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STATISTIQUE APPLIQUEE ET MODELISATION STOCHASTIQUE

Simulating interest rate structure evolution on a long term horizon
A Kohonen map application

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Soumis Pasadena
NNCM'96

Prépublication du SAMOS n°64
Juin 1996

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Introduction

In April 1993, the "Basle Committee on Banking Supervision" has published an article on the methods for computing the level of capital needed by financial institutions to face the interest rate risks. In April 1995, a new publication of the Basle Committee allowed banks for using internal models of market risk measurement to determine their capital adequacy. In this context, risk measurement models based on the construction of scenarii compatible with historical data becomes a crucial issue.

These recommandations have encouraged the banks to develop a conceptual framework and software designed to estimate and quantify the market risks (the Value at Risk approach). Today, the model which tends to become the benchmark and which has initiated a lot of debates is the RiskMetrics model of JP Morgan. The conceptual framework developped in RiskMetrics assumes that historical returns are normally distributed. The scenarii are on this basis generated by a structured Monte-Carlo simulation. This procedure is adapted to short term horizon but fails when applied on the long term. The generated scenarii tend to be explosive because the resulting dynamic of the interest rate structure does not have mean reverting properties. The generation of those long term scenarii, while crucial for a bank in the management of assets and liabilities (ALM)[1], remains therefore a hard to solve problem.

In this paper, we propose an alternative approach, also based on a Monte-Carlo simulation procedure, but using Kohonen map classification to construct conditional probability distributions of interest rate structure shocks. Using them, we are able to produce interest rate structure scenarii which not only are stable even over a five years horizon but also exhibit properties compatible (share common statistical features) with the historical interest rate structure evolution used to compute the conditional probability distributions.

In this summary, we will assume that the Kohonen algorithm is well-known. An extensive presentation can be found in [Kohonen 1982, 1989, 1995]. A study of its theoretical properties can be found, among others, in [Cottrell & Fort, 1987, Cottrell, Fort & Pagès, 1994]. Uses of Kohonen map as data analysis tools are proposed, for example, in [Cottrell, Letremy & Roy, 1993 or Cottrell, Ibbou, 1995].

To classify the observed shocks on the interest rate structure, we used data of the US bonds market. Our data are daily interest rate structures for maturity from 1 to 15 years. The interest rate for

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[1] For example, today, several assets have CAP or FLOOR. A CAP(FLOOR) is a clause that assure that the interest rate will not be higher (lower) than a certain level. These clauses affect the profit over a long period and this illustrate why one need to model the interest rate structure evolution over a long time horizon.

each maturity has been calculated by JP Morgan from the prices of US T-Bills and T-Bonds. The sample covers the period from 5/1/1987 to 10/5/1995, altogether 2088 entries. Using these data, we calculate the shocks which are the differences between the observed term structure at time t (given that we only have 15 rates corresponding to maturity ranging from 1 year to 15 years) and time t-10 over working days (time delay recommended by the Basle Committee on Banking Supervision).

1. Previous works

In [Cottrell, de Bodt, Grégoire, Henrion, 1996a,b], we get three conclusions :

- the Kohonen algorithm produces more powerfull classifications on our data that the classical hierarchical classification algorithm (extensive comparison has been realized using multidimensional extensions of the Fisher Test such as the Wilk's Lambda, Pillai's Trace, Hotelling-Lawley Trace and Roy's Greatest Root).
- the factors that give the best explanation of the produced classification (the mean profiles of the 9 units of the used one-dimensional Kohonen map are presented at fig. 1) are the level of the short rate and the spread (this result has been obtained by a canonical correlation analysis). This result confirms the one published by Litterman and Scheinkman (1988).
- there is a probabilistic relation between the shape of the interest rate structure and the kind of shock that follows. This relation explains that the approaches based on the normality assumption are explosive over a long term horizon and that interest rate evolutions seem to be bounded and mean reverting. Tab. 1 shows the distribution of frequences of the observed shocks (after a classification along a 30 units one-dimensionnal Kohonen map) for each of the nine classes of interest rate structures. Tab. 2 presents the Chi2 test that clearly confirms the difference between each of the nine distributions and the distribution of the global data set.

