SIMULATING INTEREST RATE STRUCTURE EVOLUTION ON A LONG TERM HORIZON

A KOHONEN MAP APPLICATION

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In the 90s, a new approach to model risk was proposed and named Value at Risk (VaR). Pioneered by JP Morgan (RiskMetrics), VaR has initiated a lot of debate. One of the major shortcoming of RiskMetrics is its inability to generate scenarios on a long term horizon (the Monte-Carlo procedure proposed does not produce the classical mean-reverting properties of interest rate structure dynamics). In this paper, we propose an alternative approach, also based on a Monte-Carlo simulation procedure, but using a Kohonen map quantization to construct conditional probability distributions of interest rate structure shocks. The procedure is not only able to produce interest rate structure scenarios which are stable on a long term horizon (five years) but also these scenarios exhibit properties compatible with the historical interest rate structure evolution used to compute the conditional probability distributions.

1 Introduction

Financial institutions run different kinds of risks. The credit risk corresponds to the risk of failure of a counterpart. The market risk results from unanticipated movements of prices, interest rates or foreign exchange rates. The liquidity risk is associated with the impossibility of closing a position within a short delay. With derivatives, the liquidity risk also consists in the inability to face the margin call. The operational risk is the consequence of human or system errors. Finally, the legal risk occurs when the contract is not enforceable. In this paper, we limit ourselves to the market risk and more precisely, the interest rate risk. Management of interest rate risk is one of the key tasks of bank and insurance financial supervision. The $500 billions cost of the Savings and Loan (S&L) mess in the 80s shows what mismanagement of interest rate risk can do to an industry and its deposit-insurance.

The latest advancement of Wall Street in risk measurement is the so called Value at Risk (VaR). VaR is an estimate of the maximum loss to be expected over a given period a certain percentage of the time (Beder 1995). The calculation of VaR depends on assumptions and methodology. There is two common methodologies, historical simulations and Monte-Carlo simulations over a single short term horizon. In our paper, we present a methodology to estimate VaR over a long term horizon. We propose an alternative approach, also based on a Monte-Carlo simulation procedure, but using Kohonen map classification to construct conditional probability distributions of interest rate structure shocks. Using them, we are able to produce interest rate structure scenarios which are not only stable even over a five year horizon but also exhibit properties compatible (sharing common statistical features) with the historical interest rate structure evolution used to compute the conditional probability distributions.

The first section of the paper is devoted to a short presentation of Value at Risk (VaR) methodology. There we will specify the precise position of this work. The second section presents the Kohonen algorithm. Some useful references on this algorithm and its theoretical properties will be given. The data set is presented in the third section. The fourth section presents the proposed approach in three steps : the previous works, the relation between initial interest rate structure and interest rate shocks and the simulation procedure. In the last section of the paper, we will indicate the direction some future work might take.

2 Value-at-Risk methodology

The classical approach to model risk is to assume that the asset's returns are normally distributed or that the prices are lognormal. With this assumption, the classical tradeoff between risk and return is represented in a two dimensional space, mean (return) and variance (risk). With the assumption that the returns have normal multivariate distribution, the risk of a portfolio only depends on the matrix variance-covariance of the returns.

Value-at-Risk (VaR) is an estimate of the maximum potential loss to be expected over a given period a certain percentage of the time. Risk is model on the maximum potential loss which can be estimated by historical or Monte-Carlo simulations. Historical simulations give the maximum loss over a period of time. The limits of the historical approach are that the results depend on a past period while the Monte-Carlo simulations depend on the matrix variance-covariance. A recent study (Beder 1995) shows that the magnitude of discrepancy among these methods deviates by more than 14 times for the same portfolio. Moreover, the VaR results that are calculated by an historical simulation depend on the behavior of the return during the period. For example, during a period of decreasing interest rates, risk may be underestimated. Historical simulations face all the problems encountered by the non experimental science while Monte-Carlo simulations are well adapted for very short periods of time (from 1 day to 1 month). For example, the conceptual framework developped in RiskMetrics (JP Morgan) assumes that historical returns are generated from a multivariate normal distribution. From this assumption, future price paths can be simulated by the Cholesky decomposition. This procedure is adapted to short term horizon but fails when applied to the long term. The generated scenarios tend to be explosive because the resulting dynamics of the interest rate structure do not have mean reverting properties. The generation of those long term scenarios, while crucial for a bank in the management of assets and liabilities (ALM)[1], still remains a problem hard to solve. In this paper, we propose a methodology which tries to solve this particular problem.

