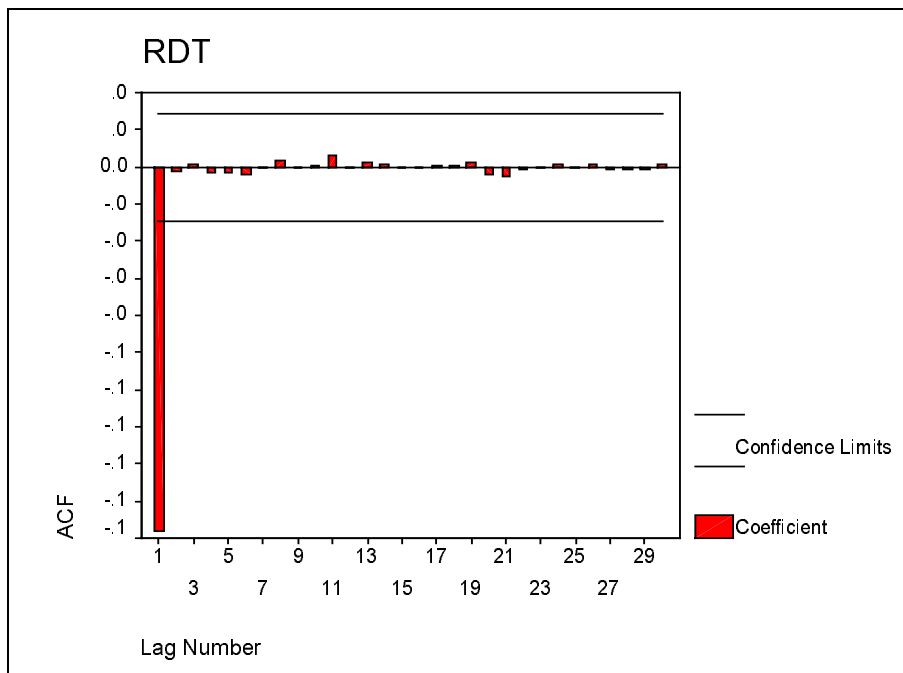

Figure 4

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[9] The augmented version of the test allows to test for the presence of random walk in higher order auto-regressive processes.

[10] Scholes and Williams [1977] analyze the consequence of non-synchronous data on the estimation of covariance of returns.

However the hypothesis of random walk is rejected by the Dickey-Fuller test at a very high level of confidence (> 99 %). Those results conduct us to conclude, at this stage, that random walk model is not satisfactory for the DJI returns. Note that the absence of random walk do not allows to conclude to stationarity. Recent developments in this area have produced what is called "Long Memory Models", characterized by alternating positive and negative significant autocorrelations at very long lag. For example, Ding, Grander and Engle [1993] find significant autocorrelations out to lags well over 2000 days in daily stock market returns. The main idea beyond these models is the notion of fractionally integrated processes, which are processes that lies somewhere between stationary and random walk.

5. Is the variance constant through time ?

Engle [1982] put into light the notion of clustering of the variance in economic time series and introduces the ARCH model. The observation of fig. 2 clearly shows this feature of returns through time. The big depression, for example, between World War I and World War II seems clearly to be a period of high volatility. The ARCH(q) model is a model of the conditional variance of the noise ε_t :

$$\begin{cases} E[\varepsilon_t|F_{t-1}] = 0 \\ E[\varepsilon_t^2|F_{t-1}] = h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 \end{cases}$$

where $E[. \mid F_{t-1}]$ is the conditional expectation on the past of the process. Note that the $(\alpha_i)_{0 \le i \le q}$ are necessarily positive to ensure that the variance is a positive number.

A sufficient condition for the stationarity of the unconditional variance ($Var(\varepsilon_t) = E(\varepsilon_t^2)$) is :

$$\sum_{i=1}^{q} \alpha_i < 1.$$

For example, if we consider an ARCH(1) model, the necessary condition for the existence of a stationary solution of our model is $0 \le \alpha_0$, $0 \le \alpha_1 < 1$ and the unconditional variance of the noise will be

$$Var(\varepsilon_t) = \frac{\alpha_0}{1-\alpha_1}.$$

Bollerslev [1986] introduces a generalization of the model in the form of a GARCH(p,q), which is defined as

$$\begin{cases} E[\varepsilon_t|F_{t-1}] = 0 \\ E[\varepsilon_t^2|F_{t-1}] = h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \end{cases} \tag{3}$$

where $(\alpha_i)_{0 \le i \le q}$ and $(\beta_i)_{0 \le j \le p}$ are positive constants. As in the previous case, a sufficient condition for the existence of a stationary solution of (3) is

$$\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1. \tag{4}$$

Note that this condition is not necessary since Nelson [1990] shows in the case of GARCH(1,1) model that for greater coefficients the stationarity can be ensured, but with a stationary solution which has an infinite variance.