2. Simulating Interest Rate Structure Evolution on a Long Term Horizon

Using those empirical conditional distributions of frequence, we propose a Monte-Carlo procedure to simulate the interest rate structure evolution. The procedure is the following :

- first, we randomly draw an initial interest rate structure.
- the winning unit of the Kohonen map associated to this interest rate structure is then determined.
- using the conditional distribution of frequences of the interest rate shocks, we randomly draw a shock.
- we then apply the shock to the interest rate structure.
- the procedure is repeated 125 times to construct an interest rate structure evolution on a five years horizon (125 times the 10 days covered by the interest rate shock).
- for each simulation, we then repeat 1000 times the procedure to build the distribution of probability of interest rate structures, starting from the same initial interest structure.

Fig. 2 and 3 respectively show the distribution of the short rate and the long rate for three simulations. The two first have been realized using the same interest rate initial shape (for which unit 6 is the winning one). The third one has been done using an initial interest rate structure attached to unit 1 (the only inverted interest rate structure mean profile). We see on those figures that the procedure is stable and that, on a five year basis, the initial interest rate structure mainly influences the short rate level. We also see that, for all simulations, the level of the short rate and the long rate are compatible with the historical one. Fig. 4 presents five interest rate structures obtained in simulation 1, drawn among the 1000 produced. We see that they are well-shaped. This property has been verified on all the results. Fig. 5 presents, always in the case of simulation 1, one trajectory of the short and the long rate over 5 years. They clearly represent possible paths.

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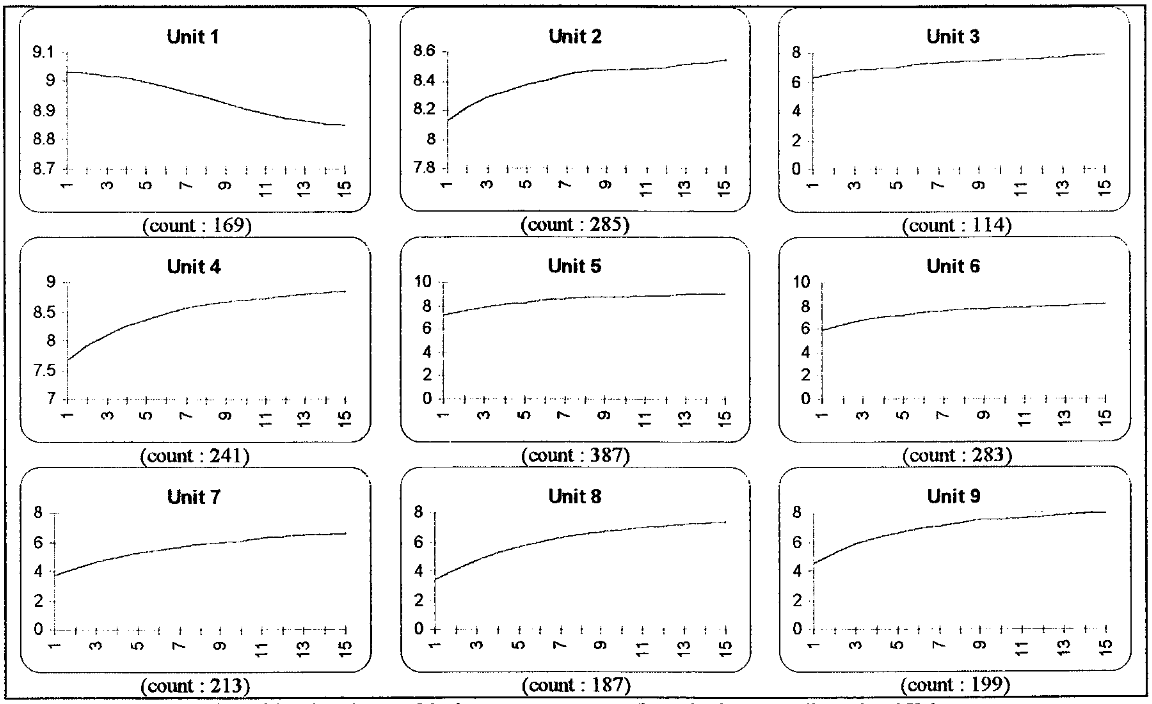