In the first step, we will propose the use of the Kohonen algorithm

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[1] For example, today, several assets have CAP or FLOOR. A CAP(FLOOR) is a clause assuring that the interest rate will not be higher (lower) than a certain level. These clauses affect the profit over a long period and this illustrate why one needs to model the interest rate structure evolution over a long term horizon.

to overcome the limitations due to the use of the variance-covariance matrix. Using it, we will show that it is possible to empirically estimate the relation between the interest rate structure and the distribution of movements it can undergo.

In the second step, the Monte-Carlo technique will be used to generate a large number of future interst rate paths on a long term horizon.

3 The Kohonen algorithm

The Kohonen algorithm (Kohonen, 1995; Cottrell, Fort, 1987; Cottrell, Fort, Pagès, 1994) is a well-known unsupervised learning algorithm which produces a map composed by a fixed number of units. A physical neighborhood relation between the units is defined and for each unit i, $Vr_{(i)}$ represents the neighborhood with radius r centered at i.

Each unit is characterized by a parameter vector W_i of the same dimension as the input space.

After learning, each unit represents a group of individuals with similar features (this group is named Voronoi region of the unit). The correspondence between the features and the units (more or less) respects the input space topology : similar features correspond to the same unit or to neighboring units. The final map is said to be self organized map which preserves the topology of the input space. The learning algorithm takes the following form :

- at time 0, $W_{i(0)}$ is randomly defined for each unit i;
- at time t, we present a vector $x_{(t)}$ randomly chosen among the rows of the data matrix and we determine the winning unit i*, which minimizes the Euclidean distance between $x_{(t)}$ and $W_{i(t)}$;
- we then modify the W_i in order to move the weights of the winning unit i* and its physical neighbors towards $x_{(t)}$, using the following relations :

$$W_{i(t+1)} = W_{i(t)} + \left[e_{(t)} \times \left[x_{(t)} - W_{i(t)} \right] \right] \text{ for } \; i \in Vr_{(t)}(i^*) \tag{1}$$

$$W_{i(t+1)} = W_{i(t)} \tag{2}$$

where $\varepsilon_{(t)}$ is a small positive adaptation parameter, $r_{(t)}$ is the radius of $Vr_{(t)}$ and $\varepsilon_{(t)}$ and $r_{(t)}$ are progressively decreased during the learning.

The results have been obtained after 100 learning cycles (at each one, we present all the individuals composing the data). The input space dimension is 15. We have used a one-dimension map.

While the asymptotic properties of this algorithm remain partly unknown, some of its theoretical properties have been demonstrated during the last 10 years. One of them is of particular interest as regards this paper and concerns the density approximation property of this algorithm. In (Pagès, 1993), the Kohonen algorithm terminating with a 0 neighbor at the end of learning is studied. According to the comment at the end of section 3, the convergence of this algorithm is thus equivalent to the convergence of a classical VQ technique as "competitive learning". The author shows that the units after VQ are a good discrete skeleton for reconstructing the initial density f(x), provided that each unit is weighted by the probability (estimated by the frequency) of its Voronoi region. In other terms, if y_1, $y_2,\ldots,y_n$ are the units after learning, and C_1, $C_2,\ldots,C_n$ the corresponding Voronoi regions, the following convergence (in law) is guaranteed:

$$\sum_{i=1}^{n} P(C_i) d_{y_i} \xrightarrow{\text{law}} P \tag{3}$$

when n goes on to infinity, and d_{y_i} is a Dirac function on y_i. This is equivalent to saying that the empirical measurement defined by units y_1, $y_2,\ldots,y_n$, weighted by the probabilities of the associated Voronoi regions, converges (in law) on the initial probability P.

Provided that units are adequately weighted, this result shows that it is possible to reconstruct the initial law, and the result is exact when the number of units goes on to infinity. Pagès also showed that the speed of convergence is better than with data obtained by independent random drawings.

This remarkable theoretical property justifies the choice of the Kohonen algorithm to quantify the distributions of interest rate structures and interest rate shocks (see section 5) and largely explains the compatibility of the generated scenarios with the historical data set used.