With condition (4) for a GARCH(1,1), one has

$$Var(\varepsilon_t) = \frac{\alpha_0}{1 - (\alpha_1 + \beta_1)}$$

Table 2 presents results of a GARCH(1,1) estimate on the DJI returns series[11].

ALPHA0 = .762366	ALPHA1 = 1.000000	BETA1 = 0.

Table 2

The estimation is problematic. The inequality (4) is not verified. The persistence seems so high (the persistence is defined, in the framework of GARCH models, as the sum of α and β parameters) that we meet the limit of domain of validity of the model. In order to try to better understand the problem, we have decided to divide the period of analysis in 4 sub periods (1915-1928, from World War I to the Great Depression, 1929-1944, from the 1929 Crash to the end of World War II, 1945-1969, from the reconstruction up to the end the sixties and 1970-1990, from the petroleum crisis up to now). To identify each sub period, we have added three dummies in the variance equation (D_1, D_2 and D_3), each one representing a shift in the unconditional variance. The model of the conditional variance is

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3 \tag{5}$$

and the results are presented at table 3.

ALPHA0 = .508330E-04	ALPHA1 = 1.000000	BETA1 = 0
GAMMA1 = .276468E-04	GAMMA2 = .110899E-03	GAMMA3 = 0

Table 3

The same problem is met. Interestingly, the coefficients of the first and the second dummies (for which the estimation is realized) are highly significant. This corroborates the hypothesis of shifts in unconditional variances between those periods (with the unconditional variance being five times superior during the second period – the Great Depression than the first one). This result clearly contradicts the one of the

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[11] The GARCH(1,1) specification is the one that seems to best fit the behavior of stock market returns on most of published empirical works.

previous section. If the unconditional variable changes through time, the DJI returns series is not weakly stationary on the observation period.

Numerous extensions of these models have been proposed in the econometric literature (see for example Hamilton [1994] for a review of them). The estimation is realized by the maximization of the log-likelihood function under the hypothesis of normality of the disturbance process[12]. Let us emphasize at this point that GARCH models and their extensions are really non linear models of the process generating the data (y_t), allowing second order dependence in the variance. One of the interesting variants of the GARCH model is the one proposed by Glosten, Jagannathan and Runkle [1993]. The variance equation is

$$h_t = \alpha_0 + (\alpha_1 + S_{t-1})\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad (6)$$

where S_{t-1} is a dummy variable which is equal to 1 if sign of ε_{t-1} is negative and 0 otherwise. That integrates an asymmetric effect of the bad news (which is captured by the sign of the previous disturbance) on the volatility.

To go one step ahead on the problem of high (too high) persistence, we present at table 4 the results obtained for GARCH(1,1) model with asymmetric effect (equation (6). The estimation is presented on the 500 first observations (something like 2 years of data).

ALPHA0 = 45.2732   ALPHA1 = .281309   BETA1 = .425584

Table 4

This table shows a level of persistence far from one. The sum of α_1 and β_1 is 0.7 (interestingly, if we take the 1000 first observations, the sum is 0.9). It appears clearly that the longer is the estimation period, the higher the persistence. This raises in fact the question of structural breaks on the variance (the same kind of behavior is observed for ARMA models). Table 4 also shows that bad news have a lot more impact on the returns than good news (remember that S_t is equal to 1 when the disturbance is negative). This asymmetric effect of news is highly significant.

