Curves Based Kohonen Map and Adaptative Classification: An Application to the Convergence of the European Union Countries

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Abstract— This paper aims to study the convergence of the current European Union countries within the Maastricht criteria framework for the 1980-2002 period. We propose two methodologies based on the Kohonen Map approach adapted to deal with cross–country time–series data. Each methodology allows us to define homogeneous classes of countries with respect to their trajectories.

1 Introduction

The Economic and Monetary Union (EMU) was a logical step towards a fully integrated market and a major political milestone on the road to a united Europe. The Treaty signed in Maastricht on February 1992 has made the progress towards a single currency irreversible. The Maastricht Treaty defines 5 macroeconomic convergence criteria that the European Union member states have to respect in order to integrate the EMU: government deficit (at most 3% of GNP), government debt (at most 60% of GNP), prices (consumer price inflation rate must be no more than 1.5% above the average of those of the three EU member states with the lowest inflation rates), long-term nominal interest rate (the long-term interest rate must be on average no more than 2% above the average of those of the three EU member with the lowest one) and exchange rate (the country should have observed the normal fluctuation margins of the Exchange Rate Mechanism of the European Monetary System). One can note that these convergence criteria solely define nominal convergence. None of them are based on real variables, such as GNP per capita or unemployment rate.

During the transition towards the EMU (which began in 1990), the member states would draw up convergence programs designed to promote improvements and convergence of economic performances. On the basis of their economic situation in 1998, 11 European countries formed the EMU and began using a common currency, the Euro, on January 1, 1999[1].

The aim of the paper is to study the convergence of the European economies, and more specifically the transition of these economies towards the criteria fixed by the treaty of Maastricht to participate in the EMU. It is indeed interesting to study the convergence processes of the different European economies and see if there is a certain homogeneity in the transition processes towards common standards or if one can observe the idea of a Europe with different speed levels.

We study the 15 European Union countries although only twelve of them have adopted the single currency and belong to the Euro area. The study is based on the period 1980-2002. We then use cross–country time–series data for four variables (we omit the exchange rate criterion which is not quantified in level): the percentage of government deficit in GNP, the percentage of debt in GNP, the inflation rate (the annual variation of Harmonized Indices of Consumer Prices in percentage) and the long-term nominal interest rate[2].

In order to study the trajectories of the European countries and define classes of European countries, we propose to adapt the Kohonen Map[3] (or Self-Organizing Map) to deal with cross–country time–series data. The next sections present the two methodologies adopted and combined to deal with our problem: a constrained Kohonen map and a trajectories approach.

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[1]In 1999, the Euro area is composed of: Belgium, Germany, Spain, France, Ireland, Italy, Luxembourg, the Netherlands, Austria, Portugal and Finland. Greece has integrated the Euro area in 2001. Denmark and United-Kingdom have opt-out clauses, which imply that they are not obliged to adopt the Euro. Sweden will join the Euro area as soon as it has fulfilled all the conditions.

[2]Data obtained from OECD Economic Outlook.

[3]We suppose that the reader is familiar with the Kohonen algorithm. See for example [4], [3] for an introduction to the algorithm and to its applications to data analysis.

2 A constrained Kohonen map

2.1 The principle

In order to deal with the time series dimension, one can compute as many Kohonen maps (for example a string, *i.e.* a one-dimensional topology) as years and classify the 15 countries according to the 4 variables by year[4]. The problem with this type of computational method is the large instability of the associated organization year by year. Thus we choose another approach which consists in computing a constrained Kohonen map with as many rows as years and a fixed number of units by year. As with the traditional Kohonen Map, the number of units needs to be decided arbitrarily. We have chosen a one-dimensional structure of 8 units. The algorithm is as follow:

- The initialization of the SOM algorithm is a random selection of 8 countries in the dataset. In the unit (i, t) of the map we put the four values of the variables[5] of the country i for the year t.
- For each iteration a country i_0 and a year t_0 are randomly selected in the initial dataset. Then for all $i \in [1, 8]$ we look for the unit (i, t_0) that is the nearest to the selected observation.
- We update this winning unit and the nearby units. The neighborhood decreases on the row dimension during the iterations from r to 0. To force the temporal organization, for a given row neighborhood r, the size of the temporal neighborhood decreases from r to 0.
- Finally, to guarantee convergence, there is no neighbor at the last third of iterations.