4 The data set

To classify the observed shocks on the interest rate structure, we used data from the US bonds market. Our data are daily interest rate structures for maturity from 1 to 15 years. The interest rate for each maturity has been calculated by JP Morgan from the prices of US T-Bills and T-Bonds. The sample covers the period from 1/5/1987 to 5/10/1995, altogether 2088 entries. Using these data, we calculate the shocks which are the differences between the observed term structure at time t (given that we have only 15 rates corresponding to maturity ranging from 1 year to 15 years) and time t–10 working days (time delay recommended by the Basle Committee on Banking Supervision). Figure 1 shows the evolution the short-rate (1-year) et the long-rate (15-year) during the period 1987-1995. Using several other data sets, results presented in this paper have been confirmed.

Fig. 1: Short-rate and long-rate evolution during the period 1987-1995 on the US market.

5 Using Monte-Carlo on a Kohonen Map

5.1 Previous works

The approach proposed in this paper is based on several previous works. The first work (Cottrell, de Bodt, Henrion, Grégoire, 1996a) was oriented towards the classification of the observed shocks on the interest rate structure. We used the Kohonen Algorithm and compared its performances to the classical hierarchical clustering approach (HCA). To compare the two algorithms, we used several statistics. First, for each dimension (maturity), we compare the ratio of the intra-class sum of squares to the total sum of squares and the classical Fisher statistic. Then, to compare the clustering power of each approach with all sets of maturities, we have used four multidimensional extensions of the Fisher test : Wilk's Lambda, Pillai's Trace, Hotelling-Lawley Trace and Roy's Greatest Root. These preliminary results clearly show the power of the Kohonen algorithm as compared to HCA.

On the basis of these results, in (Cottrell, de Bodt, Henrion, Grégoire, 1996b), we analyzed the information used by the Kohonen algorithm to form clusters of interest rate structure shocks. Using a canonical correlation analysis, we show that the level, spread and curvature are the three variables most correlated with the unsupervised classification produced.

It also seems important to mention that the structural stability of the data set have been tested and that, in all our analysis, we choose the number of units of the Kohonen map by studying the decrease of the intra-class sum of squares on the total sum of squares ratio as the number of units increase.

5.2 The relation between initial interest rate structure and interest rate shocks

A large number of term structure models are based on specific assumptions about the stochastic process determining the state variables, especially the instantaneous interest rate. Various empirical studies (Chan et al. 1992 ; Dalquist 1996) conclude that a class of models seems to outperform other models. This class of models takes into account the mean-reverting feature of the interest rates and the fact that the variance in interest rate changes appears to be dependent on the interest rate level. The method we have proposed in this article does not impose a priori an analytical form of the process governing the instantaneous interest rate. If there is a mean-reverting feature or a link between the level of the interest rates and the variance, our approach will take it into account. In fact, as we characterized the relation between the class of shocks and the initial structure, we take into account the magnitude of the shocks (variance) in respect to the initial structure (conditional variance) and the lower (higher) probability of an upper (downer) shock when the level of the initial structure is high (mean-reverting). Moreover, as we do not impose the processes governing the state variables, we hope to reproduce the statistical properties of the historical evolution of the interest rate structure.

To study the empirical relation between the initial interest rate structure and the interest rate shocks, we proceed in three steps. First, a clustering of the initial interest rate structure has been realized. A 9 unit one-dimensional Kohonen map has been used. Then, a clustering of the shocks on the interest rate structure has been done, in this case using a 30 unit one-dimensional Kohonen map. While it is behind the scope of this paper to present in full detail those results, it is interesting to mention the Fisher statistics obtained, for each maturity, on the clustering of shocks (Table 1). These statistics confirm the highly discriminate cluster formed by the Kohonen algorithm.

Dimension	Fisher		Dimension	Fisher
1	898		8	2840
2	1579		9	2803
3	2099		10	2840
4	2472		11	2544
5	2853		12	2221
6	3161		13	1894
7	3038		14	1574
			15	1279

Tab. 1 : Fisher statistics for each maturity obtained after clustering by a 30 unit one-dimensional Kohonen map.

Analysis of the relation between the initial interest rate structure and the shocks that apply to them has been conducted on this basis. To understand this procedure, we have to remember that each shock to the interest rate structure is related by its date to a specific initial interest rate structure. In other words, a specific deformation of the interest rate structure is obtained by the difference between the state of the interest rate structure at a specific date and the state of the interest rate structure 10 days later. It is therefore possible to identify the shock subset associated with each of the 9 interest rate structure classes and to calculate the conditional frequency distributions. We then test the statistical independence between the 9 empirical conditional distributions of shocks and the global population of shocks using a χ^2 test. Results are shown in Table 2. All 9 tests are statistically significant.