Two questions remains: why is there clustering in the variance of returns and is it possible to forecast future variance ? Concerning the first point, several hypotheses have proposed and tested. It seems that the main reason is not clustering in the arrival of news on the market but well heterogeneity in the anticipations between investors (see for example Mordecai and Motolese [1999]). Can we forecast the

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[12] The case of non normality of disturbances is, under some restrictions, resolved by White [1982] and Gourieroux, Montfort and Trognon [1982], who proposed the peudo (or quasi) maximum likelihood approaches. Concerning the estimation procedure, several procedures have proposed in the literature to simplify the needed calculation (see for a review of them McCullough and Renfro [1999].

variance? The title of the paper of Andersen and Bollerslev [1998], "Answering the Skeptics : Yes, Standard Volatility Models Do Provide Accurate Forecasts", seems to indicate that it is the case. Behind the title, the authors admit that, using only daily data and being interested in ex-post observed variance, it is clearly not the case (at least by classical GARCH models). As shown for example by Blair, Poon and Taylor [1999] on the S&P100 index, the use of implied volatilities of options or the use of intra day data allows to get better results. Theses approaches impose however a stringent constraint on data availability.

5. Is it possible to forecast them ?

Since the beginning of this century, the question of the predictability of financial series (at least of stock market prices) has been the subject of a highly controversial debate in finance. Fama [1965], in its seminal paper, recalls the meaning of the random walk hypothesis (first proposed by Bachelier [1900]) and presents different empirical tests of it. He concludes in those terms: "The main conclusion will be that the data seem to present consistent and strong support for the model. This implies, of course, that chart reading, though perhaps an interesting pastime, is of no real value to the stock market investor." The main theoretical argument of Fama is the notion of efficiency. On efficient market, all participants receive all information at any time. In such circumstances (very improbable in fact), the only reason for a stock price to move is the arrival of a new information, which is impossible to forecast by definition. With such an ideal representation of financial markets, what is surprising, it is the fact that most empirical works, mainly based on linear statistical tests, have conducted to the same conclusion in the years sixties and seventies, despite the heavy use of charts and technical indicators by the professional community.

However, as underlined by Campbell, Lo and MacKinlay [1997], "Recent econometric advances and empirical evidence seem to suggest that financial asset returns are predictable to some degree". Among those works, three of them have constituted main advances in this field. Brock, Lakonishok and Le Baron [1992] test two popular technical trading rules on the Dow Jones market index on the period going from 1897 to 1986. They use a bootstrap methodology to validate their results and conclude: "their results provide strong support for the technical strategies". Sullivan, Timmerman and White [1999] propose new results on the same data set (extended with 10 new years of data). Their methodology, still relying on heavy use of bootstrap, allows avoiding (at least to some extend) the data-snooping bias (cfr infra) and is applied to a universe of 26 trading rules. They confirm that the results of Brock, Lakonishok and Le Baron stands up to inspection against data-snooping effects. The recent contribution of Lo, Mamaysky and Wang [2000], using a new approach based on nonparametric kernel regression, confirms that "several technical indicators do provide incremental information and may have some practical value".

On the basis of all of those empirical evidences, we can consider that there is some interest in trying to predict the evolution of financial asset prices and that is probably one of the potential field of application of neural networks[13]. This work must however be done very cautiously. The concept of data-snooping bias has been proposed by Lo and MacKinlay [1990] (at least in the financial literature). This bias in the evaluation of forecasting method appears as soon as a data set is used more than once for purposes of inference of model selection. It can be illustrated by the "Give me the data, I will give you the model" sentence. From our experience of the neural network modeling process, at our opinion, it is almost always present in our field. We do not present here an empirical results on the DJI index, partly by lack of place, partly because this index has been so much studied that any new result on past values of this series would be suspect from this point of view.

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[13] We have to stress also that a clear distinction must be made between the prediction of the evolution of a financial series and the possibility to win money using this prediction. This last point implies to build a trading strategy which, when we take into account all the transaction costs and the normal risk remuneration of the investor, is winning. This is a highly more complicated task than simply the prediction one, which must take into account the recent theoretical and empirical advances in finance on the subject of the time varying risk premia (see for example Chordia and Shivakumar [2000]).

References

[1] Andersen T.G., Bolllerslev T., 1998, *Answering the Skeptics : Yes, Standard Volatility Models Do Provide Accurate Forecasts*, International Economic Review, vol. 39, n°4, p. 885-905

[2] Azoff E.M., 1994, *Neural Network Time Series Forecasting of Financial Markets,* John Wiley & Sons, New-York.

[3] L. Bachelier, 1900, *Théorie de la spéculation*, Gauthier-Villars, Paris.