Mean profiles of the nine classes of the interest rate structure formed using a one-dimensional Kohonen map

fig. 1

	Classes of IIRS									
Classes of IRS	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Population*
1	0.00%	0.00%	0.00%	0.00%	1.81%	0.00%	0.00%	0.00%	0.00%	0.34%
2	2.37%	0.00%	0.00%	0.41%	2.33%	0.00%	0.00%	0.00%	0.00%	0.67%
3	7.10%	0.00%	0.00%	0.83%	0.26%	0.71%	0.00%	1.07%	2.01%	1.11%
4	1.18%	0.00%	0.88%	4.15%	2.07%	0.00%	1.88%	2.14%	2.01%	1.59%
5	1.78%	2.46%	1.75%	2.49%	3.10%	0.35%	0.00%	3.74%	0.00%	1.83%
6	5.33%	2.11%	1.75%	3.32%	3.36%	3.53%	0.47%	3.21%	4.02%	3.03%
7	1.18%	0.00%	0.00%	0.83%	2.33%	0.00%	1.88%	2.67%	0.00%	1.06%
8	7.10%	0.00%	0.00%	0.41%	0.00%	1.41%	0.00%	0.00%	6.03%	1.40%
9	7.69%	9.47%	7.89%	5.81%	3.10%	7.07%	8.45%	2.14%	6.03%	6.21%
10	6.51%	8.77%	13.16%	8.71%	8.53%	6.36%	7.04%	5.35%	2.51%	7.36%
11	1.18%	7.02%	7.89%	5.39%	6.72%	8.13%	4.69%	5.35%	2.01%	5.63%
12	6.51%	3.16%	3.51%	8.71%	4.65%	1.77%	5.16%	6.95%	6.03%	5.00%
13	3.55%	5.61%	1.75%	7.47%	2.58%	4.59%	0.47%	5.35%	5.53%	4.19%
14	1.78%	3.16%	0.00%	3.32%	2.84%	0.35%	1.41%	7.49%	1.51%	2.50%
15	5.33%	2.46%	3.51%	2.07%	0.78%	3.53%	0.47%	5.35%	4.52%	2.79%
16	0.00%	0.35%	0.00%	0.00%	2.33%	0.35%	0.00%	1.07%	0.00%	0.63%
17	2.96%	5.26%	4.39%	3.32%	6.72%	9.19%	7.98%	5.88%	3.02%	5.73%
18	3.55%	5.61%	10.53%	6.64%	3.36%	8.13%	15.49%	4.28%	4.52%	6.54%
19	7.69%	4.91%	6.14%	4.98%	2.07%	4.95%	4.23%	4.28%	6.03%	4.67%
20	4.73%	3.86%	5.26%	7.05%	2.84%	8.13%	4.69%	5.88%	9.55%	5.58%
21	1.18%	1.75%	0.88%	3.73%	4.91%	2.12%	1.88%	1.60%	1.51%	2.50%
22	5.33%	4.21%	0.88%	0.41%	1.55%	3.89%	1.41%	5.35%	6.53%	3.18%
23	4.73%	6.67%	3.51%	4.15%	6.46%	7.77%	8.45%	4.81%	7.54%	6.26%
24	1.18%	3.86%	1.75%	2.49%	8.01%	2.47%	4.23%	5.35%	5.03%	4.23%
25	4.14%	5.61%	7.89%	6.64%	3.10%	8.48%	4.69%	3.21%	7.54%	5.53%
26	1.78%	3.86%	0.00%	0.41%	2.58%	2.47%	3.76%	3.74%	4.02%	2.65%
27	2.96%	3.51%	4.39%	2.90%	4.91%	2.47%	4.23%	1.60%	0.00%	3.13%
28	1.18%	4.56%	3.51%	3.32%	2.58%	0.35%	5.16%	0.00%	2.01%	2.55%
29	0.00%	1.75%	2.63%	0.00%	3.10%	1.06%	1.41%	1.60%	0.50%	1.44%
30	0.00%	0.00%	6.14%	0.00%	1.03%	0.35%	0.47%	0.53%	0.00%	0.67%
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