In other words, this is equivalent to saying that the conditional distributions of shocks are statically different from the distribution of the global population of shocks, with a very high level of confidence. The empirical relation between shocks and interest rate structure is confirmed.

Initial interest rate classes	*Independence test*
1	140
2	42.04
3	85.48
4	54.58
5	151.07
6	244.39
7	55.38
8	73.37
9	59.87

Tab. 2 : χ^2 tests between each of the nine conditional distributions of shocks and the global population of shocks.

5.3 Simulating interest rate structure evolution's on a long term horizon

Using these empirical conditional distributions of frequencies, we propose a Monte-Carlo procedure to simulate the interest rate structure evolution. The procedure is the following :

- first, we randomly draw an initial interest rate structure.
- the winning unit of the Kohonen map associated with the interest rate structure is then determined.
- using the conditional distribution of frequencies of the interest rate shocks, we randomly draw a shock.
- we then apply the shock to the interest rate structure.
- the procedure is repeated 125 times to construct an interest rate structure evolution on a five year horizon (125 times the 10 days covered by the interest rate shock).
- for each simulation, we then repeat the procedure 1000 times to build the distribution of probability of interest rate structures, starting from the same initial interest structure.

Figure 3 and 4 respectively show the distribution of the short-rate and the long-rate for three simulations. The first two have been realized using the same interest rate initial shape (for which unit 6 is the winning one). The third one has been done using an initial interest rate structure attached to unit 1 (the only inverted interest rate structure mean profile). Based on these figures we see that the procedure is stable and that, on a five year basis, the initial interest rate structure mainly influences the short rate level. We also see that, for all simulations, the level of the short-rate and the long-rate are compatible with the historical one. Figure 5 presents five interest rate structures obtained in simulation 1, drawn from among the 1000 produced. We see that they are well-shaped. This property has been verified in all the results. Figure 6 presents, still in the case of simulation 1, one trajectory of the short and the long rate over 5 years. They clearly represent possible paths. While not presented here, we should also mention that in all simulations and at all steps all forward interest rate are positive.

6 Future works

In this paper, we have described a method for generating different scenarios of the evolution of the interest rate structure. The scenarios reproduce the characteristics of the historical data and seem to be stable over a long term horizon. To measure the interest rate risk of financial institutions by the VaR numbers, we need to calculate the prices, at some future date, of the securities whose payoff are dependent on the interest rate process. The standard techniques for valuation of these securities are, implicitly or explicitly, application of a risk-neutral valuation principle. The compensation for risk is made explicitly by adjusting the discount factors or implicitly when we impose the martingale property on the simulated sample paths of the underlying security price. Our future work will focus on the valuation of the contingent claims.

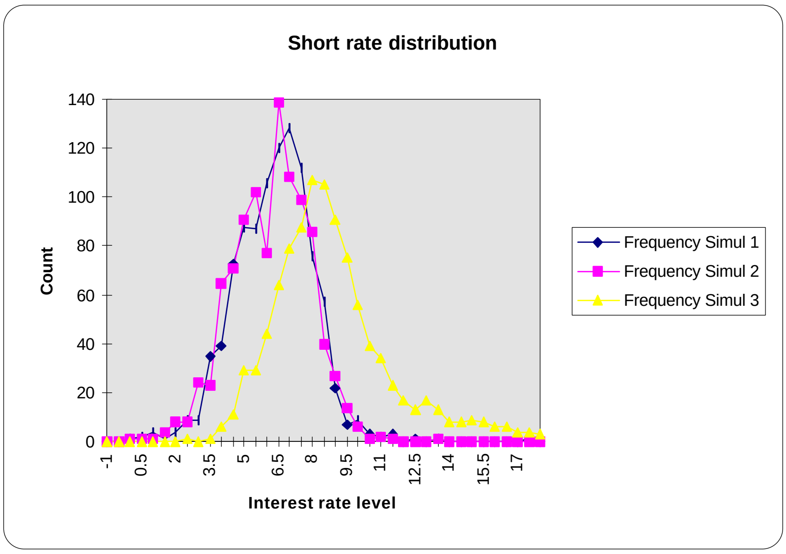

Fig. 3 : The short rate distributions produced by simulation 1 and 2 (starting from the same initial interest rate structure) highlight the stability of simulation procedure.