[4] Blair B., Poon S.H., Taylor S.J., 1999, *Forecasting S&P 100 Volatility : The Incremental Information Content of Implied Volatilities and High Frequency Index Returns*, FEN Working Paper.

[5] Bollerslev T., 1986, *Generalized Autoregressive Conditional Heteroscedasticity*, Journal of Econometrics, 31, p. 307-327

[6] Brock W., Dechert W. and Scheinkman J., 1987, *A Test for Independence Based on the Correlation Dimension*, Working Paper, University of Chicago.

[7] Brock W., Lakonishok J., Le Baron B., 1992, *Simple Technical Trading Rulesand the Stochastic Properties of Stock Returns*, The Journal of Finance, vol. XLVII, n°5, pp. 1731-1764

[8] Campbell J.Y., Lo A.W. & MacKinlay A.C, 1997., *The Econometrics of Financial Markets*, Princeton, New Jersey.

[9] Chordia T., Shivakumar L., 2000, *Momentum, Business Cycle and Time-Varying Expected Returns*, FEN Working Paper

[10] Deboeck G., Kohonen T. (ed), 1998, *Visual Explorations in Finance with Self-Organizing Maps*, Springer-Verlag.

[11] Ding Z., Granger C. and Engle R., 1993, *A Long Memory Property of Stock Returns and a New Model*, Journal of Empirical Finance, 1, p. 83-106

[12] Engle R., 1982, *Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflations*, Econometrica, 50, 1982, p. 987-1008

[13] Fama E., 1965, *The Behavior of Stock Market Prices*, Journal of Business, 38, p. 34-105

[14] Glosten L.R., Jagannathan R., Runkle D., 1993, *Relationship between the Expected Value and the Volatility of the Nominal Excess Return on Stocks*, Journal of Finance, 48, p. 1779-1801

[15] Gourrieroux C., Monfort A., Trognon A., 1984, *Pseudo Maximum Likelihood Methods : Applications to Poisson Models*, Econometrica, 52, p. 701-720

[16] Grassberger P. and Procaccia I, 1983, *Measuring the Strangeness of Strange Attractors*, Physica, 9D, p. 189-208.

[17] Greene W.H., 2000, *Econometric Analysis*, , 4°edition, Prentice Hall, London.

[18] Hutchinson J., Lo A.W. and Poggio T., 1994, *A Nonparametric Approach to Pricing and Hedging Derivative Securites Via Learning Networks*, The Journal of Finance,vol. XLIX, n°3, p. 851-889

[19] Lintner J., 1965, *The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets*, Review of Economics and Statistics, 47, p. 13-37

[20] Lo A., MackKinlay A.C. 1990, *Data-snooping Biases in Tests of Financial Asset Pricing Models*, The Review of Financial Studies, vol. 3, p. 431-467.

[21] Lo A., Mamaysky H., Wang J., 2000, *Foundations of Technical Analysis: Computational Algorithms, Statistical Inference, and Empirical Implementation*, The Journal of Finance, vol. 55, issue 4

[22] Markovitz H., 1959, *Portfolio Selection : Efficient Diversification of Investments*, John Wiley, New York

[23] Mordecai Kurtz, Motolese Maurizio, 1999, *Endogenous Uncertainty and Market Volatility*, Working paper, Stanford University.

[24] McCullough B., Renfro C., *Benchmarks and Software Standards : A Case Study of GARCH Procedures*, Journal of Economic and Social Measurement, 1999, forthcoming.

[25] Nelson D.B., 1990, *Stationarity and persistence in the GARCH(1,1) model,* Econometric Theory Vol 6, p. 318-334.

[26] Refenes A.P. (ed), 1995, *Neural Networks in the Capital Markets*, John Wiley & Sons, New York.

[27] Scholes M., Williams J., 1977, *Estimating Betas from Nonsynchronous Data*, Journal of Financial Economics, 5, p. 309-327

[28] Sharpe W., 1964, *Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk,* Journal of Finance, 19, p. 425-442

[29] Sullivan R., Timmerman A. White H., 1999, *Data-Snooping, Technical Trading Rule Performance, and the Bootstrap*, The Journal of Finance, vol. 54, issue 5, pp. 1647-1691

[30] Volterra V., 1959, *Theory of Functionals and of Integro-Differential Equations*, Dover, New York.

[31] White H. *Maximum Likelihood Estimation of Mispecified Models*, Econometrica, 53, p. 1-16