Once the algorithm converges, we situate the countries on the map in order to identify their position.

2.2 The classification

The map thus obtained allows us to show a continuity in the temporal dimension as well as in the row dimension (see figure 1). The map opposes the top right-hand side which corresponds to high levels of the 4 variables (exceeding Maastricht criteria) to the bottom left-hand side which corresponds to low levels. For each row (year) the quality of the performance of each unit is decreasing from left to right. We can see a contraction between the extreme average profiles by year during the period 1980-2002. The difference between the best and the worst performers decreases over the years. These results illustrate the processus of convergence of the European countries to the Maastricht criteria.

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[4]See [1] and [2] for some direct applications of the Kohonen algorithm which deal with transitions of individuals on the map.

[5]Data are centered and normalized by variable over the whole period.

Figure 1: The Kohonen Map

Legend: The first row of the map corresponds to 1980 and the last one to 2002. In each unit, the associated code vector, which is composed of the value of respectively government deficit, debt, inflation and interest rate, is represented.

We then reduce the number of classes by using a hierarchical classification algorithm on the 8×23 code vectors. The number of classes selected is 5 (see figure 1 where the classes are defined on a grey scale and are delimited by bold lines).

First, one can observe that the classes gather units corresponding to different years and that the number of classes by year varies between 1980 and 2002. Every year, from 1980 until 1994, the 15 countries are grouped in 3 classes (except 1985, which is characterized by 4 classes, and 1986 which has 4 classes of which one contains no country hence 3 classes). 1995 onwards only 2 classes are left. The decrease of the number of classes during the period illustrates once again the convergence of the European countries.

Second, we observe that the classes move towards the right-hand side of the map, indicating that the bad profiles in a given year previously correspond to medium profiles. For example, the class grouping Italy, Ireland, Greece and Portugal in 1980-81 disappears in 1987; the profile of Italy, Greece and Portugal in 1990 gets in the same class as Sweden, United Kingdom, Denmark and Belgium's profile in 1980. Thus the large differences in performance between countries in the 80's have progressively disappeared and one can note a larger homogeneity among European countries at the end of the period according to the Maastricht criteria.

More precisely, in the beginning of the 80's, the most virtuous class according to the Maastricht criteria is composed of Luxembourg, Austria, France, Germany, Spain and Netherlands. This class is close to the countries which constituted the core of the European Economic Community, except Spain and Austria. The medium class is composed of Sweden, United Kingdom, Denmark and Belgium. And the third class is composed of Italy, Ireland, Greece and Portugal. We can see that Italy, the third most powerful European country, remains in the worst performing class, revealing difficulties to equilibrate its public finances and to slowdown inflation.

A large change occurs in the early 90's with the beginning of a large recession and the Exchange Rate Mechanism crisis. More precisely in 1993 the medium class becomes larger grouping countries formerly belonging to the best performing class. In 1994 and 1995, the best performing class is only composed of Luxembourg. This clearly illustrates the effect of the recession in Europe causing a larger public deficit and the end of the European Monetary System. After 1995, only two classes are left. Unsurprisingly, the best performing class progressively enhances a larger number of countries as in 1997-1998 the EU members are obliged to respect the Maastricht criteria in order to integrate the EMU. However, in 2002, three countries are left in the worst performing class (Italy, Greece and Belgium) as they continue to have a debt ratio exceeding 100%. Nevertheless, as they manage to substantially decrease it, it has not constituted an obstacle to their integration to the EMU.

One can note that France and Germany in 2002 are very close to the worst performing class due to problems with their public finances. Indeed, the adoption of a single currency has its costs as well as benefits. The most obvious benefit is that it reduces the costs and difficulties of completing international transactions within the EMU. But the individual countries in the EMU can no longer conduct an autonomous monetary policy or have any control over the foreign exchange value of their national currencies. Meaning that they have to resort to other methods to combat problems of recession and inflation within their own countries. Thus the budgetary policy is the only policy which is not centralized. However, since 1997 the Stability and Growth Pact limits the ability of a member country to employ expansionary fiscal policy during a recession by implementing an excessive deficit procedure. The European Commission has implemented the deficit procedure to France and Germany in 2002, as well as to Portugal in 2001 (which is close to the worst performing class in 2001), as the possibility that they may not respect the deficit criterion has occurred.