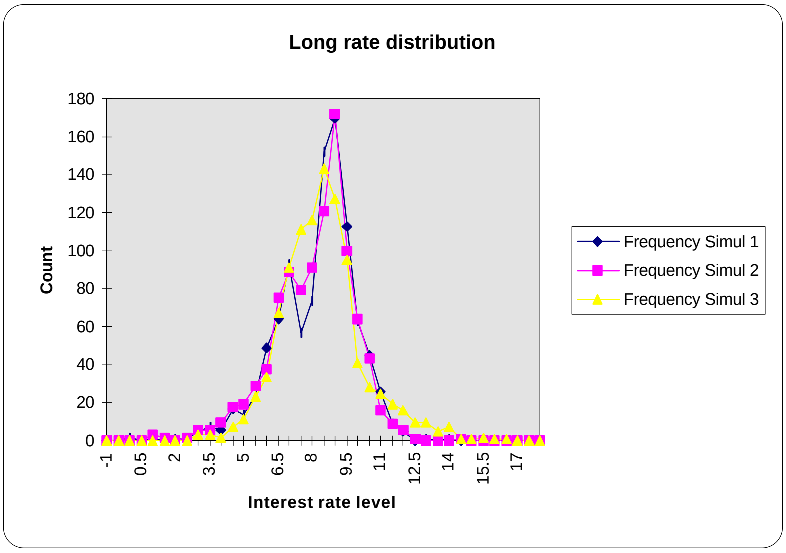

Fig. 4 : The long-rate distributions produced by simulation 1 and 2 (starting from the same initial interest rate structure) highlight the stability of simulation procedure.

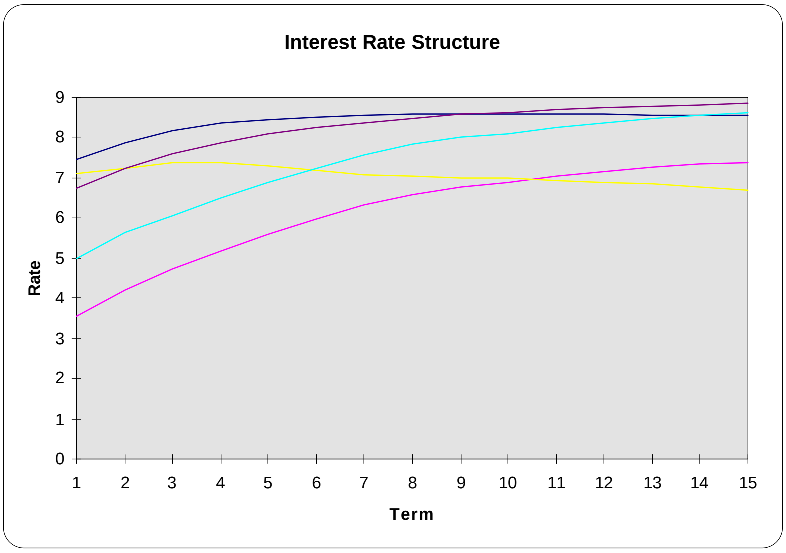

Fig. 5 : Five interest rate structures obtained in simulation 1, randomly drawn among the 1000 produced.

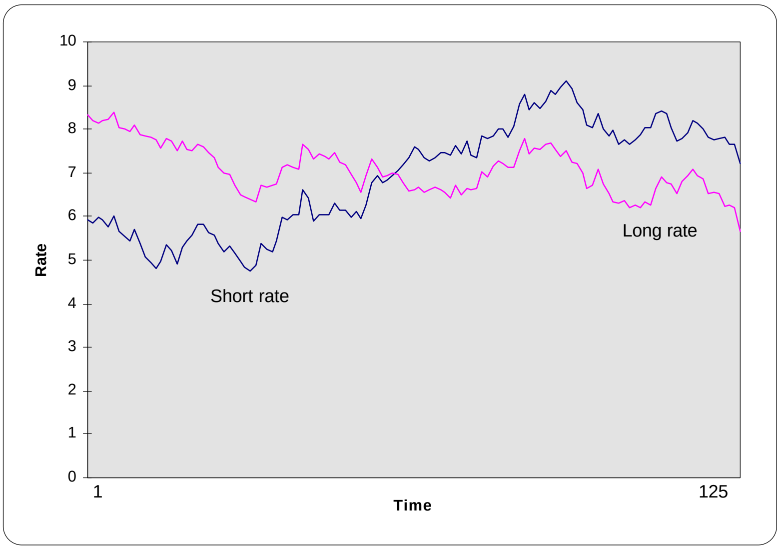

Fig. 6 : One trajectory of the short and long rate over 5 years, chosen among the 1000 produced by simulation 1.

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