Université Paris I, Panthéon - Sorbonne

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Time Series Tutorial n^0 3 : Correlogram and parameter estimation of ARMA and GARCH processes

The aims of this tutorial is a to exhibit the asymptotic behaviors of correlograms of ARMA and GARCH processes as well as providing first overview of the parametric estimation for these processes.

Correlograms

X=arima.sim(100,model=list(ar=-.3,ma=.7))	Generate a Gaussian ARMA process.
R=acf(X)	Correlogram of X
R[1]	What this? Why this value could be expected?

We are going to replicate this computation of the sample correlation for lag 1 many times for observing the asymptotic behavior of this estimator: this is the aim of the following Monte-Carlo experiment:

```
Res=c()
for (j in c(1:200))
{X=arima.sim(1000,model=list(ar=-.3,ma=.7))
R=acf(X)
Res=c(Res,as.numeric(R[1]$acf))}
hist(Res)
```

Explain what is done with this program and connect the result with theoretical results. Replace the law of the innovation by a uniform law on $[-1, 1]$. Conclusion?

Do the same Monte-Carlo experiments for a GARCH[1,1] process. Conclusion?

Estimation of the parameters of an AR[1] process

In the sequel, we define and use an estimator of the parameters of a AR[1] process. If $X_{t+1} = \theta\, X_t + \xi_t$ for $t \in \mathbf{Z}$, with $|\theta| < 1$, then $\theta = \text{cov}(X_0, X_1)/\text{var}(X_0)$. Hence, an estimator of θ could be obtained:

$$\widehat{\theta} = \frac{\frac{1}{n}\sum_{i=1}^{n-1} X_i X_{i+1}}{\frac{1}{n}\sum_{i=1}^{n} X_i^2}.$$

Use the previous program for exhibiting the asymptotic behavior of $\widehat{\theta}$. Consider the cases $n = 100$, $n = 500$ and $n = 1000$. How to check the $\sqrt{n}$ convergence rate of this estimator?

For estimating the variance σ_ξ^2 of ξ_t, a natural estimator is:

$$\widehat{\sigma}_\xi^2 = \frac{1}{n}\sum_{i=1}^{n-1}\big(X_{i+1} - \widehat{\theta}\, X_i\big)^2.$$

Exhibit also the asymptotic behavior of this estimator. Which convergence rate could be expected?

Write the conditional log-likelihood of (X_t) and deduce the previous estimators.