Finally, we can draw the trajectories of every country through the map (see figure 2 which represents the trajectories of each country through the transposed map 1).

The trajectory of Luxembourg is very close to the left-hand side of the map indicating that it has been the most virtuous in the sense that it has always been characterized by the lowest value for the 4 criteria.

The trajectories of Italy, Belgium and Greece are opposed to that of Luxembourg since they have remained close to the worst position in the map throughout the period.

The trajectories of Austria, France, Germany, the Netherlands, United Kingdom, Denmark, Finland and Sweden are quite similar in the sense that they begin in the most virtuous class in 1980, cross a medium position in the 90's during the recession and end up in the most virtuous class.

The trajectory of Spain is very fluctuating indicating difficulties to converge to Maastricht criteria.

Two trajectories are quite particular. Whereas Portugal and Ireland begin in the worst situation among European countries in 1980, they manage to switch in a medium position in the middle of the 90's –whereas at the same period most of the European countries have their situation worsened because of the recession– finally they end up in the most virtuous class. Our study clearly illustrates the exceptional transition of these countries.

Figure 2: The trajectories of the countries on the trans-
posed Kohonen Map
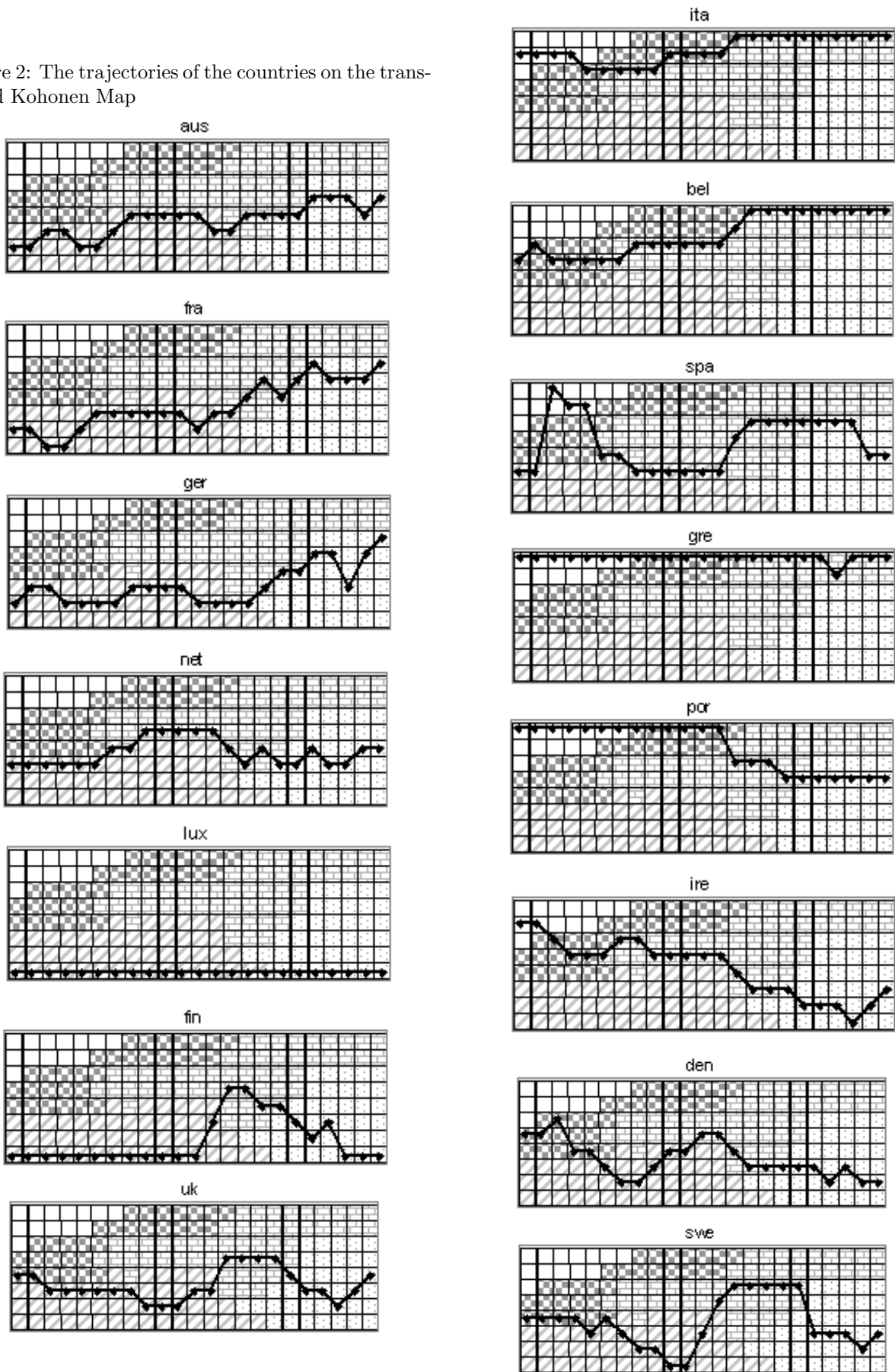
3 The trajectories approach