IIRS : Initial Interest Rate Structure

IRS : Interest Rate Shock

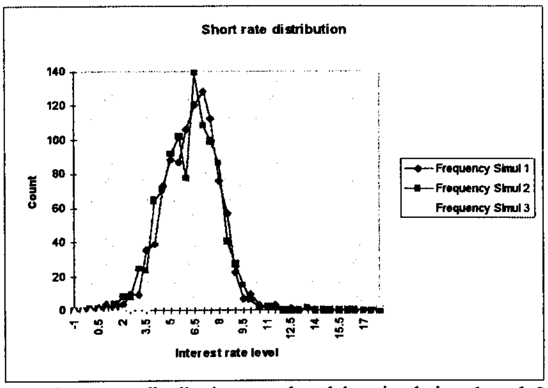
Conditional distribution of IRS for each of the nine IIRS classes.

Tab. 1

	Chi2
Unit1	140.00
Unit2	42.04
Unit3	85.48
Unit4	54.58
Unit5	151.07
Unit6	244.39
Unit7	55.38
Unit8	73.37
Unit9	59.87

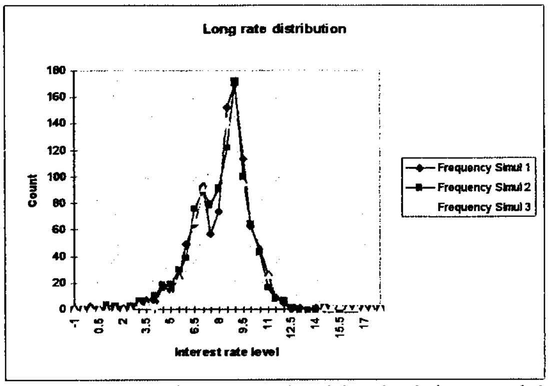
The Chi^2, applied to probability distributions of tab. 1, clearly confirms the difference between each of the nine distributions and the distribution of the global data set.

Tab. 2



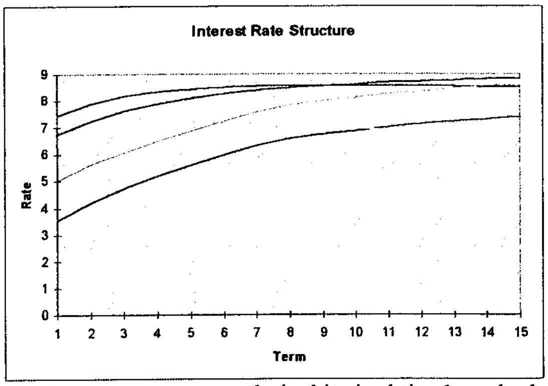
The short rate distributions produced by simulation 1 and 2 (starting from the same initial interest rate structure) highlight the stability of simulation procedure. The comparison the of the short rate distributions produced but simulation 1 and 2 to the one produced by simulation 3 shows the impact of the initial interest rate structure on the short rate evolution over a 5 years time horizon.

Fig. 2



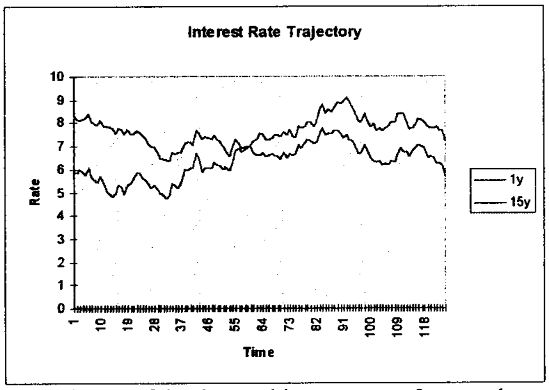
The long rate distributions produced by simulation 1 and 2 (starting from the same initial interest rate structure) highlight the stability of simulation procedure. The comparison the of the long rate distributions produced but simulation 1 and 2 to the one produced by simulation 3 shows that the initial interest rate structure seems to have no impact on the long rate evolution over a 5 years time horizon.

Fig. 3



Five interest rate structures obtained in simulation 1, randomly drawn among the 1000 produced.

Fig. 4



One trajectory of the short and long rate over 5 years, chosen among the 1000 produced by simulation 1.

Fig. 5

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