3.1 The principle

The main idea of this approach is to consider the whole trajectory of each country. We can consider a trajectory as realizations of a multivariate function (4 dimensions) of time (23 years). The SOM algorithm allows us to get a classification of these functions, because it is well adapted to high dimensional data.

Since there are only 15 countries we use a small Kohonen string to get a summary of the main trajectories. During the learning process, the 4×23 components of the units are updated using all the components of the nearest countries. This approach leads to a constrained learning process since there are only 15 possible directions to update the coordinates of the centroids.

In the SOM algorithm, a neighborhood structure is defined for the units and is respected throughout the beginning of the learning process, then the neighborhood is decreased to 0. With such learning rules, one can first observe the self-organization of the units, and later the convergence of the components of the units in order to quantify the input space. The SOM keeps the topology of the input space, and such ordering property of Kohonen yields a classification of the countries from the most to the less virtuous countries with respect to the Maastricht criteria.

However, since we get a classification of the whole trajectories, we can not observe the individual trajectory of each country. Then we compute the individual trajectories by classifying the countries year by year, using the computed centroids. Hence, during the period of interest some countries are switching from one trajectory to another and a closer look of these changes gives us an insight of the evolution of the economic situation of the countries with respect to the chosen criteria.

3.2 The classification

In order to obtain a meaningful classification the best choice seems to choose a Kohonen string with 6 units. We later reduce the number of classes by applying a hierarchical classification algorithm to these units. The final trajectory computed by the SOM algorithm clusters the European countries into the 6 following groups:

(1) Finland and Luxembourg
(2) France, Germany, Austria and United Kingdom
(3) Sweden, Denmark, Spain and Netherlands
(4) Belgium and Ireland
(5) Italy
(6) Portugal and Greece

We can see that the string (see figure 3) once again classifies countries from the most virtuous countries to the worst ones. Indeed, we observe that the curves by variable are more or less classified from the highest to
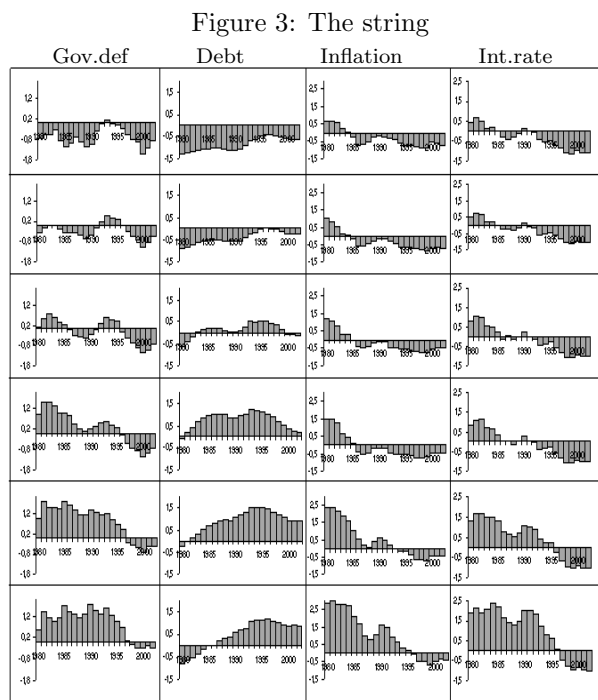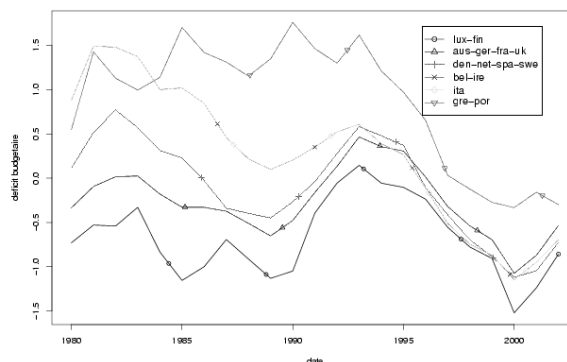
Figure 3: The string

The first row corresponds to the first unit of the string and the last row to the 6th unit.

Figure 4: Quantification of the budget deficit criterion

the lowest through the 6 units. To illustrate this order, figure 4 reports the representation of the trajectory of the different units for the budget deficit criterion. These trajectories are representative of the behaviors of the countries.

We observe that even the most virtuous countries have relatively larger government deficits during the recession of the 90's. We can see a significative convergence especially concerning inflation and interest rate criteria since the values are all below the average in the end of the period[6].

Since this method groups countries on the basis of their whole trajectories, one can not directly observe

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[6]Since the data are centered and standardised by variable, the negative values correspond to values below the average.

different transitions for two countries of the same unit (as for Portugal and Greece). We propose to compute the individual trajectories in the next section.

3.3 Countries trajectories

On the basis of the trajectories quantified above, we can compute the nearest centroid (or the nearest class) for each country at each year between 1980 and 2002. Doing so, we can distinguish the individual evolution of each country.

Then we use a hierarchical classification to group the closest centroids by year. The study of the previous section gives us the insight to use 3 classes for the period 1980-1994 and only 2 classes for the period 1995-2002. Table 1 gives the classification thus obtained, for each year. Clearly, some countries do not always belong to the same class. Results are quite similar to those found with the first method. The three classes in the 80's as well as the two classes at the end of the period are similar for both methods adopted. However, we obtain different classes during the recession period (1993-1997).

Table 1: Classification by years (1980-1995)

year	Class 1	Class 2	Class 3
1980	aus, fin, fra, ger lux, net	bel, den, spa swe, uk	gre, ire, ita por
1981	aus, fin, fra, ger lux, net, spa	bel, den swe, uk	gre, ire, ita por
1982	aus, fin, fra, ger lux, net, uk	bel, den, spa swe	gre, ire, ita por
1983	aus, fin, fra, ger lux, net, uk	bel, den, ire spa, swe,	gre, ita, por
1984	aus, fin, fra, ger lux, net	bel, den, ire spa, swe, uk	gre, ita, por
1985 1986	aus, den, fin, fra ger, lux, net, spa swe, uk	bel, ire,	gre, ita, por
1987	aus, den, fin, fra ger, lux, spa, swe uk	bel, ire, ita net	gre, por
1988 1989 1990	aus, den, fin, fra ger, lux, spa, swe uk	bel, ire, net	gre, ita, por
1991	fin, fra, ger, lux spa, swe, uk	aus, bel, den ire, net	gre, ita, por
1992 1993	aus, fin, fra, ger lux, spa, uk	bel, den, ire net, swe	gre, ita, por
1994	aus, fin, fra, ger lux, uk	bel, den, ire, net por, spa, swe	gre, ita

year	Class 1		Class 2
1995 1996 1997	aus, den, fin, fra, ger, ire lux, net, por, spa, swe, uk		bel, gre, ita
1998	aus, den, fin, fra, ger, ire lux, net, por, swe, uk		bel, gre, ita, spa
1999 2000 2001 2002	aus, den, fin, fra, ger, ire lux, net, por, spa, swe, uk		bel, gre, ita

We then compute the average value for each variable of each class year by year and define a theoretical tra-

jectory for a country and a variable by affecting the average value of the class to the country. Figures 5 and 6 show the theoretical trajectories of Portugal and Ireland. On the graph, we also report their actual trajectories (called "data" on the figures). This allows us to observe the quality of the quantification.

Once again, we find that Portugal and Ireland have clearly converged to the most virtuous class (class 1) since they begin in the worst class (class 3) in 1980, go through the medium class (class 2) and end up in the most virtuous class in 2002 (see table 1 and figures 5 and 6). We also observe that Ireland has converged more rapidly to Maastricht criteria than Portugal.

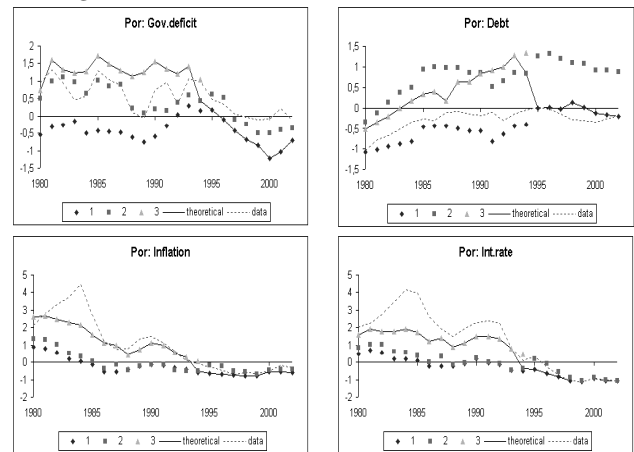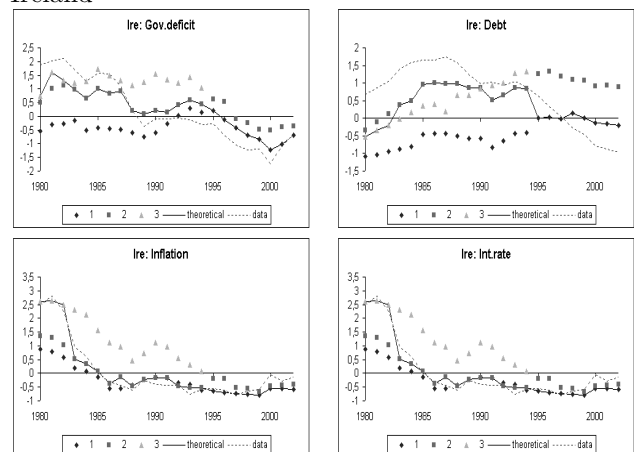Figure 5: The theoretical and actual trajectories of Portugal

Figure 6: The theoretical and actual trajectories of Ireland

4 Conclusion

We have developed two methods based on the Self-Organizing Map adapted to deal with cross–country time–series data and to define individual trajectories.

Our results show that the two methods give quite similar results but that the second one is more constrained since it groups countries over their trajectories. The results confirm the convergence of the European countries to Maastricht criteria since the number of classes has diminished since 1980. However the case of Belgium, Greece and Italy reveals some difficulties with the convergence criteria. One can note the good performances of Ireland and Portugal in the process of the European convergence. Our results indicate that United Kingdom, Denmark and Sweden are in the same class as the Euro area countries that respect the Maastricht criteria although they did not join the Euro area. This reveals that other criteria have to be integrated in order to understand their economic policy choices.

Although a certain convergence can be observed among the European countries on the basis of the Maastricht criteria, substantial difficulties remain as regards their real performances. Consider the large increase of the unemployment rate or the difficulties encountered during the 2001 crisis, clearly a nominal convergence is far from being enough. Coordination and stabilization policies need to be taken into account for a more effective policy mix.

5 Acknowledgements

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